UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

BOULDER CANYON PROJECT
FINAL REPORTS

PART V—TECHNICAL INVESTIGATIONS

Bulletin 1

TRIAL LOAD METHOD OF ANALYZING ARCH DAMS

UNITED STATES
DEPARTMENT OF THE INTERIOR
HAROLD L. ICKES, Secretary

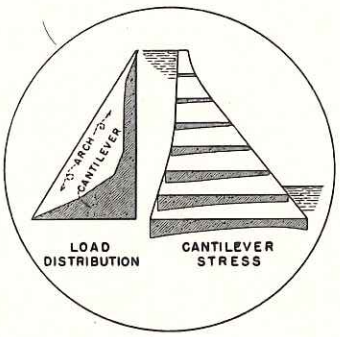
BUREAU OF RECLAMATION
JOHN C. PAGE, Commissioner
R. F. WALTER, Chief Engineer

BOULDER CANYON PROJECT
FINAL REPORTS

PART V—TECHNICAL INVESTIGATIONS

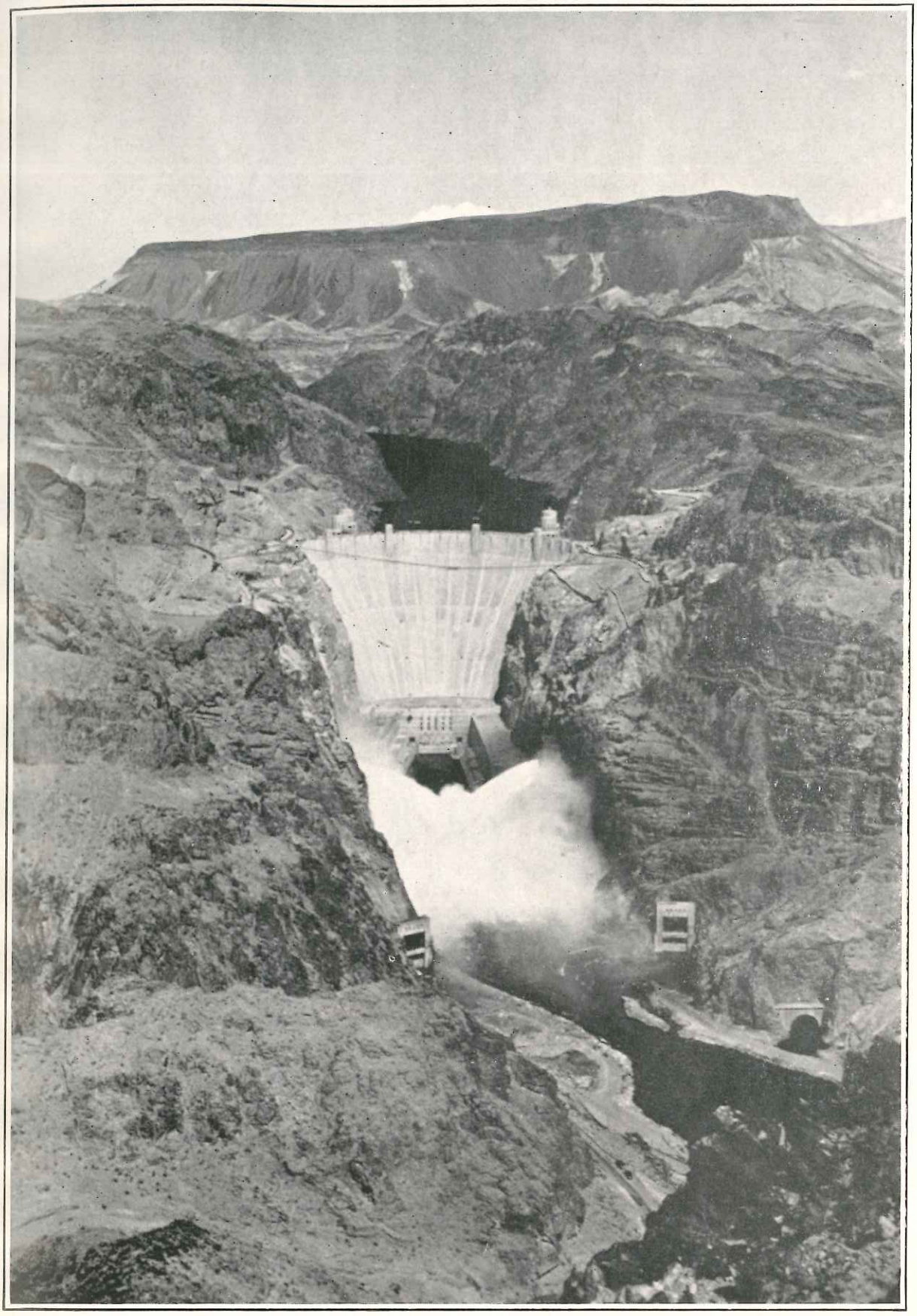
Bulletin 1

TRIAL LOAD METHOD OF ANALYZING ARCH DAMS

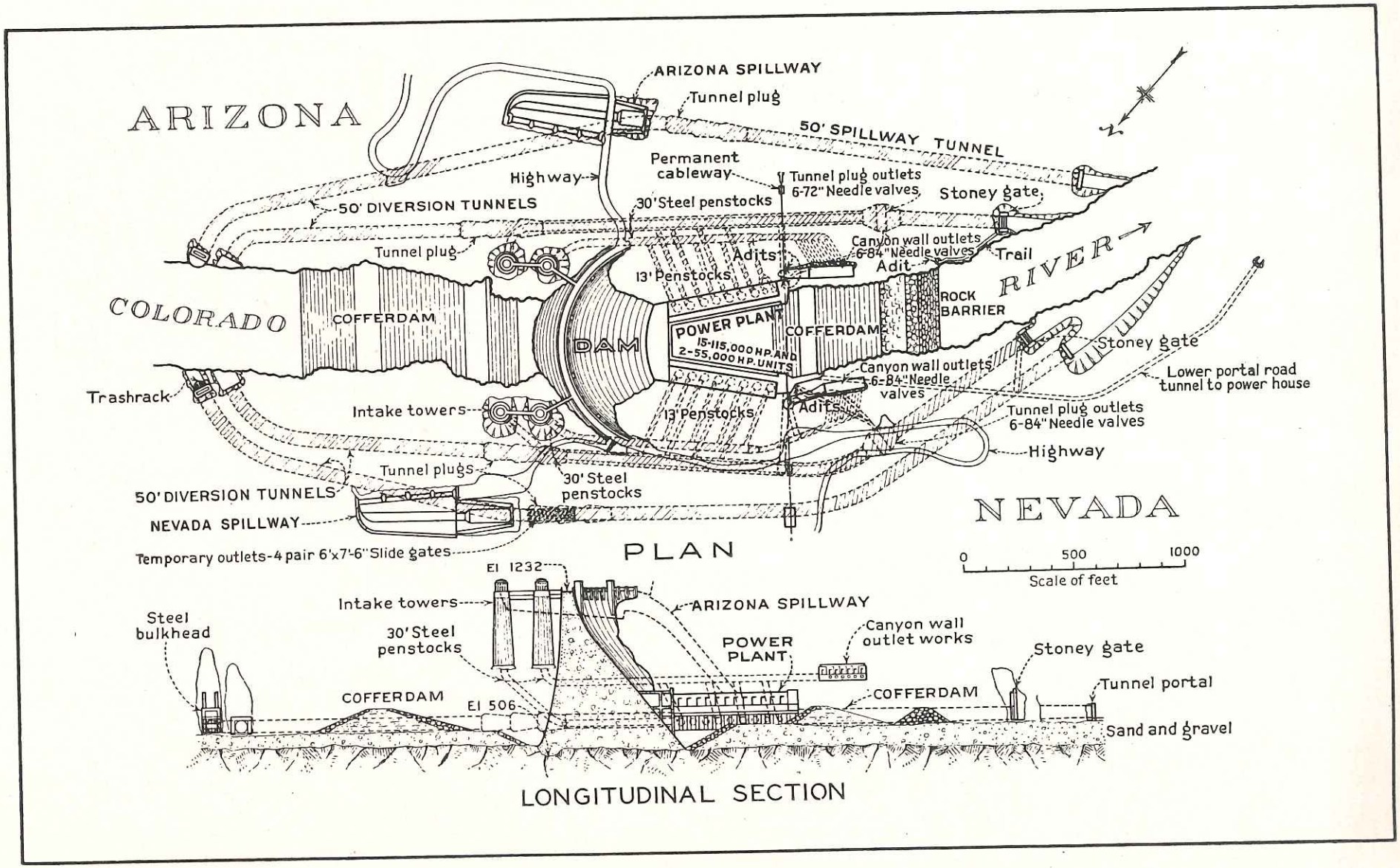


DENVER, COLORADO
1938

This bulletin is one of a series prepared to record the history of the Boulder Canyon Project, the results of technical studies and experimental investigations, and the more unusual features of design and construction. A list of the bulletins available and tentatively proposed for publication is given at the back of this report.

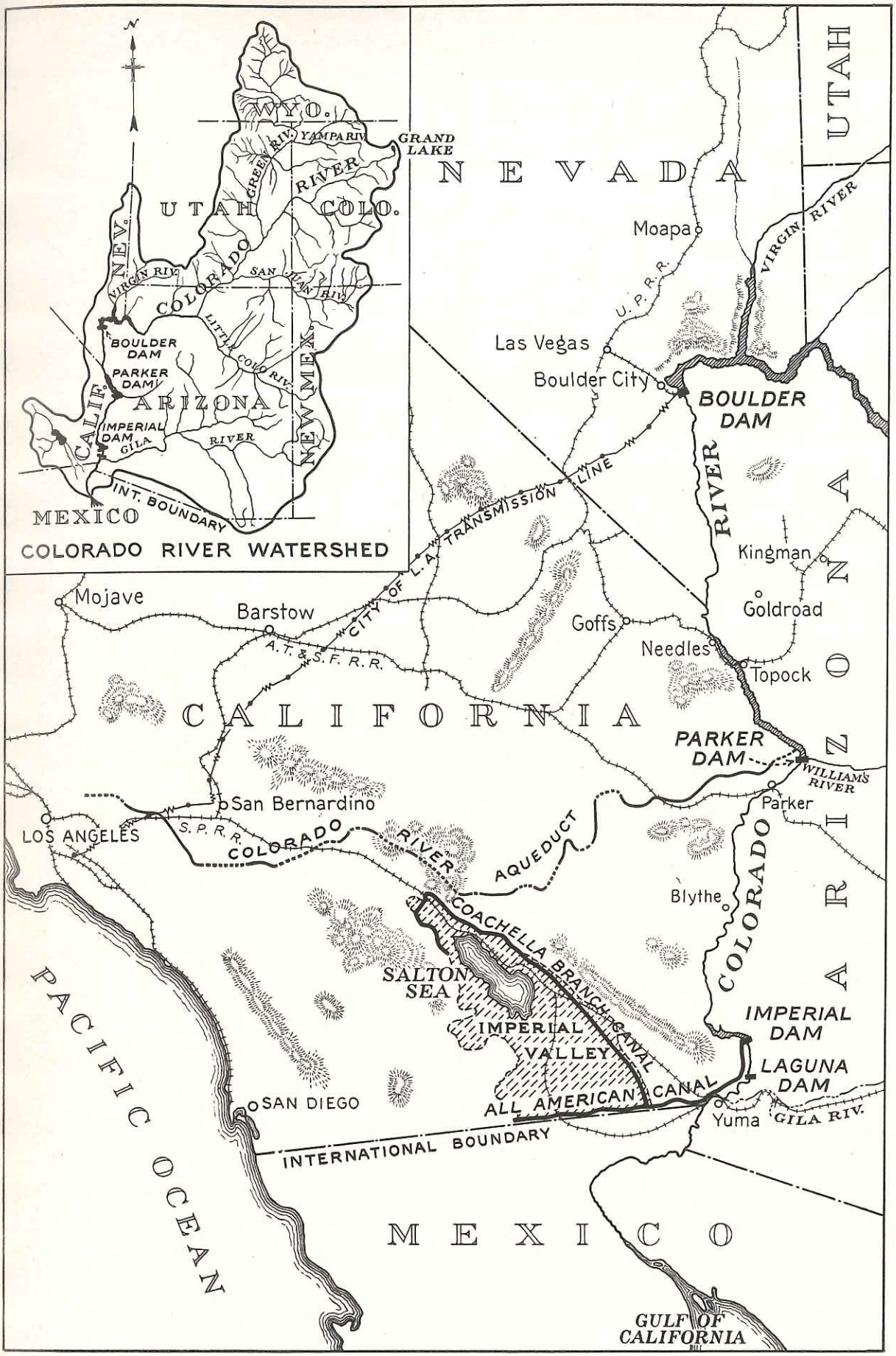


FRONTISPIECE—BOULDER DAM



LONGITUDINAL SECTION

BOULDER DAM AND APPURTENANT WORKS



BOULDER CANYON PROJECT—LOCATION MAP

BOULDER CANYON PROJECT
ENGINEERING ORGANIZATION DURING CONSTRUCTION

ADMINISTRATIVE OFFICE, WASHINGTON, D. C.

Elwood Mead*	Commissioner of Reclamation

ENGINEERING AND EXECUTIVE OFFICE, DENVER, COLORADO

Raymond F. Walter	Chief Engineer
Sinclair O. Harper	Assistant Chief Engineer
John L. Savage	Chief Designing Engineer
Leslie N. McClellan	Chief Electrical Engineer
William H. Nalder	Assistant Chief Designing Engineer
Byram W. Steele	Civil Engineer-Dams
Erdman B. Debler	Hydraulic Engineer
Charles M. Day*	Mechanical Engineer
Ivan E. Houk	Senior Engineer-Technical Studies
Harry R. McBirney	Civil Engineer-Canals
Kenneth B. Keener	Assistant Civil Engineer-Dams
Harvey F. McPhail	Electrical Engineer
Robert A. Monroe	Structural Engineer
Porter J. Preston	Senior Engineer-Investigations
Howard C. Stetson	Senior Engineer-Specifications

PROJECT OFFICE, BOULDER CITY, NEVADA

Walker R. Young	Construction Engineer
Ralph Lowry	Field Engineer
John C. Page	Office Engineer

CONSULTANTS

Colorado River Board:	William L. Sibert*, Chairman
Charles P. Berkey	Warren J. Mead
Daniel W. Mead	Robert Ridgway
Boulder Dam Board:	Andrew J. Wiley*, Chairman
William F. Durand	Arthur P. Davis*
David C. Henny*	Louis C. Hill
Concrete Research Board:	Phaon H. Bates, Chairman
Franklin R. McMillan	Raymond E. Davis
Herbert J. Gilkey	William K. Hatt
Special Consultants:	
Harald M. Westergaard	Mechanics
Frederick L. Ransome*	Geology
Gordon B. Kaufmann	Architectural Features

PRINCIPAL CONTRACTORS

General Contractors	Six Companies, Inc., San Francisco
Penstock & Outlet Pipe	The Babcock & Wilcox Co., New York

* Deceased.

FOREWORD

Colorado River, originating in the melting snows of the Wyoming and Colorado Rockies and augmented by rapid run-off from spasmodic rains and cloudbursts over a vast arid region, has menaced life and property in its descent to the Gulf of California since the days of the first covered wagon.

With increased population along the lower reaches of the river the problem of controlling the Colorado became more important. During recent years millions of dollars have been spent in mitigating the evils of silt deposition and in protecting the highly cultivated Imperial Valley lands from annual threats of inundation.

The need for a comprehensive plan of development to check the ravages of Colorado River, to regulate its flow, and to utilize a part of its enormous energy led, first, to investigations by the Reclamation Service of all water storage possibilities; next, to the Colorado River Compact, a mutual agreement for the protection of the seven basin states; and, finally, to the adoption of the Boulder Canyon Project, as the initial development.

The Boulder Canyon Project Act, approved December 21, 1928, authorized a total appropriation of $165,000,000 for the various features involved. These include Boulder Dam and appurtenant works, the power plant, the reservoir, and the All-American Canal System. The purposes of the project are: (1) flood and silt control for protection of lands along the lower river; (2) improvement of navigation; (3) river regulation and storage of water for irrigation and municipal use; and (4) development of electric power for domestic and industrial purposes. The project is self-liquidating, largely through contracts for disposal of electrical energy. It was constructed and is being operated under the supervision of the Bureau of Reclamation, United States Department of the Interior.

Boulder Dam is located on the Nevada-Arizona boundary near Las Vegas, Nevada, at a place where Colorado River has carved a deep gorge between towering rock cliffs, known as Black Canyon. The dam is a concrete arch gravity structure with a maximum height of 726 feet above foundation rock, a maximum base thickness of 660 feet, and a crest length of 1,244 feet. The dam and appurtenant works contain 4,400,000 cubic yards of concrete, of which 3,250,000 cubic yards were required in the dam.

During construction the river was diverted through four 50-foot

diameter, concrete-lined tunnels, two on each side of the river. These tunnels were subsequently plugged near the upstream ends. The spillways, each of 200,000 second-feet capacity, are connected through inclined shafts to the two outer tunnels. A 30-foot diameter steel power penstock is installed in each of the inner tunnels. Discharge from the reservoir is controlled by cylinder gates in four intake towers, founded on the canyon walls near the upstream face of the dam. Four 30-foot steel penstocks, connected to the bases of the intake towers, conduct water to the power plant and to the outlet valves for release of flood, irrigation, and domestic water supply when the power plant discharge is insufficient for such purposes. The reservoir above the dam is 115 miles long and has a capacity of 30,500,000 acre-feet, the equivalent of two years' normal river flow.

The power plant is in a U-shaped, reinforced concrete structure, over 200 feet high and 1,500 feet long, located immediately downstream from the dam. The plant is designed for an ultimate installation of fifteen 115,000 and two 55,000 horsepower units, making a total installed capacity of 1,835,000 horsepower.

The All-American Canal, located near the Mexican border, will carry water to irrigate lands in the Imperial and Coachella Valleys. The canal proper, with a diversion capacity of 15,000 second-feet, is the largest ever constructed for irrigation purposes in America.

The entire Boulder Canyon Project is characterized by the extraordinary. The height and base thickness of the dam, the size of the power units, the dimensions of the fusion-welded, plate-steel pipes, the novel system of artificially cooling the concrete, the speed and coordination of construction, and other major features of the project are without precedent. The magnitude of the undertaking introduced many new problems and intensified many usual ones, requiring investigations of an extensive and diversified character to insure structures representing the utmost in efficiency, safety, and economy of construction and operation.

The major credit for the conception of the project and the initiation of investigations leading to its adoption must be given to the late Arthur P. Davis, former Director of the Reclamation Service. Dr. Elwood Mead, Commissioner of Reclamation during the greater part of the construction period, passed away January 26, 1936, four months after the dedication of Boulder Dam. In commemoration of his untiring services on the Boulder Canyon Project, the reservoir created by the construction of the dam has been officially named "Lake Mead".

CONTENTS

SECTION		PAGE
	Introductory statement	ii
	Engineering organization during construction	vi
	Foreword	vii
	Contents	1
	List of figures	9
	List of tables	11

CHAPTER I—INTRODUCTION

THE TRIAL LOAD METHOD

1	General	13
2	Development of method	14

BASIS OF TRIAL LOAD METHOD

3	Assumptions	15
4	Accuracy of assumptions	16

EFFECTS INCLUDED IN ANALYSES

5	Adjustments considered	17
6	Poisson's ratio effects	18
7	Importance of supplemental effects	18
8	Temperature effects	19

TRIAL LOAD ANALYSES IN DAM DESIGN

9	Procedure	20
10	Design considerations	21

TRIAL LOAD CALCULATIONS

11	General methods	22
12	Notation	23

USE OF THE METHOD

13	Trial load dams	23
14	Personnel	24

BIBLIOGRAPHY 24

CHAPTER II—DESCRIPTION OF THE METHOD

THE STRUCTURE

SECTION		PAGE
15	Types of dams analyzed	25
16	Linear stress assumption	25
17	Coordinates	25
18	Arches and cantilevers	25
19	Trial load adjustments	26
20	Vertical displacements	27

LOAD CONDITIONS

21	General	28
22	External loads	28
23	Internal loads	29
24	Application of loads	29

CANTILEVERS

25	Description	31
26	Unit radial loads	31
27	Cracked cantilevers	31
28	Unit tangential loads	31
29	Unit twist loads	33
30	Secondary movements	33
31	Deflections of unit loads	33

ARCHES

32	General description	34
33	Unit radial loads	34
34	Unit tangential loads	34
35	Unit twist loads	34
36	Concentrated loads	36
37	Temperature loads	36
38	Secondary movements	36
39	Cracked arches	37

TRIAL LOAD ADJUSTMENTS

40	General statement	37
41	Radial adjustment	37
42	Tangential adjustment	37
43	Twist adjustment	38
44	Sequence of adjustments	38
45	Adjustments for Poisson's ratio	39

RESULTS OF ANALYSIS

46	Movements	39
47	Stresses	39

CHAPTER III—FOUNDATION AND ABUTMENT DEFORMATIONS

INTRODUCTION

SECTION		PAGE
48	General statement	41
49	Assumptions	41

DEFORMATION OF FOUNDATION SURFACE

50	Notation	44
51	Deformation equations	44
52	Curves	47

MOVEMENTS OF UNIT ELEMENTS

53	Notation	53
54	Unit horizontal element	56
55	Unit vertical element	58
56	Application to cantilevers	60
57	Computation form	62

CHAPTER IV—ANALYSIS OF CANTILEVER ELEMENTS

INTRODUCTION

58	General	63
59	Description of cantilever element	63

ANALYSIS OF UNCRACKED CANTILEVER

60	Notation	64
61	Evaluation of properties	64
62	Concrete and vertical water loads	68
63	Unit triangular loads	68
64	Unit radial loads	69
65	Unit tangential loads	70
66	Unit twist loads	70

MOVEMENTS OF UNCRACKED CANTILEVER

67	Notation	70
68	Foundation movements	71
69	Radial deflections	72
70	Tangential deflections	73
71	Angular movements	73
72	Secondary movements	74
73	Tabulations	75

ANALYSIS OF CRACKED CANTILEVER

SECTION		PAGE
74	General statement	75
75	Notation	75
76	Cracking at upstream face	76
77	Cracking at downstream face	79
78	Deflections of cracked cantilever	81

COMPUTATION FORMS

79	General	83
80	Use of forms	83

CHAPTER V—ANALYSIS OF ARCH ELEMENTS

INTRODUCTION

81	Preliminary statement	89
82	Arch elements	89
83	Loads	90
84	Assumptions	92

GENERAL ARCH THEORY

85	General statement	92
86	Notation	93
87	Convention of signs	95
88	Movements of arch at crown	95
89	Solution of crown forces	98
90	Arch deflections	99

UNIFORM THICKNESS CIRCULAR ARCH

91	General statement	103
92	Arch constants	103
93	Load constants	104
94	Uniform radial load	106
95	Triangular radial load	107
96	Uniform tangential load	108
97	Triangular tangential load	109
98	Uniform twist load	110
99	Triangular twist load	111
100	Temperature load	111
101	Summary of load formulas	111

VARIABLE THICKNESS ARCH

SECTION		PAGE
102	General statement	113
103	Notation	114
104	Arch constants	114
105	Load constants	116
106	Radial load constants	116
107	Tangential load constants	117
108	Twist load constants	117
109	Application of arch formulas	118

ARCH WITH INTRADOS FILLETS

110	General statement	118
111	Notation	119
112	Arch constants	119
113	Load constants	120
114	Application of formulas	121

OTHER TYPES OF ARCHES

115	General statement	121
116	Three-centered arch	121
117	Polycentered arch	123
118	Noncircular arches	124
119	Cracked arches	125

CHAPTER VI—CALCULATION OF ARCH DATA

INTRODUCTION

120	Preliminary statement	127
121	Quantities required for analysis	127

COMPUTATION FORMS

122	General	128
123	Arch and abutment data	128
124	Crown constants and forces	133
125	Moments, thrusts, shears, and radial deflections	134
126	Tangential and angular movements	135
127	Use of forms	135

UNIFORM THICKNESS CIRCULAR ARCH

128	General	136
129	Symmetrical arch with symmetrical loads	136
130	Symmetrical arch with nonsymmetrical loads	141
131	Nonsymmetrical arch	142

VARIABLE THICKNESS ARCH

SECTION PAGE

132 General 143
133 Arch constants 143
134 Load constants 143
135 Other calculations 149

ARCH WITH FILLETS AT DOWNSTREAM FACE

136 General statement 149

CURVES AND TABULATIONS

137 General 150
138 Symmetrical unit loads 150
139 Nonsymmetrical unit loads 157
140 Arch forces and deflections 157

CHAPTER VII—LOADS AND ADJUSTMENTS

INTRODUCTION

141 General statement 159
142 Arches and cantilevers 159
143 Constants and structural data 159

FORCES ACTING ON A DAM

144 General 160
145 Concrete weight 160
146 Temperature 160
147 Water pressure 162
148 Uplift pressure 162
149 Silt 163
150 Ice 163
151 Earthquake shocks 163

APPLICATION OF EXTERNAL LOADS

152 General 164
153 Horizontal water loads 165
154 Vertical water loads 165
155 Temperature loads 165
156 Concentrated loads 165

PREPARATORY STEPS

SECTION		PAGE
157	General	166
158	Foundation	166
159	Cantilevers	166
160	Arches	167

RADIAL ADJUSTMENT

161	Procedure	168
162	Readjustments	170

TANGENTIAL ADJUSTMENT

163	General	170
164	Movements due to radial adjustment	171
165	Application of loads	171
166	Results of tangential adjustment	171

TWIST ADJUSTMENT

167	General	173
168	Application of loads	173
169	Radial cantilever deflections due to twist loads	175
170	Effects of adjustment	176

SEQUENCE OF ADJUSTMENTS

171	Readjustments required	176
172	Complete analysis	176

ADJUSTMENTS FOR POISSON'S RATIO

173	General	176
174	Cantilever movements	177
175	Arch movements	177
176	Adjustments	177
177	Effects of grouting joints	178
178	Effects on stresses and movements	178

CHAPTER VIII—EVALUATION OF STRESSES

INTRODUCTION

179	General statement	179
180	Notation and definitions	179

STRESSES AT FACES OF DAM

181	Procedure	182
182	Stresses at downstream face	183
183	Stresses at upstream face	188

STRESSES ON HORIZONTAL AND VERTICAL PLANES

SECTION		PAGE
184	Procedure	189
185	Normal cantilever and arch stresses	189
186	Cantilever shear stresses	189
187	Radial shear in arch elements	191
188	Horizontal radial stresses	191
189	Summary of stresses	193

STRESSES ON INCLINED CIRCUMFERENTIAL AREA

190	General	194
191	Stress equations	194

STRESSES ON OBLIQUE PLANE

192	General	195
193	Stress equations	197
194	Principal stresses in interior of dam	198

CHAPTER IX—TABLES USED IN ARCH ANALYSES

INTRODUCTION

195	General statement	199
196	Arrangement	199

CONSTANTS FOR CIRCULAR ARCHES

197	Use of table	199
198	Functions included	204

TRIGONOMETRIC FUNCTIONS AND INTEGRALS

199	Explanation	204
200	Arrangement	204
201	Formulas in trigonometric form	205
202	Formulas in power series	206

LOAD CONSTANTS FOR CIRCULAR ARCHES

203	General statement	229
204	Evaluation	229
205	Tabulations	234

TRIGONOMETRIC TABLE FOR ARCH ANALYSIS

206	General statement	260

LIST OF FIGURES

FIGURE		PAGE
Frontispiece	Boulder Dam	iii
	Boulder Dam and appurtenant works	iv
	Boulder Canyon Project, location map	v
1	Location of arches and cantilevers	27
2	Deformation of arch and cantilever	30
3	Typical unit cantilever loads	32
4	Typical unit arch loads	35
5	Values of k in equation 19	48
6	Values of k in equation 20	49
7	Values of k in equation 21	50
8	Values of k in equation 22	51
9	Values of k in equations 23 and 24	52
10	Loaded area of foundation surface	53
11	Unit horizontal element at left abutment	54
12	Unit vertical element at left abutment	54
13	Abutment at left side of dam	55
14	Computation form for unit foundation movements	61
15	Cross section of radial-side cantilever	65
16	Properties of radial-side cantilever	67
17	Parallel-side cantilever equivalent to radial-side cantilever	68
18	Basic unit triangular loads	69
19	Convention of signs, radial-side cantilever	73
20	Horizontal section of cantilever cracked at upstream face	76
21	Properties of cantilever cracked at upstream face	78
22	Horizontal section of cantilever cracked at downstream face	80
23	Properties of cantilever cracked at downstream face	82
24	Computation form for properties of horizontal sections	85
25	Computation form for weights and moments of vertical loads	85
26	Computation form for deflections of uncracked cantilever	86
27	Computation form for forces and moments, cracked cantilever	87
28	Computation form for deflections and stresses, cracked cantilever	87
29	Unit arch load patterns	91
30	Circular arch cut at crown	93
31	Direction of positive arch properties	95
32	Left part of arch	96
33	Radial arch loads	107

FIGURE		PAGE
34	Tangential arch loads	108
35	Twist loads on arch elements	110
36	Variable thickness arch	113
37	Arch with intrados fillet	119
38	Three-centered arch of uniform thickness	121
39	Polycentered arch of variable thickness	124
40	Computation form for arch and abutment data	129
41	Computation form for crown constants and forces	130
42	Computation form for moments, thrusts, shears, and radial deflections	131
43	Computation form for tangential and angular movements	132
44	Computation form for eccentricities and multipliers, for arch and load constants	144
45	Computation form for arch or load constants, for variable thickness arches	145
46	Determination of formulas for load 5, table 9	147
47	Computation form for moments due to eccentricity, tangential loads	148
48	Typical deflections, symmetrical radial loads plus temperature change	151
49	Typical deflections, symmetrical tangential loads	152
50	Typical deflections, symmetrical twist loads	153
51	Typical deflections, nonsymmetrical radial loads plus temperature change	154
52	Typical deflections, nonsymmetrical tangential loads	155
53	Typical deflections, nonsymmetrical twist loads	156
54	Maximum range in concrete temperature	161
55	Contact of arch and cantilever with abutment	165
56	Radial loads and deflections—Boulder Dam	169
57	Tangential loads and deflections—Boulder Dam	172
58	Twist loads and angular movements—Boulder Dam	174
59	Intersection of arch and cantilever elements	182
60	Stresses at tetrahedron *ABCO*	183
61	Tetrahedron *ABCO*, showing axes and angles	184
62	Stresses at section *ADO*	185
63	Stresses in plane *BCO*	185
64	Shear stress on plane *BCO*	186
65	Shear stress on section *ADO*	186
66	Stresses in *X′ Z′* plane	187
67	Vertical cantilever element	190
68	Horizontal arch element	190
69	Stresses on inclined circumferential area	194
70	Elemental tetrahedron in interior of dam	196
71	Stresses on oblique plane	197

LIST OF TABLES

TABLE		PAGE
1	Movements of horizontal element	56
2	Movements of vertical element	58
3	Constants for solution of crown forces	100
4	Formulas for circular arch load constants	112
5	Formulas for moments, thrusts, and shears of radial loads	137
6	Formulas for moments, thrusts, and shears of tangential loads	138
7	Formulas for moments, thrusts, and shears of twist loads	139
8	Algebraic signs of quantities	141
9	Circular arch *D*-terms	146
10	Constants for circular arches	200
11	Trigonometric functions for arch load constants	209
12	Trigonometric functions for arch load constants, continued	213
13	Trigonometric functions and integrals for arch load constants	217
14	Integrals for arch load constants	221
15	Integrals for arch load constants, continued	225
16	Computations for load constants	230
17	Constants for radial deflections, radial loads	235
18	Constants for twist and tangential deflections, radial loads	240
19	Constants for tangential deflections, tangential loads	245
20	Constants for radial and twist deflections, tangential loads	250
21	Constants for twist, radial, and tangential deflections, twist loads	255
22	Trigonometric table for arch analysis	261

CHAPTER I—INTRODUCTION

THE TRIAL LOAD METHOD

1. **General.**—Development of the trial load method of analyzing arch dams was begun in the Denver office of the Bureau of Reclamation in 1923. Prior to that time most concrete dams had been designed on the assumption that the entire water load would be carried vertically to the foundations by gravity action in both straight and curved gravity dams; and horizontally to the abutments by arch action in arch dams. The use of the trial load method now enables the designing engineer to analyze load distributions, deflections, and stresses in curved concrete dams of all sizes and shapes, whether of the massive arched gravity type or the relatively thin, monolithic, arch type.

In a few arch dams designed prior to 1923, also a few designed by other agencies than the Bureau of Reclamation since that time, stress analyses were made on the assumption that the water load would be divided between arch and cantilever elements in such a way as to produce equal arch and cantilever deflections in a radial direction at the crown cantilever. However, in such cases the division of load was assumed to be constant from abutment to abutment at each horizontal element analyzed; and no vertical elements were considered except at the crown section. This method might logically be called the "arch and crown cantilever" method.

The trial load method is essentially an amplification of the arch and crown cantilever method. The trial load method, as now used by the Bureau, assumes that the water load is divided between arch and cantilever elements; that the division may or may not be constant from abutment to abutment at each horizontal element; and that the true division of load is the one which causes equal arch and cantilever deflections at all points in all arches and cantilevers, instead of at the crown cantilever only. Furthermore, the trial load method assumes that the distribution of load must be such as to cause equal arch and cantilever deflections in all directions; that is, in tangential and rotational directions as well as in radial directions. Since the required agreement of arch and cantilever deflections can only be obtained by assuming different distributions of load and

calculating resulting arch and cantilever movements until the specified criterion is fulfilled, the procedure is logically called the "trial load" method.

The true distribution of load between horizontal and vertical elements having been determined, resulting arch, cantilever, and principal stresses may be calculated. Arch and cantilever stresses are usually calculated at upstream and downstream faces of the dam. Arch stresses are usually calculated at crown and abutment locations but may also be calculated at other sections along arch rings. Cantilever stresses are generally calculated at elevations of arches and at foundation levels. Magnitudes and directions of principal stresses are calculated along lines of contact between faces of the dam and rock profiles. Principal stresses usually act in approximately horizontal planes at the abutment ends of the top arch, in approximately vertical planes at the base of the crown cantilever, and in gradually changing directions at intervening locations between the bottom of the canyon and the ends of the top arch. The calculated arch, cantilever, and principal stresses are considered to be the true stresses caused by the assumed condition of reservoir loading.

2. Development of Method.—In the first use of the trial load method, foundation and abutment formations were considered rigid, and both arches and cantilevers were assumed to have constant unit thicknesses. Arch elements were considered as horizontal layers having a uniform vertical thickness of one foot, and cantilever elements as vertical slices having a uniform horizontal thickness of one foot. In the earlier analyses, calculations included effects of bending and thrust in arch elements; and bending, thrust, and shear in cantilever elements. The next step in the development was the introduction of radial shear effects in arch computations. Since that time the method has been gradually amplified until now the formulas consider effects of radial sides of cantilever elements, twist action, tangential shear, Poisson's ratio effects, and movements of foundation and abutment rock, as well as the more commonly considered effects of thrust, shear, and flexure in concrete elements.

Formulas used in computing movements of horizontal and vertical elements are the usual arch and cantilever beam equations derived on the basis of the elastic theory. Since the trial load method brings arch and cantilever movements into agreement in all parts of the structure, the procedure automatically makes proper allowances for irregularities in rock profile and lack of symmetry on the two sides of the canyon.

BASIS OF TRIAL LOAD METHOD

3. **Assumptions.**—The trial load method of analyzing curved concrete dams is based on the following assumptions:

1. Rock formations which make up the foundation and abutments at the site are homogeneous and uniformly elastic in all directions, and are strong enough to carry applied loads with stresses well below the elastic limit.

2. The concrete in the dam is homogeneous and uniformly elastic in all directions and strong enough to carry applied loads with stresses well below the elastic limit.

3. The dam is thoroughly keyed into the foundation and abutment rock throughout its contact with the canyon profile; so that arches may be considered as fixed with relation to abutments, and cantilevers as fixed with relation to the foundation.

4. Vertical construction joints in the dam are grouted, or open joints filled, before the water load is applied; so that the structure may be considered to act as a monolith and arch action to begin as soon as the reservoir begins to fill.

5. The horizontal water load is carried by two systems of structural elements; namely, a system of horizontal arch elements and a system of vertical cantilever elements.

6. Horizontal elements have a constant vertical thickness of one foot from abutment to abutment and from upstream to downstream face of dam.

7. Vertical elements are bounded by vertical radial planes a horizontal distance of one foot apart at the axis of the dam. The axis is defined as the upstream face when the upstream face is vertical, or as a vertical circumferential plane passing through the upstream edge of the top arch when the upstream face is built on a batter.

8. Both horizontal and vertical elements are considered free to move under reservoir loads, without restraint from adjacent elements, and then brought into geometric continuity with adjacent elements by the application of self-balancing tangential shear and twist loads or couples, one set of loads or couples being applied to arch elements and the balancing set of loads or couples being applied to cantilever elements.

9. Total horizontal water loads are divided between the two systems of elements in such a way as to satisfy conditions of equi-

librium and produce equal arch and cantilever deflections in all directions in all parts of the loaded structure.

10. Total vertical loads, including weights of water on the faces of the dam as well as weights of concrete, are assigned to cantilever elements and are assumed to be transmitted to the foundation without any transfer of load laterally to adjacent cantilevers by vertical arching.

11. Effects of flow can be adequately considered by using somewhat smaller values of modulus of elasticity than would otherwise be adopted.

4. Accuracy of Assumptions.—The first of the basic assumptions listed above is probably the most uncertain of all, since it is doubtful if rock formations at any dam site are ever sufficiently free from cracks, seams, and fissures to be considered satisfactorily homogeneous. It is also doubtful if such formations are ever uniformly elastic in all directions; so that their movements under applied loads can be accurately calculated by mathematical formulas based on the theory of elasticity, or that the amount of plastic flow or joint slippage can be determined. However, for the present there seems to be no better way to estimate these movements which are important and which, therefore, should be considered in the design of curved concrete dams.

It is not believed that much uncertainty is involved in basic assumptions listed as numbers 2 to 5, inclusive. The uncertainty regarding homogeneity and uniform elasticity of concrete, assumption 2, which may have been important in some of the older structures, is gradually becoming less important, due to recent researches in design of concrete mixes together with improvements continually being made in methods of manufacturing and placing concrete.

Assumption 4, regarding closing of the dam before application of water load, can probably be violated in most cases without causing serious errors in maximum stresses calculated for the full condition of the reservoir. In the case of Boulder Dam, a series of careful trial load analyses for successive stages during the construction period, taking into account actual grouting operations, eccentric vertical loads produced by adding successive increments of concrete weight, intermediate partial-depth water loads, and effects of Poisson's ratio, did not change final maximum stresses more than 10 percent from those determined by the usual trial load analysis for full reservoir loading. The stress distribution was changed somewhat; some of the smaller stresses were altered appreciably, particu-

larly in the arch elements; and elevations of maximum arch stresses were shifted slightly at both crown and abutment sections. However, the general effect on magnitude of maximum stresses was not important in either arch or cantilever elements.

As regards assumption 11, recent results of laboratory investigations indicate that, for stresses lower than those usually permitted in dam design, concrete flow is substantially proportional to stress. Consequently, the effect of flow on the action of a dam under continuous full reservoir load is primarily an increase in downstream deflection, although some effect on stress may result from the fact that the flow of the foundation and abutment rock may not be the same as the flow of concrete.

EFFECTS INCLUDED IN ANALYSES

5. Adjustments Considered.—In calculating deflections of a curved dam for use in a trial load analysis the various arch and cantilever elements are first assumed free to move independently and are then brought into radial, tangential, and twist agreements by varying magnitudes of loads applied. In the radial adjustment the vertical component of the water load is assigned to the cantilevers, and the horizontal component of the water load is divided by trial into two components, both radial, one acting on the arches and the other on the cantilevers. The desired division of load is that which gives the arches and cantilevers a certain common deflection at each point. This process involves the use of certain pattern loads, the effects of which combine conveniently, but which set up twisting and tangential influences in other parts of the structure. These influences must be considered later in the analysis by means of other self-balancing torsional and tangential loads.

It is expedient to introduce the new loads in such a manner that torsional and tangential shearing stresses are assigned to the cantilevers only. Arches are subject to twisting, bending, and compression only, and may be treated as though they had no resistance to tangential sliding. Cantilevers must resist bending, compression, twisting, and tangential shear.

At each stage in the analysis it is necessary to determine displacements of arch cross-sections relative to corresponding cantilever cross-sections. These relative displacements are deflections and rotations. New self-balancing loads must be applied in such a manner that not only relative radial deflections but also relative rotations and tangential deflections of arches and cantilevers are removed.

6. Poisson's Ratio Effects.—The influence of Poisson's ratio is introduced into the trial load analysis, indirectly, in calculations of shear, twist, and tangential deflections, through the use of the shearing modulus of elasticity. Additional Poisson's ratio effects are analyzed by special readjustments after preliminary trial load calculations are completed. Corrections of stress analyses to allow for additional Poisson's ratio effects are not particularly important, inasmuch as the general tendency is toward a more uniform distribution of arch and cantilever stresses without causing changes of more than from 5 to 10 percent at locations of maximum stress.

7. Importance of Supplemental Effects.—With the exception of Poisson's ratio adjustments, all supplemental effects included in trial load analyses since the first use of the method, have been found worthy of consideration, the importance of the effects usually varying with the size and height of structure.

Effects of including radial shear in calculating arch deflections were found particularly important in thick arches, as would naturally be expected, the importance increasing rapidly as the ratio of arch thickness to radius of curvature increases. Radial shear deflections in the lower arches of a massive curved concrete dam frequently equal or exceed moment and thrust deflections at practically all locations along the arch ring.

The importance of making proper allowances for radial sides of cantilever elements increases with the size of the dam and sharpness of curvature. The downstream base width of a cantilever with radial sides, one foot apart at the upstream face, was only 0.496 feet at the crown section of Gibson Dam, a 199-foot, constant radius, arch dam, constructed on the Sun River Project, Montana; and only 0.490 feet at the crown section of Owyhee Dam, a 421-foot, constant radius, arch dam, constructed on the Owyhee Project, Oregon. Proper allowances for radial sides tend to increase flexibility of cantilever elements, thus throwing more load on arch elements. Consequently, the net result of such allowances is a new load distribution with slightly increased radial deflections and slightly higher maximum stresses in both arch and cantilever elements.

Allowances for foundation and abutment deformations naturally tend to increase calculated deflections of the structure. The maximum downstream movement of the foundation rock calculated for the base of the crown cantilever of Owyhee Dam amounted to about 10 percent of the maximum radial deflection at the top of the dam. The corresponding movement of the foundation rock at the base of the crown cantilever of

Boulder Dam amounted to about 30 percent of the maximum radial deflection. The general result of including foundation and abutment movements in trial load calculations seems to be somewhat lower arch and cantilever stresses along foundation and abutment locations, without material stress changes in central and upper portions of the structure.

The general result of considering tangential shear in trial load analyses is a decrease in radial deflections near the crown section, an increase in radial deflections near the abutments, and a slight increase in arch stresses at the abutments without appreciable stress changes at the crown. The importance of tangential shear effects varies primarily with the ratio of length of dam to height. Such effects are not important in an arch dam built in a narrow canyon, as in Shoshone Dam, where the length of the top arch is less than the height of the structure. However, they are important in a relatively long arch dam like Gibson, where the top length is about four and one-half times the height of the structure.

The general effect of considering twist action is a decrease in radial deflections at practically all locations, a decrease in maximum arch stress, a decrease in cantilever stress at the downstream face of the dam, and an increase in cantilever stress at the upstream face of the dam. Twist effects are important in most high dams of the curved gravity type, such as Owyhee and Boulder. They are also important in arch dams of more usual size.

8. **Temperature Effects.**—Effects of temperature changes in the interior of the dam are included in trial load analyses by adding temperature deflections of arch elements to water load deflections before adjusting arch and cantilever movements. No temperature corrections are necessary in cantilever elements since uniform temperature changes in vertical sections do not cause appreciable horizontal movements. Increases in temperature of arch elements cause upstream movements which work against the water load, while decreases in temperature cause downstream movements which work with the water load. Consequently, the maximum probable decrease in temperature, below the temperature existing at the time of grouting construction joints, is the change which usually must be included in the arch analysis. Of course, proper allowances must be made for any setting heat which may not have been generated or dissipated at the time of grouting the joints.

Temperature changes in arch elements vary with the thickness of the arch and temperature changes at the faces of the dam. For the purpose of trial load analyses, temperature changes in the interior of the structure

are assumed to be uniform from abutment to abutment and from upstream to downstream face of dam at each elevation analyzed. In unusually important structures, such as Boulder Dam, effects of nonuniform temperature changes in arch elements may be made by special analyses after trial load studies are completed.

TRIAL LOAD ANALYSES IN DAM DESIGN

9. Procedure.—Several trial load analyses usually are required in designing an arch dam by the trial load method. This is particularly true if the site possesses a shape definitely different from any section previously investigated, or if the profile at the site contains irregularities which cannot economically be removed in the base excavation and which, therefore, may cause stress concentrations in the finished structure.

Ordinarily, the procedure is to adopt a tentative cross-section and make a preliminary plan of the dam, introducing as much horizontal curvature as possible; and then to calculate maximum stresses in the structure by a simplified trial load method. The simplified method may assume tension stresses to transmit internal forces without cracking the concrete, neglect effects of uplift, tangential shear, and twist action, and make an adjustment of arch and cantilever deflections in the radial direction only. Usually a total of five or six cantilever elements and a like number of arch elements, spaced fairly uniformly throughout the dam, is sufficient in preliminary trial load studies. If at all possible, it is desirable to smooth out irregularities of the canyon profile in assuming excavation lines, and to adopt a design symmetrical about the crown section. Deep holes or relatively narrow gorges in the bottom of the canyon can sometimes be considered as plugged with concrete and treated as parts of the foundation instead of parts of the dam.

When the first trial load analysis is completed, tentative dimensions of the structure can be modified as indicated by preliminary stress calculations, and the modified design reanalyzed. This procedure can be continued, using the simplified, radial adjustment method, until dimensions of the structure are such as to give satisfactory stresses in both arch and cantilever elements. The final design can then be carefully analyzed by the amplified trial load method, including effects of uplift, tangential shear, twist action, cracking in the tension areas when calculated tension stresses are more than about 50 pounds per square inch, and, if necessary, effects of unsymmetrical conditions at the site. In the final trial load studies it usually is desirable to analyze about eight or ten cantilever elements and about the same number of arch elements.

10. **Design Considerations.**—Tentative designs for curved concrete dams are sometimes based on computations of arch thickness by the thin cylinder formula, using a relatively low value of maximum allowable stress. However, experience has shown that designs prepared on such a basis must be materially modified when stresses are analyzed by the trial load method. Consequently, it seems better to base tentative plans on a careful study of actual dimensions of arch dams built at similar sites, where dimensions and curvatures were determined by careful trial load analyses.

Experience gained from analyses of important dams has shown that cracking in the lower arch elements has no important effect on the distribution of water load, or on radial arch deflections. This simply means that the opening and closing of radial contraction joints at lower elevations along the upstream face of the dam, during the filling and emptying of the reservoir, has no important effect on the results of the trial load analysis as long as tension stresses are properly considered.

In some recent trial load dams, tension stresses and resultant cracking in arch elements were largely eliminated by sub-cooling the concrete and grouting radial construction joints while concrete temperatures were below ultimate mean annual values. Cracking in cantilever elements is analyzed whenever preliminary trial load studies indicate tension stresses greater than about 50 pounds per square inch.

Uplift forces are included in analyses of cracked cantilevers, since they tend to increase radial cantilever deflections, thus changing the distribution of water load. Effects of uplift pressure, either at the base of the dam or within the body of the structure, are not especially important in dams carrying the major part of the water load by arch action. However, they must be carefully and properly considered in the design of all masonry dams carrying the principal part of the water load by gravity action, even though the plans may provide elaborate grouting and draining installations in the foundation rock along the upstream edge of the base. Effects of uplift pressure in the pores of the concrete are more than adequately allowed for by using the same uplift assumptions as at the base of the dam.

In making trial load analyses for determining the design of an arch dam it usually is sufficient to consider the maximum water and temperature loads expected. After design dimensions are determined, trial load analyses can be made for water loads occurring during normal full reservoir operation and for ice loads combined with partial-depth loads. Tailwater loads should be considered, although they seldom have an appreciable effect on the design.

If the dam is located in a region subject to earthquake shocks, effects of an assumed earthquake can be combined with the above-mentioned loads, including the one for which the dam was designed. If the dam is located on a silt-bearing stream, it may be necessary to consider additional loads caused by silt deposits at the upstream face of the structure.

A new trial load analysis for maximum load conditions should be made as soon as the excavation of the base is completed, since limits of such excavation can seldom be accurately predicted in preliminary designs.

TRIAL LOAD CALCULATIONS

11. General Methods.—During the early work on the development of the trial load method, arch and cantilever deflections were calculated in the radial direction only. Then the tangential shear deformations were analyzed, and next the twist deformations. During some of the earlier analyses tangential shear and twist effects were evaluated by successive trial load adjustments and readjustments after arch and cantilever movements had been brought into radial agreement. However, tangential shear and twist adjustments are now being made simultaneously with radial adjustments.

In the early use of the trial load method arches were divided into voussoirs, and cantilevers into vertical increments. Total loads, moments, shears, slopes, and deflections were then calculated by summation methods. Vertical increments and summation methods, much amplified, are still used in analyzing cantilever elements. However, mathematical formulas, based on circular curves at upstream and downstream edges of horizontal elements, are now being used in analyzing arch elements. If extrados and intrados curves are not concentric, so that the arch thickness varies from crown to abutment, the half arch is divided into four segments, a uniform thickness assumed for each segment, and the analysis made by the use of special formulas derived for the shorter sections. If appreciable tensile stresses are indicated in the arch elements, so that investigation of cracked arches is advisable, analyses may be made by the original summation method.

During recent years analyses of uncracked arches have been greatly facilitated by the compilation and use of tables of arch constants. Analyses of both arch and cantilever elements have also been greatly facilitated by calculating effects of unit loads of rectangular and triangular shape, covering different parts of the loaded surface, so that effects of practically any shape of arch or cantilever load desired can be obtained by combining effects of unit loads already calculated.

Concrete stresses are computed on the assumption of a straight line distribution of stress from upstream to downstream face of dam in both arch and cantilever elements. In unusually important dams, such as Boulder, effects of nonlinear distribution of stress are determined by subsequent trial load analyses which bring horizontal and vertical slopes and deformations in the interior of the arches and cantilevers into agreement in all parts of the dam. Effects of nonlinear distribution of stress have been found important at locations near the foundation and abutments. However, they seem to be negligible in the upper central portions of the dam; that is, in portions extending from the crown section horizontally to about the quarter points of the arch elements, and from the top of the dam down about half way to the foundations of the cantilever elements.

12. Notation.—In the development of the trial load method many individuals have contributed new formulas and simplified forms for computation. This has resulted in notations not entirely consistent for different phases of the analysis. For this reason the notation used is given in each chapter, and the notation in a particular chapter does not refer to other chapters unless specifically so stated.

USE OF THE METHOD

13. Trial Load Dams.—Thus far, the Bureau has designed seven arch dams on the basis of trial load analyses; namely, Boulder, Owyhee, Parker, Seminoe, Gibson, Deadwood, and Cat Creek Dams. Cat Creek Dam was designed for the Bureau of Yards and Docks, United States Navy Department. All dams mentioned have been built except the Parker and Seminoe Dams which are now under construction. The Bureau has also made preliminary trial load designs for several contemplated arch dams where construction has not yet become feasible or where other types of dams have been found more desirable; also trial load analyses for several additional existing arch dams, including Gerber, Stevenson Creek, O'Shaughnessy, Mormon Flat, Horse Mesa, and Stewart Mountain Dams. Existing arch dams designed on the basis of trial load analyses and built by agencies other than the Bureau include Ariel, Calles, and Eleven Mile Canyon Dams. Dimensions and descriptive data regarding most of the dams mentioned may be found in articles published in the Reclamation Era or in various engineering journals.

14. Personnel.—The development of the trial load method of analyzing arch dams was begun by C. H. Howell, Julian Hinds, and A. C. Jaquith, former engineers in the Denver office of the Bureau. The method of analyzing circular arches by substituting integrals for the laborious summations of the original method was developed by R. E. Glover and R. S. Lieurance. Others who aided in the development of theoretical equations, organization of computations, and preparation of this report included A. W. Simonds, F. D. Kirn, W. A. Waldorf, R. W. Whinnerah, M. G. McCleery, L. V. Downs, E. R. Dexter, R. T. Larsen, F. A. Houck, Clarence White, and Edwin Rose.

BIBLIOGRAPHY

The following bibliography includes the principal articles on the trial load method and its application in the design of curved concrete dams, which have been published in engineering literature up to the present time. Some of the articles by engineers of the Bureau have been freely drawn upon in preparing the foregoing material.

1. HOUK, IVAN E.
Technical Design Studies for Hoover Dam, (now Boulder Dam), West. Const. News, Apr. 10, 1933, pp. 187-193.

2. ———
Technical Design of High Masonry Dams, The Eng., Aug. 4 and 11, 1933, pp. 105-106 and 128-130.

3. ———
Trial Load Method of Masonry Dam Analysis, West. Const. News, Nov. 1933, pp. 455-460.

4. ———
Trial Load Analyses of Curved Concrete Dams, The Eng., July 5 and 12, 1935, pp. 3-5 and 28-31.

5. ———
Arch Deflections and Temperature Stresses in Curved Dams, The Eng., Apr. 2 and 9, 1937, pp. 382-384 and 414-415.

6. ———
The Trial Load Method of Analyzing Arch Dams, Dams and Control Works, 2nd ed., Bur. of Recl., 1938, pp. 235-242.

7. HOWELL, C. H., AND JAQUITH, A. C.
Analysis of Arch Dams by the Trial Load Method, Trans. Am. Soc. C. E., Vol. 93, 1929, pp. 1191-1316.

8. LARNED, A. T., AND MERRILL, W. S.
Actual Deflections and Temperatures in a Trial Load Arch Dam, Trans. Am. Soc. C. E., Vol. 99, 1934, pp. 897-961.

9. WESTERGAARD, H. M.
Arch Dam Analysis by Trial Loads Simplified, Eng. News-Rec., Jan. 22, 1931, pp. 141-143.

CHAPTER II—DESCRIPTION OF THE METHOD

THE STRUCTURE

15. Types of Dams Analyzed.—The trial load method of analyzing arch dams as described in this report may be used in studying the structural action of any concrete dam that is arched in plan and is capable of transferring the reservoir water load to the supporting rock by both arch and cantilever action. The method is applicable to thick dams with gravity sections and small amounts of curvature; to thin arch dams of concentric or constant angle types; and to all designs that come between these two classifications.

16. Linear Stress Assumption.—The assumption of linear distribution of stress through the thickness of both arch and cantilever elements is used in computing stresses in the structure. It is known that in thick dams this assumption does not hold. However, it is possible to correct for the nonlinear distribution of stress by additional mathematical studies after the load distribution has been determined by trial load analyses.

17. Coordinates.—A dam of any of the types mentioned is essentially a three-dimensional elastic body. By elastic theory, conditions of deformation of a three-dimensional structure may be expressed in terms of three mutually perpendicular linear displacements and three angular displacements. In trial load analyses a system of cylindrical coordinates is ordinarily used. The axes are, first, radial along the radius of the arch; second, tangential along the tangent to the arch center line; and, third, vertical. Linear displacements are referred to these axes and angular displacements are measured as rotations about the axes.

Other coordinates are used in some phases of the analyses. In such cases the system of coordinates used is stated at the beginning of the chapter.

18. Arches and Cantilevers.—For the purpose of the analysis, the dam is replaced by two systems of elements; a system of vertical cantilevers and a system of horizontal arches. Both of these substituted struc-

tures occupy the entire volume of the dam. One structure carries the load supported by cantilever action, the other carries the load supported by arch action. Each arch and each cantilever may move independently of other arches and cantilevers, but at the conclusion of the analysis geometrical continuity must be restored at all points in the structure. That is, the arch and cantilever structures must have identical linear and angular displacements throughout, so that the continuous structures of arches and cantilevers occupy the position of the loaded dam. Between the initial state of the unloaded dam and its final position, elements are free to deflect as they may under the loads applied. However, being component parts of the dam, there are certain restrictions on the nature of their movements. For example, tangential movement of a cantilever takes place by shearing deformation instead of bending. The secondary tangential bending caused by change in vertical stress is considered negligible. These limitations will be cited, later, when movements of individual arches and cantilevers are discussed.

Instead of investigating a great number of vertical and horizontal elements, only a few sample arches and cantilevers are analyzed. The analyzed structure then becomes a grid of intersecting sample arches and cantilevers, as shown in figure 1. Obviously, the greater the number of elements considered the more accurate the results of the analysis. The complexity of resulting operations, however, holds the practical limit to a moderate number of elements. Chapter VII gives information of aid in determining the proper number and locations of sample arches and cantilevers.

19. Trial Load Adjustments.—The trial load analysis is carried out in steps, or adjustments as they are called. At present three adjustments are made; radial, tangential, and twist. As the names imply, these serve to bring the arch and cantilever movements into agreement in radial and tangential directions and in vertical and tangential rotations. Needless to say, with a small number of elements, exact geometrical congruity of the two systems is an ideal which can only be approximated. When the adjustments are completed, the grid occupies very closely the position assumed by the loaded dam. It is then a relatively simple matter to compute stresses in the arch and cantilever elements.

It will be noted that by simplifying the structure, requirements for a solution have been somewhat modified. In the original structure a solution would require complete geometrical congruence between arch and cantilever elements in all parts of the dam. With the simplified structure, it is sufficient to secure coincidence of sample arches and canti-

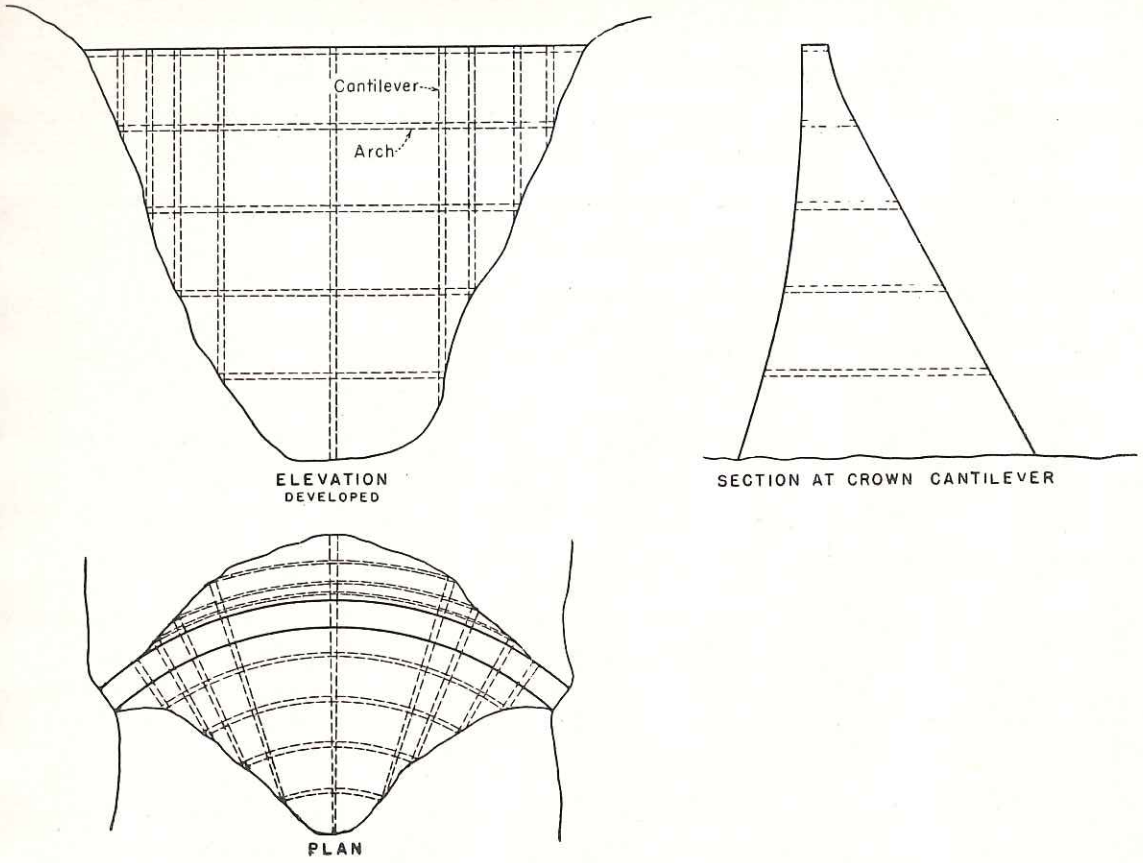


FIGURE 1—LOCATION OF ARCHES AND CANTILEVERS

levers at their points of juncture. Such coincidence is achieved when there is equality of the three linear and three angular displacements of the arch with those of the cantilever.

This equality of displacements, or an approach to it, is the condition for a complete solution. However, the adjustments previously mentioned bring only four of the six movements into agreement. These are the radial and tangential displacements, and tangential and vertical rotations. Tangential and vertical rotations refer to rotations about the tangential and vertical axes, respectively. The present method is, therefore, incomplete, since it lacks adjustments for vertical displacements and radial rotations. Of these two, the radial rotation seems of minor significance.

20. Vertical Displacements.—Consideration of vertical displacements would require an additional adjustment. Some investigations of vertical movements have been made, but thus far no satisfactory method of adjusting such movements has been developed. During the past few years trial load analyses, neglecting vertical movements, have been made for model dams, and the results compared with laboratory measurements of model action. The comparisons have shown satisfactory agreements of calculated and measured deflections, indicating that for reservoir water

loads, at least, the vertical adjustment is not of great importance. Furthermore, the columnar method of construction of large dams eliminates, or at least minimizes, the effects of several factors which would cause vertical movements. The weight is, of course, taken directly by the columns as they are built. Thus the initial deformations due to weight have taken place in the cantilevers before the arches are brought into existence by grouting the dam. In addition, the major part of the initial shrinkage takes place while the dam is still in columnar form. In Boulder Dam, another element, effects of setting heat, was also removed prior to grouting. This leaves, as the more important elements directly concerned in a vertical adjustment, the seasonal temperature change, and possibly plastic flow and Poisson's ratio effects.

LOAD CONDITIONS

21. General.—So far the discussion has been in terms of results rather than methods. Adjustments have been considered in terms of displacements without examining the nature of the loads producing the displacements. As the name of the analysis implies, deformations of individual arches and cantilevers are produced by loads determined by trial. These loads may be freely chosen with the provision that the sum of the arch and cantilever loads must equal the external load at every point. Loads are classified for convenience as external loads and internal loads.

22. External Loads.—External loads include all external forces acting on the dam. Loads for which a dam is commonly analyzed are the weight of the structure, temperature changes, reservoir water, tailwater, silt, concentrated ice forces, and earthquake shocks. In the analysis these loads are applied as follows. The weight is assigned to the cantilever elements. As stated previously, dams are commonly built in vertical sections which are pressure grouted to form a monolithic structure. Since the vertical sections are in equilibrium for the weight loading, prior to grouting, the weight may be regarded as solely a cantilever loading and omitted from the trial load analysis until stresses are calculated. After completing deflection adjustments, vertical stresses in the cantilevers are obtained by adding trial load stresses to initial weight stresses.

Temperature changes, assumed to occur in a dam, are applied to the arches as an initial condition. This procedure ignores the vertical movements due to temperature changes which would occur in the cantilevers.

Reservoir water pressure is considered as an external radial load at the upstream face of the structure. The vertical component is applied to

the cantilevers at the upstream face of the dam. The horizontal component is divided between arches and cantilevers in the process of making the radial adjustment. Tailwater pressure is likewise an external radial load and is applied at the downstream face, with the vertical component applied to the cantilevers. Silt effects are usually confined to increases in vertical loads. Possible increases in horizontal loads are considered by using fluid pressures greater than hydrostatic. Ice pressures are represented by equivalent radial loads.

Two effects of earthquake movements are considered, first, the inertia effect due to acceleration of the mass of the dam; and, second, the hydrodynamic effect caused by movement of the dam against the water of the reservoir.[1] The inertia effect is represented by equivalent radial and tangential loads at the center line of the dam. The hydrodynamic effect results in increased water pressures at the face of the dam. Additional radial loads are used to represent this superimposed water load.

Total external loads are applied to the dam as indicated above and the horizontal loads divided between the arch and cantilever systems until a satisfactory agreement of radial deflections is obtained. All subsequent adjustments are made by means of internal loads.

23. Internal Loads.—Internal loads are commonly called self-balancing loads, since they are always applied in pairs of equal and opposite loads, one acting against the arch and the other against the cantilever. By this means, arch and cantilever deflections may be brought into agreement without changing the external load on the structure. The internal loads represent, in a physical sense, forces set up by the interaction between the assumed arch and cantilever systems in the actual dam. In the practical application of the analysis they appear as forces required to eliminate relative movements of the two systems.

24. Application of Loads.—Figure 2 shows a clearer picture of the nature and use of internal loads and the successive steps in a trial load analysis. Figure 2-a shows a plan of a typical arch which is intersected by a cantilever element at S, as indicated by the heavy lines. This is for the unloaded condition. The section which is a part of both the arch and the cantilever is hereafter referred to as the common section. In figure 2-b external loads have been placed on the dam and the first trial load distribution made between arch and cantilever elements. Note that the cantilever has moved radially to S_c while the arch movement has a tangential

[1]Westergaard, H. M., "Water Pressures on Dams During Earthquakes," Trans. Am. Soc. C. E., vol. 98, 1933, pp. 418-472.

and a rotational component, bringing the arch section to S_a. All displacements are shown to a highly exaggerated scale. In figure 2-c the radial adjustment has been completed. Although radial deflections are equal, it is evident that the arch and cantilever are out of alinement, both tangentially and in rotation. The first is corrected by the tangential adjustment in which a tangential load is applied to the cantilever, displacing it to the left, and an equal and opposite load applied to the arch. The loads are proportioned by trial to bring the centers of the two common sections together as in figure 2-d.

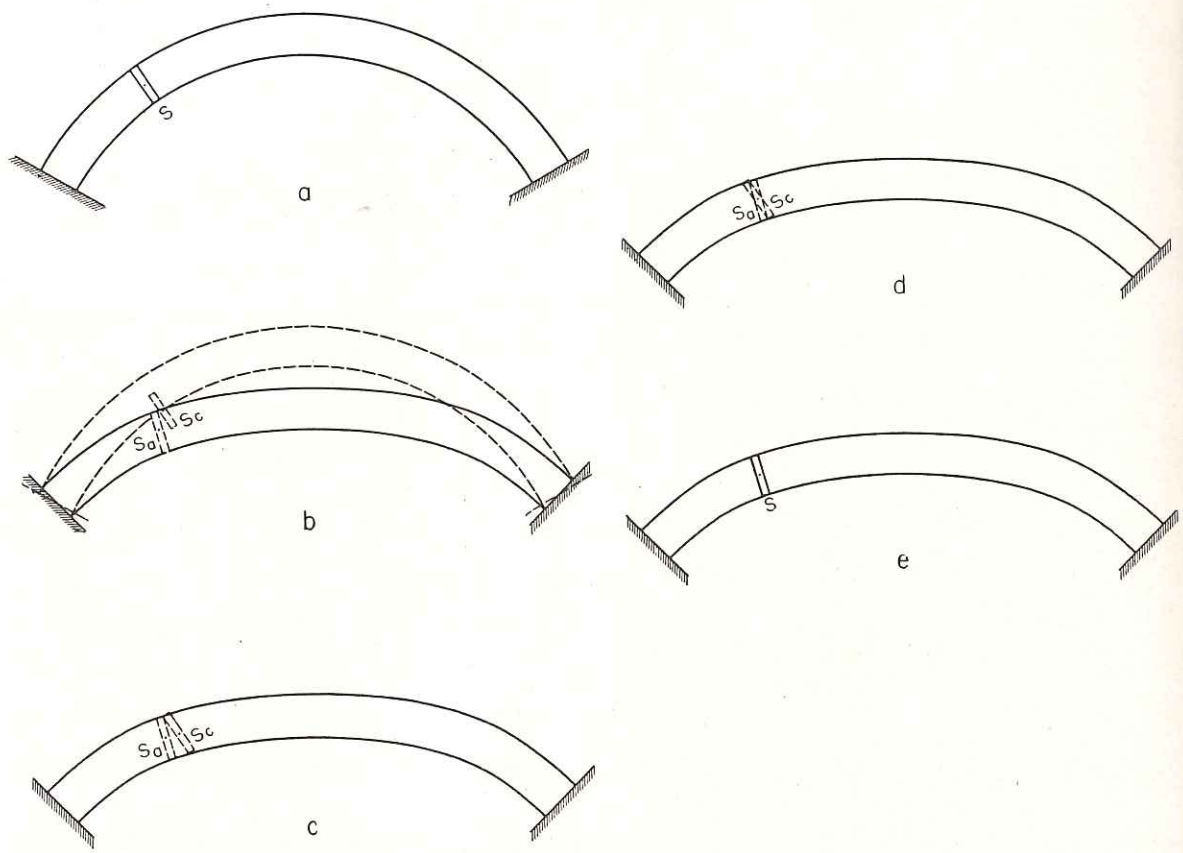


FIGURE 2—DEFORMATION OF A TYPICAL ARCH AND CANTILEVER

Next a twisting moment is applied to the cantilever and an opposing couple of equal magnitude applied as a bending moment on the arch. The magnitude of the couples is adjusted until the two common sections are in rotational agreement as in figure 2-e. This is the twist adjustment. The trial load adjustment is now complete and figure 2-e represents the loaded condition of the dam with the common sections again congruent as they were in figure 2-a.

In practice, the later adjustments throw the structure out of radial adjustment, so that a radial readjustment is necessary. In exceptional cases readjustments of tangential and twist loads are necessary. Roughly,

this is an outline of the steps necessary in a trial load analysis. The many details and modifications required in an actual analysis are discussed later. In order to simplify the computations as much as possible, both internal and external loads are represented by systems of unit loads.

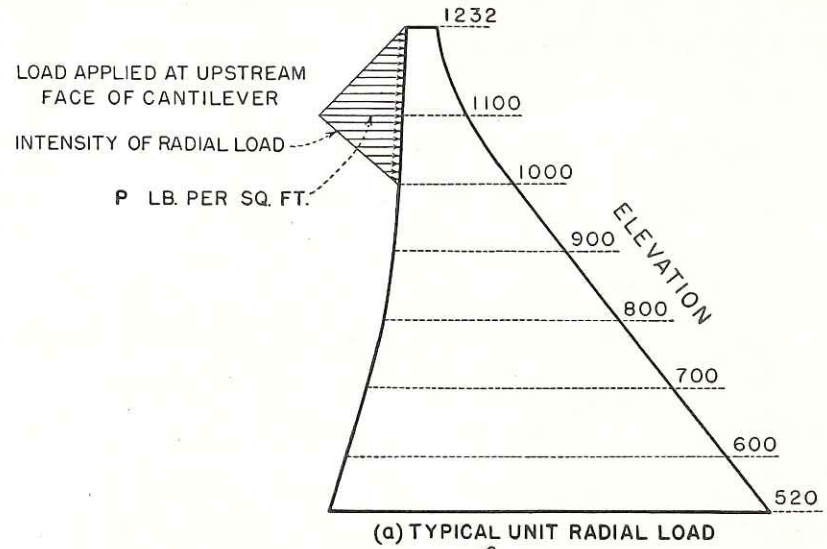
CANTILEVERS

25. Description.—Vertical cantilever elements are generally assumed to be one foot thick at the axis of the dam and to have radial sides converging from the upstream face to the downstream face. The cantilevers are elastic units, resting on elastic foundations. They resist vertical and radial forces applied at the upstream or downstream faces, tangential forces applied at the center lines, twisting and bending moments applied at the center lines, and weight. The last, being purely a cantilever loading, is considered only in the final computation of stresses.

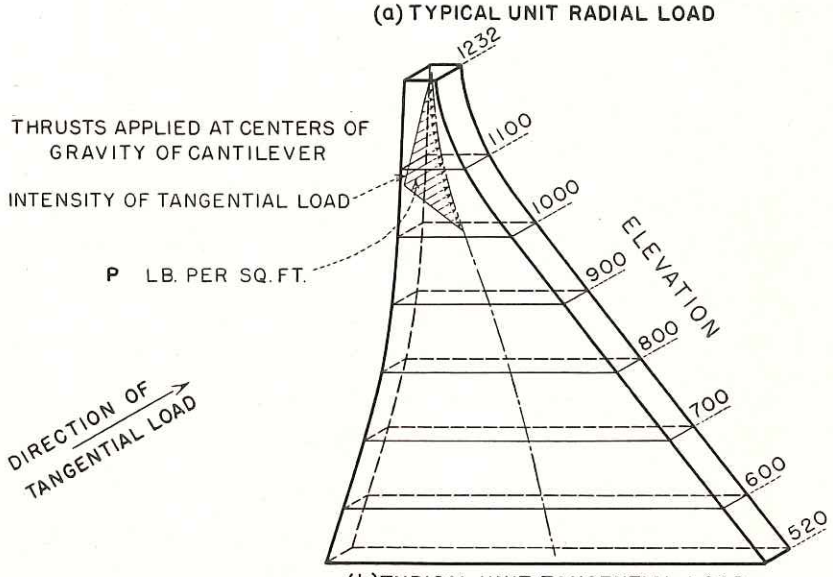
26. Unit Radial Loads.—Unit radial loads are used in applying radial forces carried by cantilevers. The unit loads are triangular in shape and vary from P, usually 1,000 pounds per square foot horizontal pressure, at one arch elevation to zero pressure at the sample arches directly above and below, see figure 3-a. With these loads it is possible to apply any horizontal force that varies as a straight line between successive elevations of sample arches. Shears and bending moments are computed for each unit load, and radial deflections due to each load are determined by the ordinary theory of flexure of beams with contributions from transverse shears included.

27. Cracked Cantilevers.—In thin arch dams, applied radial loads generally produce excessive tension in the cantilevers. Since concrete is capable of resisting only relatively small tension stresses, the cantilevers may be assumed to crack to the point of zero stress; so that the uncracked sections are entirely in compression. This assumption makes the cantilevers more flexible and transfers a greater portion of the water load to the arch elements, thus increasing the arch stresses. Cantilever stresses at the downstream face of the dam may also be increased, due to the reduced width of concrete available for transmitting cantilever forces. The assumed condition is somewhat analagous to that made in reinforced concrete beam theory.

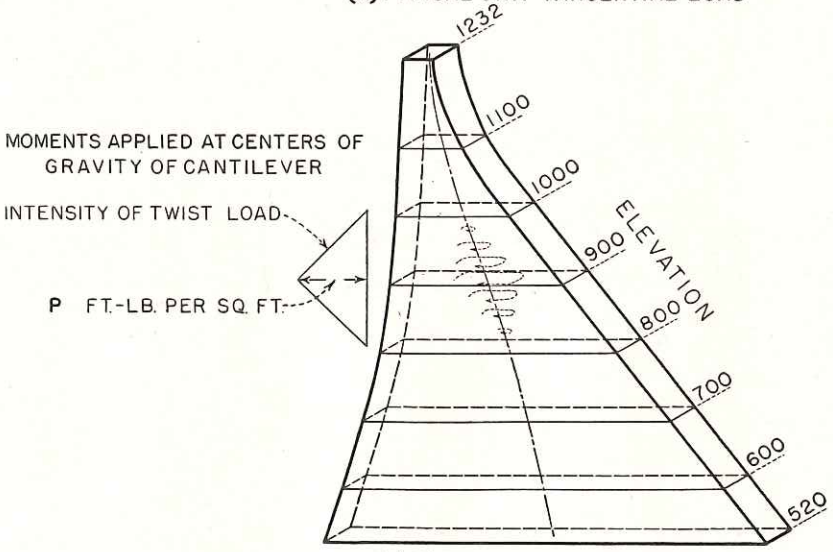
28. Unit Tangential Loads.—In the application of tangential loads, it is also convenient to use triangular unit loads. Unit tangential loads



(a) TYPICAL UNIT RADIAL LOAD



(b) TYPICAL UNIT TANGENTIAL LOAD



(c) TYPICAL UNIT TWIST LOAD

FIGURE 3—TYPICAL UNIT CANTILEVER LOADS

applied to cantilevers are similar to unit radial loads, except that they represent tangential shearing forces applied at the center lines instead of radial forces applied at the faces, see figure 3-b. In computing tangential deflections, tangential shear is assumed to be distributed uniformly through each horizontal cantilever section. Only deflections due to shear are evaluated. Tangential bending of cantilevers is not considered.

29. **Unit Twist Loads.**—As in the case of radial and tangential loads, triangular unit loads are used for twist loads. Unit cantilever twist loads represent twisting moments applied to cantilevers as shown in figure 3-c. Twisting moments act on cantilevers that are elements of a continuous structure. Therefore, shears set up by these moments act in tangential directions. They are assumed to have a linear variation from the upstream face to the downstream face. With this assumption, angular movements can be calculated by the formula used for twist in a continuous slab.[2]

30. **Secondary Movements.**—Both tangential and twist cantilever loads produce secondary movements that are generally large enough to require consideration. Tangential loads cause significant rotations in horizontal planes if the cantilever sections change considerably from the top of the dam to the base. These rotations exist because the center lines of the cantilevers are not vertical. Therefore, tangential thrusts applied near the top of a cantilever produce twisting moments in horizontal planes at all lower elevations. Twist loads cause appreciable secondary movements in radial directions because rates of change of applied twisting moments along arch center lines produce rates of change of bending moments in the cantilevers. These secondary effects are calculated after each trial and are included in the succeeding adjustments.

31. **Deflections of Unit Loads.**—After deflections are calculated for unit radial, tangential, and twist loads, the results are tabulated so they can be conveniently used in trial load adjustments. The tabulations also aid in estimating proper amounts of loads to apply to arches and cantilevers. If the cantilevers are assumed to crack, unit load deflections cannot be used because cantilever movements depend on the amount of cracking. In this case, it is necessary to calculate cantilever deflections for each trial load.

---

[2]Westergaard, H. M., "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, vol. 11, March 1930, pp. 1-23.

ARCHES

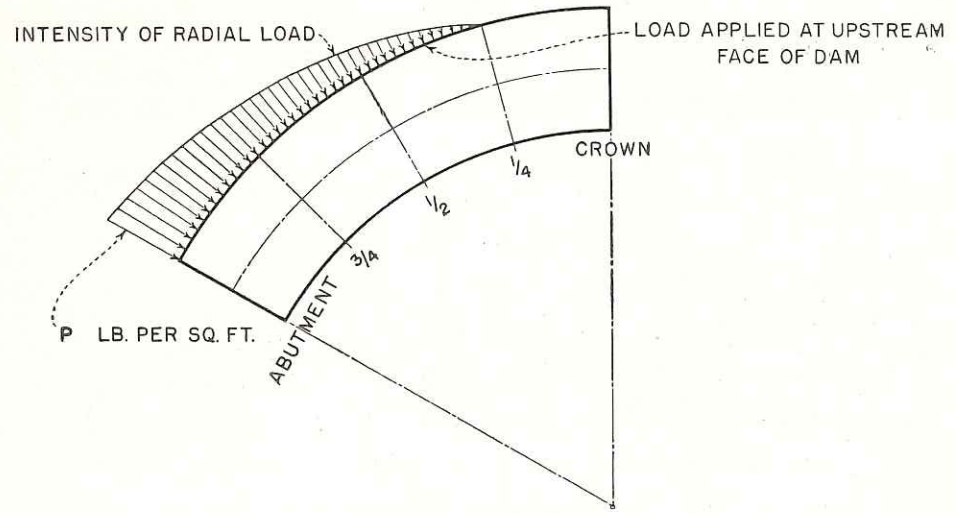
32. General Description.—Horizontal arch elements are usually assumed to be one foot high, with horizontal top and bottom faces and vertical upstream and downstream edges. Ordinarily, extrados and intrados curves are concentric circles; but fillets are often needed near the abutments at the downstream faces. The chapter on analyses of arch elements gives methods of solution for uniform thickness arches, nonuniform thickness arches, and arches with fillets, all arches having circular extrados curves. In addition, methods of solution are outlined for circular arches of the three-centered uniform thickness type and arches of the polycentered variable thickness type. For other types of arches the voussoir summation method may be used. Although ends of arch abutments are generally assumed to be radial, they may be assumed as nonradial and the analysis made on that basis.

The arches are statically indeterminate elements terminating at elastic abutments. They resist radial forces applied at the faces, tangential forces applied at the center lines, horizontal moments applied at the center lines, and twisting moments in vertical radial planes. Calculations of deflections of arch elements are made by the ordinary theory of flexure for curved beams, with effects of rib-shortening, transverse shear, and yielding abutments included.

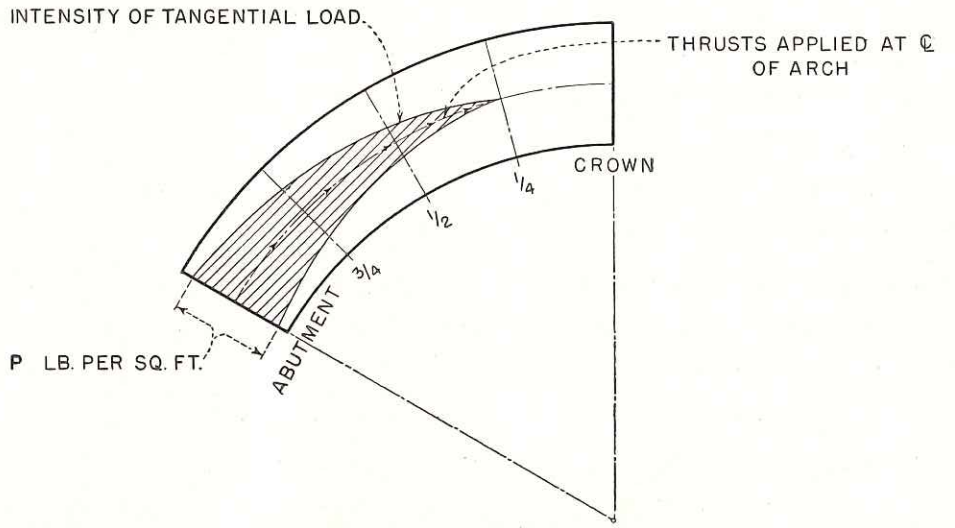
33. Unit Radial Loads.—A system of unit loads is used in applying the portion of the external radial loads resisted by the arch elements. Types of radial loads include a uniform load over the entire arch length and triangular loads varying from a maximum pressure at the abutment to zero pressure at the quarter points, as shown on figure 4-a. It is possible, with these unit loads, to apply any load that varies as a straight line between quarter points.

34. Unit Tangential Loads.—Loads consisting of tangential thrusts applied at arch center lines are required in adjustments of tangential deflections. Types of unit loads needed to build up total tangential loadings include a uniform load P, usually 1,000 pounds per square foot, and triangular loads varying from P pounds per square foot at the abutment to zero at the different quarter points, as shown in figure 4-b.

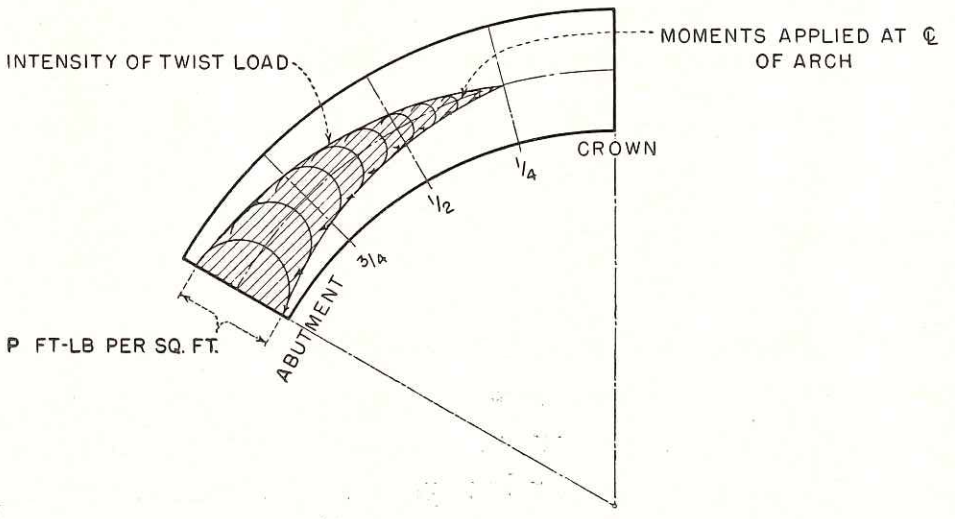
35. Unit Twist Loads.—Twist loads are required for establishing equal rotations in arches and cantilevers at conjugate points. Unit twist loads are also uniform and triangular in shape. The uniform load repre-



(a) TYPICAL UNIT RADIAL LOAD



(b) TYPICAL UNIT TANGENTIAL LOAD



(c) TYPICAL UNIT TWIST LOAD

FIGURE 4—TYPICAL UNIT ARCH LOADS

sents a moment of P foot-pounds per square foot, applied along the entire center line of the arch. Triangular loads represent moment loads varying from P foot-pounds per square foot at the abutment to zero at the quarter points, as shown in figure 4-c.

36. Concentrated Loads.—In addition to the loads just discussed, another set of loads is required by the assumption of elastic foundations and abutments. At the base of the crown cantilever of a symmetrical dam, the foundation is subjected to cantilever loads only. At the ends of the top arch the abutment is subjected to arch loads only. At all intermediate points the canyon wall acts both as an arch abutment and as a cantilever foundation. This is a direct corollary of the assumption that each system occupies the whole volume of the dam.

Both cantilever and arch loads act to deform canyon walls. That is, a part of the arch abutment movement is due to cantilever loads and a part of the cantilever foundation movement is due to arch loads. Figure 1 shows that cantilevers are located on the abutments of all sample arches. By this arrangement, the interaction of arch and cantilever loads on the abutment is brought into the analysis. Concentrated loads, representing shear, thrust and moment, are placed on the arch abutment. Using these loads, total forces and moments on the canyon wall at an arch abutment are determined. Abutment movements due to these total forces and moments are, therefore, abutment movements of corresponding arches and cantilevers. Concentrated radial, tangential, and twist loads are, respectively, a radial shear of P, usually 1,000 pounds, a tangential thrust of P pounds, and a twisting moment of P foot-pounds.

37. Temperature Loads.—Temperature changes in a dam are determined at elevations of sample arches and are assigned to arches as initial loads. A convenient unit of 1° Fahrenheit is used in applying temperature loads. Radial, tangential, and angular movements are calculated for the unit temperature change.

38. Secondary Movements.—Besides producing radial movements, radial loads cause secondary tangential and angular movements. Tangential loads cause secondary radial and angular movements, and twist loads produce secondary radial and tangential movements. In other words, each arch load, radial, tangential, twist, or temperature, produces radial, tangential, and angular movements that have to be considered in the trial load analysis. Consequently each adjustment is disrupted by loads of succeeding adjustments. However, secondary movements converge rapidly

with successive readjustments. More than a few readjustments are seldom required to secure satisfactory agreement of all deflections.

39. Cracked Arches.—Arch elements are ordinarily assumed to carry tension but may be computed as cracked arches. For this condition, concrete in tension is not considered to carry stress. Since this consideration requires the use of the more complicated voussoir summation method of arch analysis, in which tabulated unit loads cannot be used, cracked arches are computed only for unusually important cases.

TRIAL LOAD ADJUSTMENTS

40. General Statement.—In a trial load analysis, the dam is replaced by arch and cantilever systems. However, geometrical continuity must exist between the two systems at all points in the dam. This is accomplished by making radial deflections, tangential deflections, and angular movements in arches and cantilevers agree at conjugate points. Radial, tangential, and angular movements are considered respectively in radial, tangential, and twist adjustments.

Loads for which the analysis is made are first applied to the arch and cantilever structures as outlined in the preceding section on load conditions. The arch structure will have initial radial, tangential, and twist deflections due to temperature change. The cantilever structure will have initial radial deflections due to vertical water loads, and initial tangential deflections due to tangential earthquake forces, if included. All other external loads are included in the total radial loads. Starting with this partially loaded and deflected structure, the first adjustment is undertaken.

41. Radial Adjustment.—The analysis is started with the radial adjustment. This consists of dividing external radial loads between arches and cantilevers, by trial, with due consideration for effects of tangential shear and twist. The first step is to divide the horizontal component of the total radial load into two parts, one acting on the arches and the other on the cantilevers. In making the division the aim is to distribute the loads between the two systems, so that radial deflections of arches and cantilevers will be reasonably close at conjugate points.

42. Tangential Adjustment.—After radial deflections caused by radial loads have been adjusted, tangential movements are considered in the tangential adjustment. Tangential movements are tangential earthquake deflections of cantilevers and tangential movements of arches due

to temperature and to secondary movements resulting from the radial adjustment. In the tangential adjustment, equal and opposite tangential shear loads, one on the arch and the other on the cantilever, are introduced to remove relative tangential deflections of the elements. The required amounts of the loads are estimated and applied by successive trials to the two systems of elements.

43. Twist Adjustment.—In the next step of the analysis, rotations, or angular movements, of arch and cantilever elements about vertical axes are considered in the twist adjustment. A new set of loads is introduced, consisting of equal and opposite twist loads that act on arches and cantilevers. While these are termed twist loads, they are really horizontal couples which cause twisting of the cantilevers and bending of the arches.

Twist loads are required to remove relative angular movements of elements caused by radial and tangential loads. The amounts of the twist loads are estimated and gradually adjusted by trial. It will be noted that the twist adjustment corrects rotations about the vertical axis only. However, it can be demonstrated that tangential twisting moments are equal to vertical twisting moments. Also, if the structure is brought into twist adjustment in one direction and radial adjustment is maintained, then it is automatically in twist adjustment in the other direction. It has been found simpler to make the twist adjustment for rotations about the vertical axes. Horizontal twist effects on cantilever bending may be computed as shown in chapter VII.

44. Sequence of Adjustments.—After the twist adjustment is completed, as described in the preceding paragraph, radial movements due to tangential and twist loads are considered in the first radial readjustment. A tangential readjustment is then made to correct for discrepancies caused by twist loads and the first radial readjustment loads. Likewise, a twist readjustment and subsequent radial, tangential, and twist readjustments may be required, because each readjustment produces movements that have to be considered in succeeding readjustments. In most dams secondary effects converge rapidly; so that only a few readjustments are needed for the complete analysis. In exceptional cases, such as long flexible arches with short stiff cantilevers, the convergence is slow, and the process of adjustment may become long and tedious.

It has been found possible to materially lessen the number of readjustments required. After considerable experience, secondary movements can be estimated with fair accuracy before undertaking an adjustment. Estimated movements can then be allowed for in bringing arch and cantilever

structures into congruence; so that the complete series of adjustments will leave the structure in satisfactory agreement. Using this method, more than a single radial readjustment is seldom needed after completing the first radial, tangential, and twist adjustments. It should be mentioned that this procedure applies particularly to the normal case, where general characteristics of arch and cantilever structures are well known from preliminary studies.

When a completed structure is analyzed, another method may prove more effective. This consists in carrying each adjustment only to the point where errors in adjustment are within the range of estimated secondary movements. A saving in time results, even though several readjustments may be necessary, since approximate adjustments may be made rapidly. This method is often used for dams of unusual shapes, where no experience to aid in estimating secondary movements is available.

45. Adjustments for Poisson's Ratio.—Effects of Poisson's ratio may be analyzed after adjustments for external loads are completed. Poisson's ratio produces radial, tangential, and angular movements in both arches and cantilevers. Consequently, additional loads are needed to restore continuity throughout the structure. Separate adjustments for Poisson's ratio effects are usually made, using the same procedure of adjustments and readjustments as before.

RESULTS OF ANALYSIS

46. Movements.—Radial, tangential, and angular movements, determined by trial load adjustments, give displacements and rotations in the dam for assumed conditions of loading and temperature change. Effects of water soaking of the concrete at the upstream face, plastic flow, and nonlinear distribution of normal stresses are not included.

47. Stresses.—Trial load adjustments determine total loads resisted by arch and cantilever elements. For these total loads, stresses throughout the dam may be computed, providing a definite variation of stress between upstream and downstream faces of the dam is assumed. To this end, three of the stresses, the vertical cantilever stress normal to a horizontal plane, the horizontal arch stress normal to a vertical radial plane, and the horizontal tangential shear stress acting on a horizontal plane, are assumed to vary linearly between the upstream and downstream faces. As was implied earlier, a nonlinear distribution of stress across the dam might be assumed. In that case it would be necessary to carry the

same assumption through a complete additional series of adjustments, calculating arch and cantilever movements on the basis of such an assumption.

By means of the linear assumption, a direct computation of the three stresses mentioned above may be made, using total arch and cantilever loads. From these three stresses either principal stresses or stresses on any desired plane may be computed throughout the dam. Chapter VIII contains the formulas needed for such computations.

CHAPTER III—FOUNDATION AND ABUTMENT DEFORMATIONS

INTRODUCTION

48. General Statement.—At the time the trial load method was first developed, foundations and abutments of arch dams were considered rigid. Later developments included effects of elastic movements in the foundation and abutment rock. This chapter deals with the development of curves and formulas for foundation deformation. The inclusion of yielding abutments increases deflections of the dam as determined by a trial load analysis. Stresses are usually decreased at the abutments and foundations, but may be increased in other parts of the structure.

The changes required in the analysis are relatively slight. Elastic deformation of the foundation causes tilting, twisting, and displacement of arch abutments and cantilever bases. Since a cantilever normally rests on an arch abutment, the two move together as a single unit. It is not possible to compute movements of each separately, at least insofar as foundation loading and deformation are concerned. Foundation rotations and displacements must be computed separately for each trial combination of loads on the two elements. That is, any change in the loading of one element will affect foundation movements and hence the movements of both. It is necessary, therefore, to use the total abutment load from both members in computing foundation deformations. Being rigidly fastened to the foundation surface, both arch and cantilever are rotated and displaced as rigid bodies. These rigid body movements are added directly to the elastic deformations of the elements themselves, giving total rotations and displacements of the arch or cantilever at any point. Deformations of an element as used in the analysis are thus seen to be deformations of the element for a fixed base plus rigid body movements due to the foundation.

49. Assumptions.—Those familiar with foundation research will note several deficiencies in the procedure given on the following pages. It is recognized that the method of foundation analysis given here is hardly more than an approximate approach to the problem. A short discussion of the major assumptions involved serves to indicate the departure from actual conditions.

In addition to the usual assumptions of elasticity, the principal assumptions of the analysis are:

1. Foundation deformations are independent of the shape of foundation surface.

2. Movements of a unit foundation area are due only to loads applied directly on that area.

3. Water loads on the reservoir walls do not cause differential movements at the dam site.

Assumption 1 is utilized in the analysis in the following manner. Consider the surface of contact between dam and foundation developed upon a plane, as shown in figure 10-b. Take this plane as the surface of an infinite elastic solid. Deformations of this surface, after applying forces existing at the boundary of the dam, are then computed by means of the well known Boussinesq and Cerruti formulas. Computed movements of the developed surface are, of course, referred back to the abutment slopes of the actual structure before being used in the analysis.

Data on the effect of canyon shape on foundation deformations are meager, if not entirely lacking. It should be remembered that the loaded area is of the same order of magnitude as the canyon. Consequently the influence of the curvature of the canyon on deformations can hardly be considered negligible. An approximate analysis of the effects of loads applied to the sides of a U-shaped canyon on displacements at the bottom and along the sides indicates that assumption 1 departs rather widely from actual conditions.

This structural picture of the foundation is further modified by assumption 2 which makes each unit foundation area independent of the rest of the foundation. The dam is now supported by what is essentially a series of independent springs, or, more accurately, by a series of independent infinite foundations, if such can be imagined. The surface of each such foundation is the tangent plane at the point of contact. The effect of this modification on assumption 1 is, in general, to increase movements of the foundation near the ends of the dam and to reduce movements near the center. This is readily seen from a consideration of a series of uniform loads. For assumption 2, the foundation is depressed equally under all loads of the series; but for assumption 1, using the Boussinesq and Cerruti formulas, the depression under central loads is greater than under end loads, due to the additive effect of adjacent loads upon the depression under any central load.

The third assumption merely disregards effects of reservoir water loads on foundations and abutments. In view of the degree of approxi-

mation involved in the first two assumptions, it appears hardly worth while to introduce further complications at the present time. When the direct foundation analysis is in a satisfactory state this secondary effect should be considered. Although these are the assumptions of major importance, others appear as details of computations are developed. One might be mentioned here, since it determines the magnitude of the computed movements; that is, movements of any unit abutment area are average movements of a rectangular area on an infinite foundation surface. This rectangle has a width equal to the width of the dam at the point in question, a fixed ratio of length to width, and supports the load intensity, exerted by the dam, acting on the unit abutment area. The ratio of length to width of the loaded rectangle being fixed, it is seen later that the only variables in displacement equations are elastic constants and the load intensity, see equations 14 and 15. This assumption will be treated again in the second part of the chapter.

The foregoing discussion can be restated simply. The dam is assumed supported by a series of independent springs. The elastic constants of the springs are determined by reference to deformations of an infinite foundation with a plane surface.

As stated previously, the assumptions listed are made for the purpose of utilizing known formulas for deformations of an infinite foundation with a plane surface. Basing his work on these formulas, Dr. Fredrik Vogt obtained equations for average deformations of a loaded rectangular area of the foundation surface.[3] These equations express deformations due to a bending moment in a plane normal to the surface, a force normal to the surface, and a tractive or shear force in the plane of the surface. To supplement these equations, a formula has been derived for average deformation of the foundation due to a twisting moment in the plane of the surface.

The next section is devoted to the evaluation of the deformation of a plane rectangular foundation surface for various types of loadings. For ease of computation, each fundamental foundation equation is transformed into an expression that includes a function of two variables, Poisson's ratio and the ratio of the dimensions of the loaded surface. This function is multiplied by two or more constants to give foundation deformation. In order that functions may be rapidly determined, curves have been developed covering the range of values usually encountered in concrete dams. The application of these deformation equations to the analysis is covered in the remainder of the chapter. Movements of the abutments

---

[3]Vogt, Dr. Fredrik, "Ueber die Berechnung der Fundamentdeformation," Det Norske Videnskaps Akademi, Oslo, 1925.

of unit horizontal elements and of the foundations of unit vertical elements are considered, and equations are derived for movements of the elements. The forms of the equations are then changed, so they can be included directly in computations for deflections of arches and cantilevers.

DEFORMATION OF FOUNDATION SURFACE

50. Notation.—The notation used in deformation equations for a plane rectangular foundation surface is given below. Numbers given in parentheses refer to equations in the paper cited above.

α' = average rotation in a plane normal to foundation surface due to bending moment load, (16).

β' = average deformation normal to foundation surface due to normal load, (6).

γ' = average deformation in plane of foundation surface due to tractive or shear load, (23).

δ' = average rotation in plane of foundation surface due to twisting moment load.

α'' = average rotation in a plane normal to foundation surface due to tractive load, (26).

γ'' = average deformation in plane of foundation due to bending moment load, (19-a).

μ = Poisson's ratio for foundation material.

E_R = modulus of elasticity of foundation material in direct stress.

p = normal or tractive load per unit area.

a = short dimension of loaded area ab.

b = long dimension of loaded area ab.

M = bending moment, in direction of a, per unit length of $b = pa^2/6$.

H = normal force per unit length of $b = pa$.

V = shear force, in direction of a, per unit length of $b = pa$.

M_T = twisting moment per unit length of $b = pa^2/6$.

51. Deformation Equations.—The original development was based on a uniform distribution of load in the direction of the long dimension, b, of a loaded rectangular area. In the direction of the short dimension, a, the distribution of loading is linear for moment loading and uniform for shear and normal loadings. Satisfying this condition, the average deformation of the area for any type of load is a function of the shape of the area, the load per unit area, and the elastic properties of the foundation material.

The following equations give average deformations in terms of these

variables. The shape of the area is expressed by the dimensions a and b; the load per unit area, by p; and the elastic properties, by μ, Poisson's ratio, and E_R, the modulus of elasticity in direct stress.

$$\alpha' = (1-\mu^2)\frac{p}{E_R}\cdot\frac{3}{\pi}\left[\frac{4}{3}\cdot\frac{\sqrt{a^2+b^2}-a}{b} - \frac{8}{9}\cdot\frac{(\sqrt{a^2+b^2})^3-(a^3+\tfrac{3}{2}b^3)}{a^2\,b} - \frac{8}{45}\cdot\frac{(\sqrt{a^2+b^2})^5-(a^5+b^5)}{a^4\,b} + \frac{4}{3}\cdot\frac{b}{a}\cdot\log_e\left(\frac{a+\sqrt{a^2+b^2}}{b}\right)\right] \quad (1)$$

$$\beta' = (1-\mu^2)\frac{p}{E_R}\cdot\frac{2}{\pi}\left[a\cdot\log_e\left(\frac{b+\sqrt{a^2+b^2}}{a}\right) + b\cdot\log_e\left(\frac{a+\sqrt{a^2+b^2}}{b}\right) - \frac{(\sqrt{a^2+b^2})^3-(a^3+b^3)}{3ab}\right] \quad (2)$$

$$\gamma' = (1-\mu^2)\frac{p}{E_R}\cdot\frac{2}{\pi}\left[\left\{a\cdot\log_e\left(\frac{b+\sqrt{a^2+b^2}}{a}\right) + b\cdot\log_e\left(\frac{a+\sqrt{a^2+b^2}}{b}\right) - \frac{(\sqrt{a^2+b^2})^3-(a^3+b^3)}{3ab}\right\} + \frac{\mu}{1-\mu}\left\{b\cdot\log_e\left(\frac{a+\sqrt{a^2+b^2}}{b}\right) - \frac{(\sqrt{a^2+b^2})(2b^2-a^2)-(2b^3-a^3)}{3ab}\right\}\right] \quad (3)$$

$$\delta' = \frac{p}{E_R}\cdot\frac{1}{30\pi}\left[\left(90 - \frac{24a}{b} + \frac{5a^2}{b^2} - \frac{3a^4}{8b^4} + \frac{9a^6}{112\,b^6}\right) + \mu\left(90 - \frac{48a}{b} + \frac{15a^2}{b^2} - \frac{15a^4}{8b^4} + \frac{63a^6}{112\,b^6}\right) - \mu^2\left(\frac{24a}{b} - \frac{10a^2}{b^2} + \frac{12a^4}{8b^4} - \frac{54a^6}{112b^6}\right)\right] \quad (4)$$

$$\alpha'' = (1-\mu-2\mu^2)\frac{p}{E_R}\left[1 - \frac{1}{2\pi}\left\{4\cdot\frac{a^2+b^2}{a^2}\cdot\tan^{-1}\left(\frac{a}{b}\right) - \frac{3b}{a} - \left(\frac{b}{a}\right)^3\log_e\left(\frac{a^2+b^2}{b^2}\right) + \frac{a}{b}\cdot\log_e\left(\frac{a^2+b^2}{a^2}\right)\right\}\right] \quad (5)$$

$$\gamma'' = (1-\mu-2\mu^2)\frac{p}{E_R}\cdot\frac{a}{6}\left[1 - \frac{1}{2\pi}\left\{4\cdot\frac{a^2+b^2}{a^2}\cdot\tan^{-1}\left(\frac{a}{b}\right) - \frac{3b}{a} - \left(\frac{b}{a}\right)^3\log_e\left(\frac{a^2+b^2}{b^2}\right) + \frac{a}{b}\cdot\log_e\left(\frac{a^2+b^2}{a^2}\right)\right\}\right] \quad (6)$$

These equations may be transformed to a function of two variables, μ and the ratio b/a multiplied by two or more constants. The constants are $1/E_R$, p, the dimension a, and such numerical constants as are present.

The transformed equations, with the ratio b/a replaced by x for convenience, are

$$\alpha' = (1-\mu^2)\frac{p}{E_R}\cdot\frac{4}{\pi}\left[\frac{(\sqrt{1+x^2})(3-14x^2-2x^4)+15x^3+2x^5-3}{15x} + x\cdot\log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right)\right] \quad (7)$$

$$\beta' = (1-\mu^2)\frac{p}{E_R}\cdot\frac{2}{\pi}\cdot a\left[\log_e(x+\sqrt{1+x^2}) + x\cdot\log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right) - \frac{(\sqrt{1+x^2})^3-1-x^3}{3x}\right] \quad (8)$$

$$r' = \frac{p}{E_R}\cdot\frac{2}{\pi}\cdot a\left[(1-\mu^2)\left\{\log_e(x+\sqrt{1+x^2})+x\cdot\log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right)-\frac{(\sqrt{1+x^2})^3-1-x^3}{3x}\right\}\right.$$
$$\left.+(\mu+\mu^2)\left\{x\cdot\log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right)-\frac{(\sqrt{1+x^2})(2x^2-1)-2x^3+1}{3x}\right\}\right] \quad (9)$$

$$\delta' = \frac{p}{E_R}\cdot\frac{1}{30\pi}\left[\left(90-\frac{24}{x}+\frac{5}{x^2}-\frac{3}{8x^4}+\frac{9}{112x^6}\right)+\mu\left(90-\frac{48}{x}+\frac{15}{x^2}-\frac{15}{8x^4}+\frac{63}{112x^6}\right)\right.$$
$$\left.-\mu^2\left(\frac{24}{x}-\frac{10}{x^2}+\frac{12}{8x^4}-\frac{54}{112x^6}\right)\right] \quad (10)$$

$$\alpha'' = (1-\mu-2\mu^2)\ \frac{p}{E_R}\left[1-\frac{1}{2\pi}\left\{4(1+x^2)\tan^{-1}\left(\frac{1}{x}\right)-3x-x^3\cdot\log_e\left(\frac{1+x^2}{x^2}\right)+\frac{1}{x}\cdot\log_e(1+x^2)\right\}\right] \quad (11)$$

$$r'' = (1-\mu-2\mu^2)\ \frac{p}{E_R}\cdot\frac{a}{6}\left[1-\frac{1}{2\pi}\left\{4(1+x^2)\tan^{-1}\left(\frac{1}{x}\right)-3x-x^3\cdot\log_e\left(\frac{1+x^2}{x^2}\right)+\frac{1}{x}\cdot\log_e(1+x^2)\right\}\right] \quad (12)$$

If p is replaced by the forces or moments causing the deformation, see notation, and, if the product of the numerical constants and the expression containing b/a, or x, be symbolized by capital letters representing that product for definite values of a and b, the equations assume the following contracted forms.

$$\alpha' = \frac{M}{E_R a^2}\left[(1-\mu^2)A\right] \quad (13)$$

$$\beta' = \frac{H}{E_R}\left[(1-\mu^2)B\right] \quad (14)$$

$$r' = \frac{V}{E_R}\left[(1-\mu^2)B+(\mu+\mu^2)C\right] \quad (15)$$

$$\delta' = \frac{M_T}{E_R a^2}\left[E+\mu F-\mu^2 G\right] \quad (16)$$

$$\alpha'' = \frac{V}{E_R a}\left[(1-\mu-2\mu^2)D\right] \quad (17)$$

$$r'' = \frac{M}{E_R a}\left[(1-\mu-2\mu^2)D\right] \quad (18)$$

The expressions in brackets are functions of μ and b/a. Replacing these functions by k and making a equal to T, width of dam, equations 13 to 18 become, for moments and forces of unity, per unit length,

$$\alpha' = \frac{k}{E_R T^2} \tag{19}$$

$$\beta' = \frac{k}{E_R} \tag{20}$$

$$\gamma' = \frac{k}{E_R} \tag{21}$$

$$\delta' = \frac{k}{E_R T^2} \tag{22}$$

$$\alpha'' = \frac{k}{E_R T} \tag{23}$$

$$\gamma'' = \frac{k}{E_R T} \tag{24}$$

Note that expressions represented by k are different except for equations 23 and 24.

52. Curves.—Values of k were computed for a sufficient number of combinations of μ and b/a, or x, to determine the curves in figures 5, 6, 7, 8, and 9. It should be noted that only one series of curves is needed for equations 23 and 24, their k values being identical.

The diagrams on figures 5 to 9, inclusive, show a unit element whose dimension, T, is less than the dimension a of the total area. This was done in an attempt to clarify the application of the curves to the analysis of the unit elements of a dam as described in the following section.

The primary deformations are α', β', γ', and δ'. Deformations α'' and γ'' are secondary in character and are of small magnitude compared to primary deformations α' and γ'.

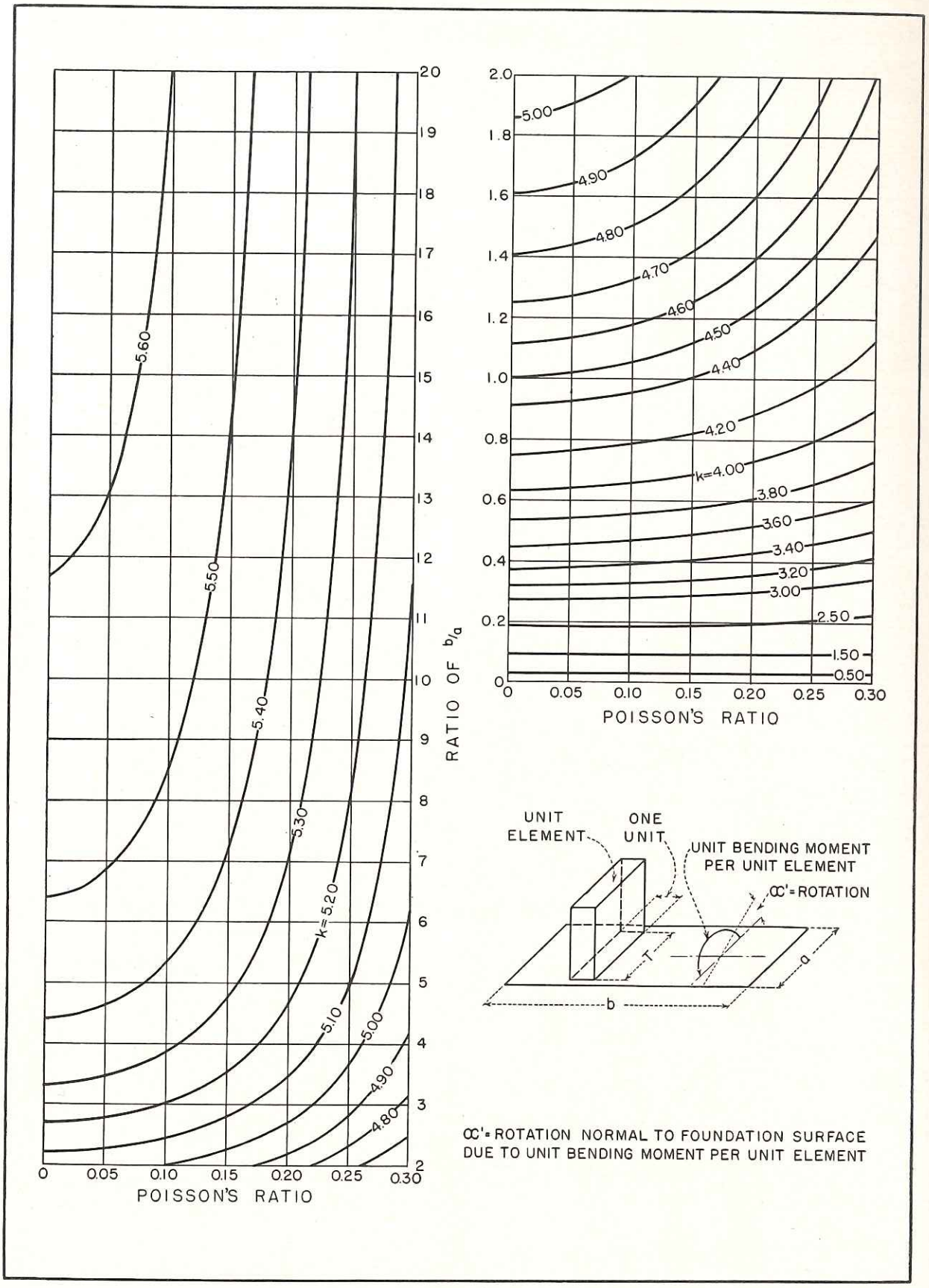


FIGURE 5—VALUES OF k IN EQUATION 19

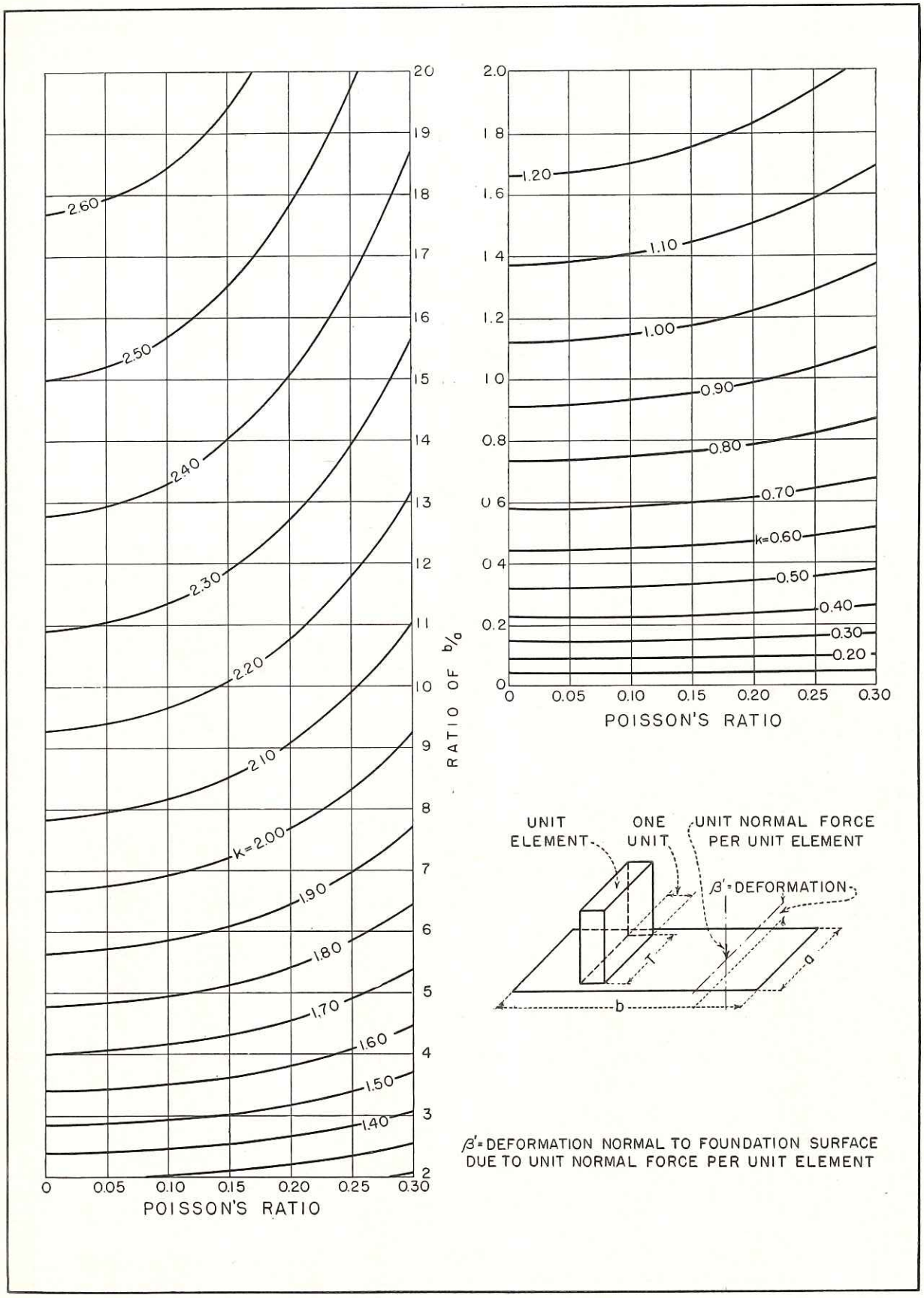


FIGURE 6—VALUES OF k IN EQUATION 20

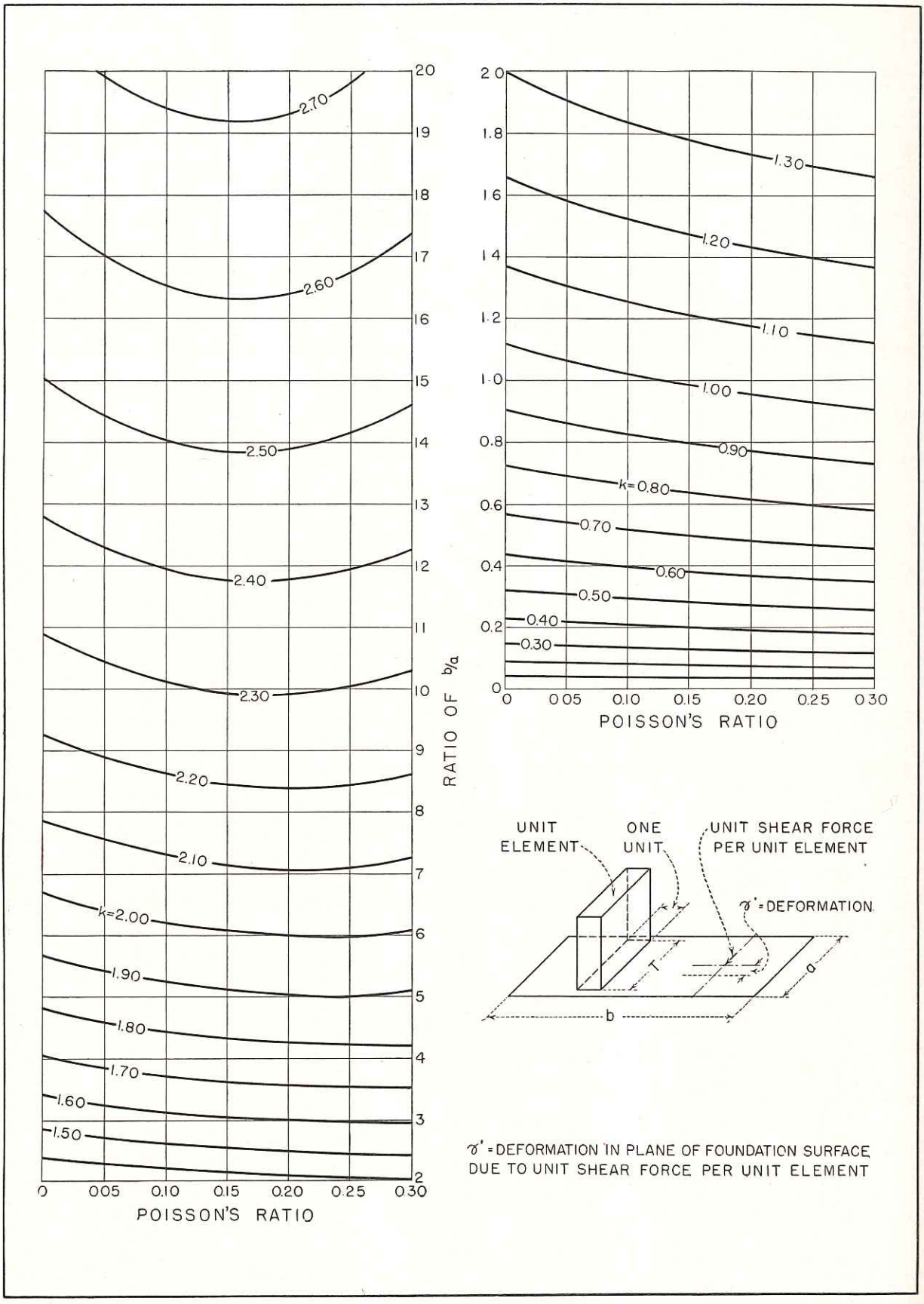


FIGURE 7—VALUES OF k IN EQUATION 21

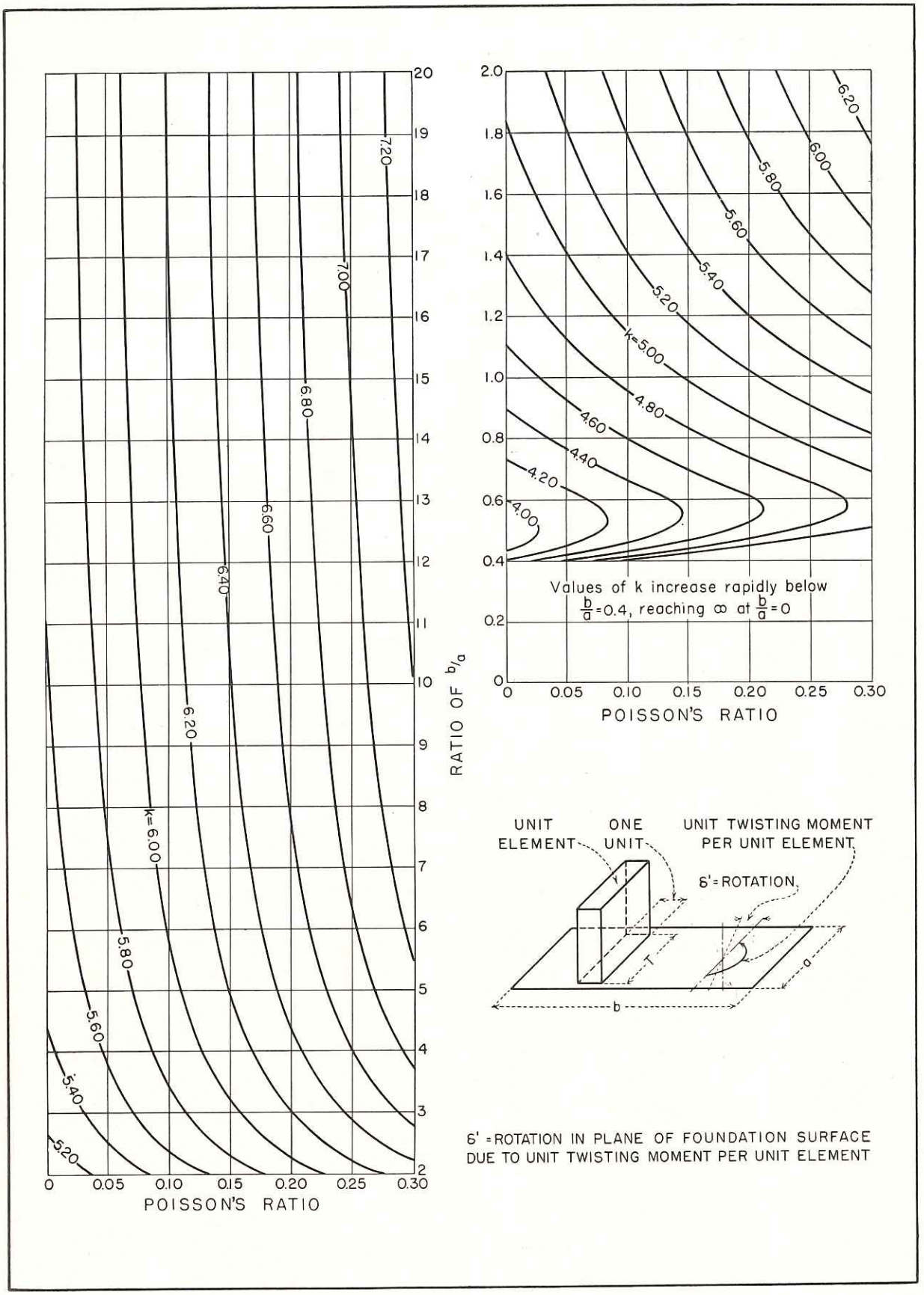


FIGURE 8—VALUES OF k IN EQUATION 22

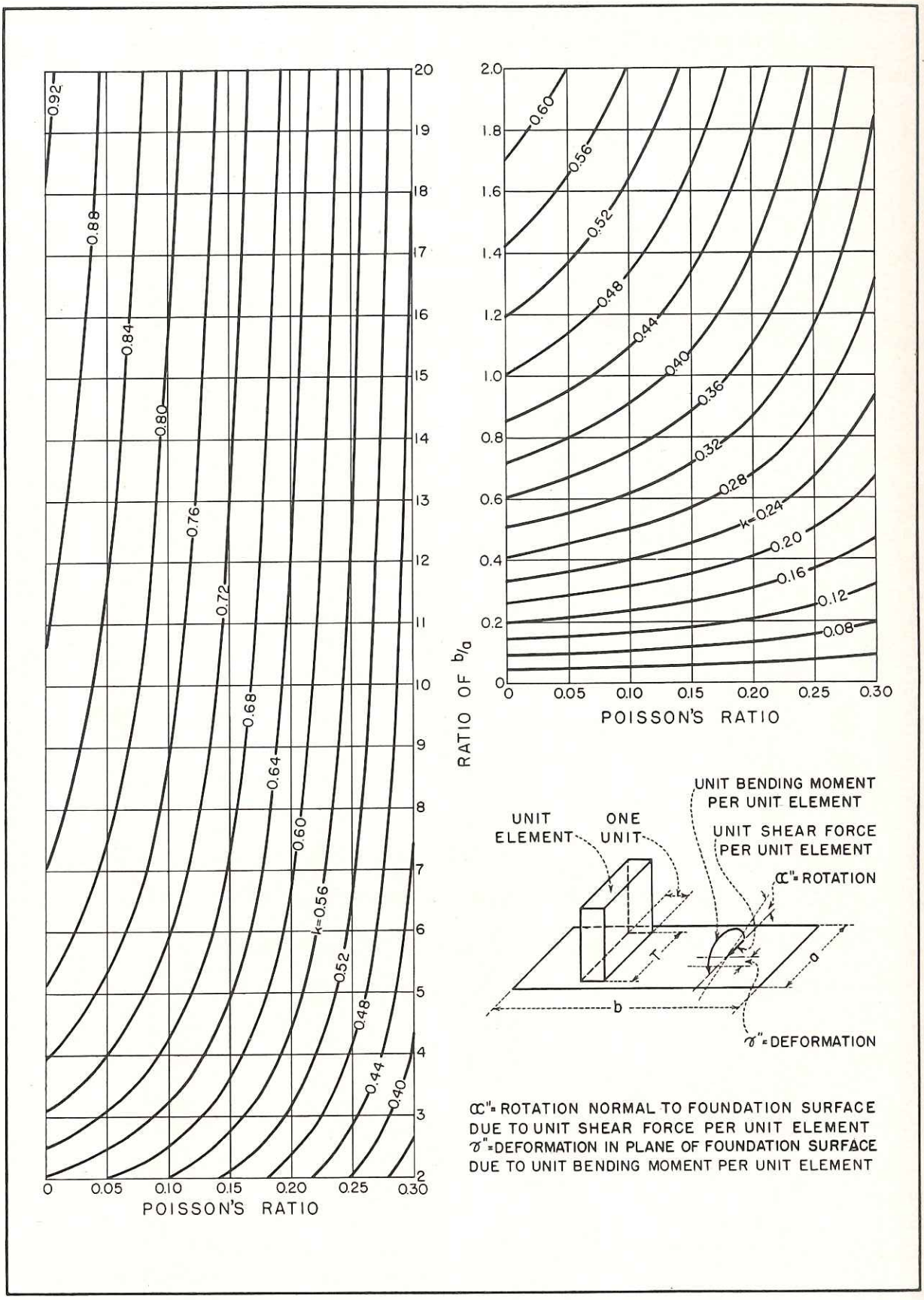


FIGURE 9—VALUES OF k IN EQUATIONS 23 AND 24

MOVEMENTS OF UNIT ELEMENTS

53. Notation.—The following list gives additional notation needed in considering movements of unit horizontal and vertical elements of a dam with a loaded foundation surface as shown in figure 10. Symbols refer to the left side of the dam. Reference planes and directions are indicated in figures 11, 12, and 13.

M_x = moment in vertical plane.

M_y = moment in tangential plane.

M_z = moment in horizontal plane.

H = tangential force.

V = horizontal force in vertical plane.

W = vertical force.

$\textcircled{\alpha'}$, $\textcircled{\gamma'}$, $\textcircled{\alpha''}$, and $\textcircled{\gamma''}$ are at right angles to the directions of the foundation deformations not designated by the circles, see figure 13-b.

θ_x = angular movement in vertical plane.

θ_y = angular movement in tangential plane.

θ_z = angular movement in horizontal plane.

Δx = tangential movement.

Δy = horizontal movement in vertical plane.

Δz = vertical movement.

ψ = angle between vertical plane and plane of foundation surface at its intersection with the unit element.

$\perp$ = perpendicular to foundation surface.

$\parallel$ = parallel to, or in plane of foundation surface.

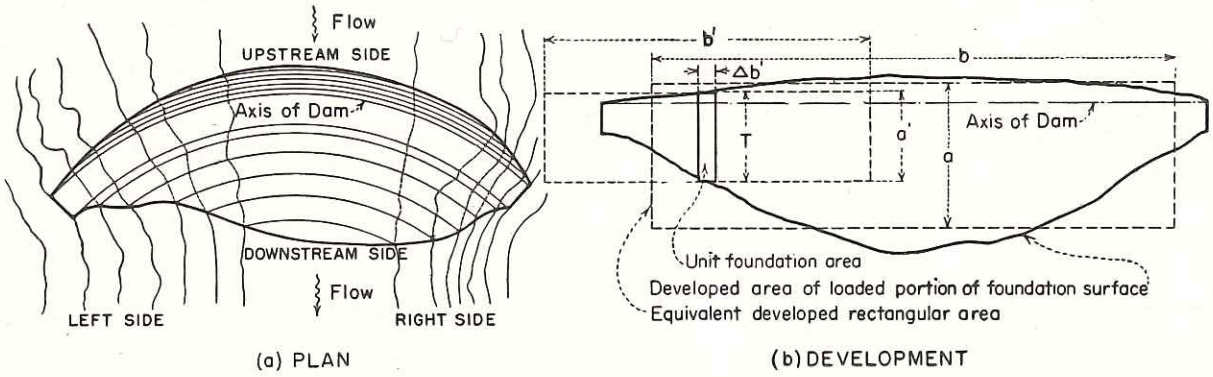
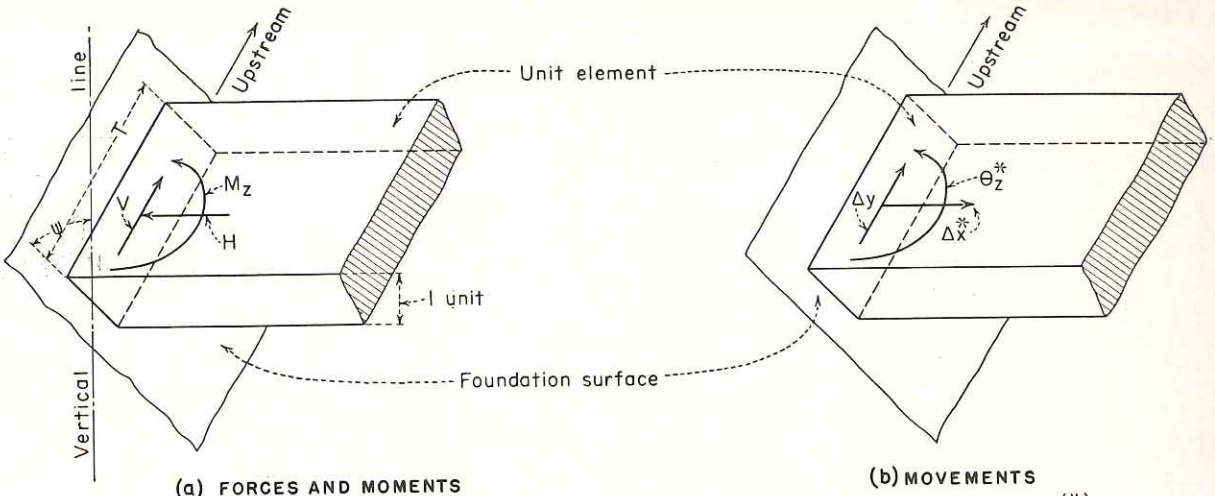
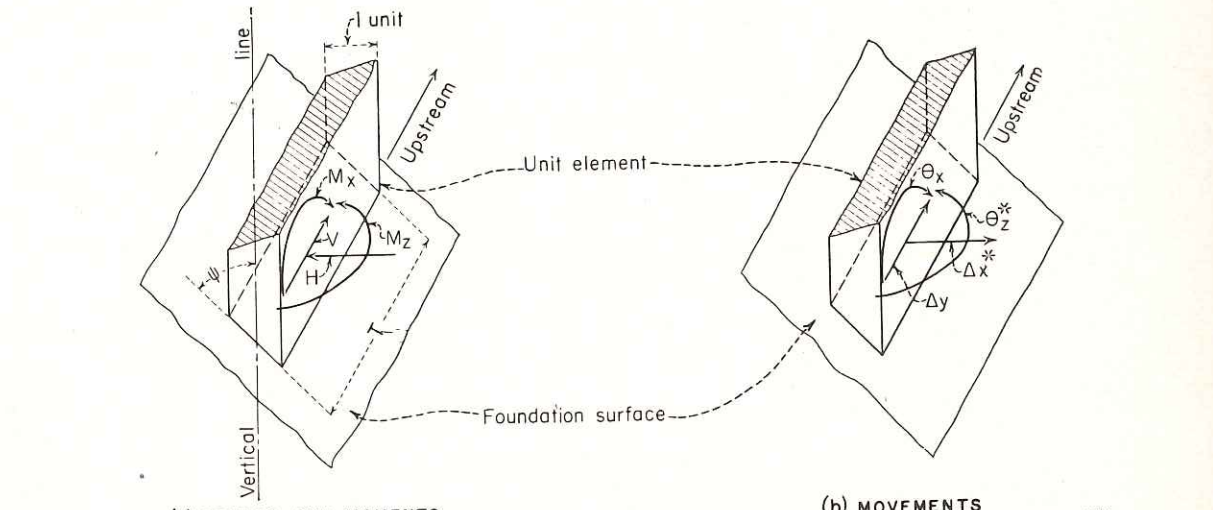


FIGURE 10—LOADED AREA OF FOUNDATION SURFACE



Note: Positive Forces, Moments, and Movements have same direction with respect to right abutment except (*) which are opposite; for example, at right abutment, positive H acts toward the abutment.

FIGURE 11—UNIT HORIZONTAL ELEMENT AT LEFT ABUTMENT



Note: Positive Forces, Moments, and Movements have same direction with respect to right abutment except (*) which are opposite, for example, at right abutment, positive H acts toward the abutment.

FIGURE 12—UNIT VERTICAL ELEMENT AT LEFT ABUTMENT

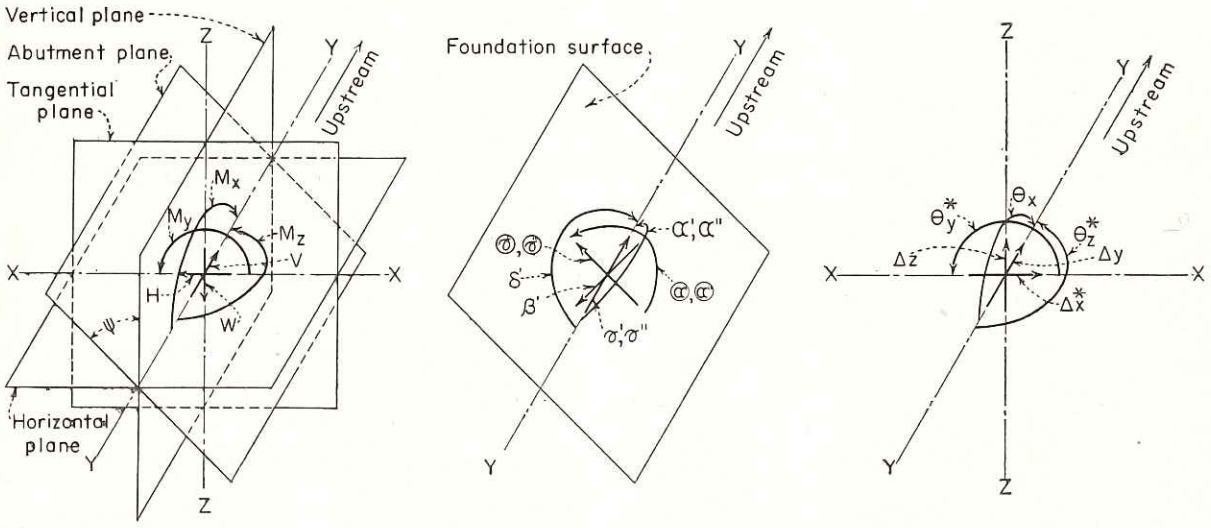
Deformation equations of the preceding section contain elastic constants E_R and μ, the ratio b/a, and the load intensity applied either normally or tangentially to the foundation plane. Of these, elastic constants are ordinarily determined by direct experimental methods.

As formulated, conditions of the problem do not fix a definite value for the ratio b/a. An assumption of some kind is necessary. At present, the value of b/a, is determined in the following manner. Consider the surface of contact between dam and foundation developed and plotted, as in figure 10-b, with the axis as a straight line. This is replaced by a rectangle of the same area and approximately the same proportions, called the "equivalent developed area." The ratio of length to width of this rectangle is taken as the b/a ratio for the foundation in question. The value of b/a is, therefore, a constant for any one dam. In computing

deformations, the width a' in b/a is made equal to T, the thickness of the dam at the element considered, making b' equal to $(b/a)T$, see $a'b'$ in figure 10-b.

The reasoning behind this is simple. Movements of the equivalent developed area are assumed equal to movements of the actual foundation. Then, by taking the ratio, length/width, of this equivalent area as the b/a value throughout, the average of the deformation of all elements equals the average deformation of the equivalent developed area. This follows directly from the fact that the only variables in the equations, exclusive of moment effects, are the b/a ratio and the load intensity.

In this discussion, loads have been considered as acting normally or tangentially to the foundation surface. Forces and moments in the dam, however, have all been measured with reference to radial, tangential, and horizontal planes, see figure 13-a. It is necessary, then, to resolve these forces and moments into normal and tangential components acting on the foundation. For this purpose, contributions of horizontal and vertical elements are considered separately, see figures 11-a and 12-a. Forces and moments transmitted by the unit element to the foundation surface are resolved into forces and moments on the unit differential area, acting both normal to and in the abutment plane. These resolved forces and moments produce deformations and rotations of the unit differential area, both normal to and in the abutment plane, as shown in figure 13-b. Deformations and rotations are resolved into components in principal planes of



(a) POSITIVE FORCES AND MOMENTS (b) FOUNDATION DEFORMATIONS (c) POSITIVE MOVEMENTS

Note: Positive Forces, Moments, and Movements have same direction with respect to right abutment except (*) which are opposite, for example, at right abutment, positive H acts toward the abutment.

FIGURE 13—ABUTMENT OF LEFT SIDE OF DAM

movements, and the summation of the components in any plane or direction is the net movement of the element in that plane or direction.

TABLE 1—MOVEMENTS OF HORIZONTAL ELEMENT DUE TO UNIT FORCES AND MOMENTS AT ABUTMENT OF ELEMENT. LEFT SIDE OF DAM

FORCE OR MOMENT	RESOLVED TO UNIT ABUTMENT SURFACE		DEFORMATION OR ROTATION OF UNIT ABUTMENT SURFACE		MOVEMENT OF UNIT ELEMENT *MOVEMENT HAS OPPOSITE SIGN FOR RIGHT SIDE OF DAM					
					Θ_x	Θ_y^*	Θ_z^*	Δ_x^*	Δy	Δz
M_x	⊥	$M_x \sin^2\psi$	⊥	$(M_x \sin^2\psi)\alpha'$	$+(M_x \sin^3\psi)\alpha'$		$+(M_x \sin^2\psi \cos\psi)\alpha'$		$+(M_x \sin^2\psi)\sigma''$	
	∥	$M_x \sin\psi \cos\psi$	∥	$(M_x \sin^2\psi)\tau''$						
	∥	$M_x \sin\psi \cos\psi$	∥	$(M_x \sin\psi \cos\psi)\delta'$	$+(M_x \sin\psi \cos^2\psi)\delta'$		$-(M_x \sin^2\psi \cos\psi)\delta'$			
M_y	⊥	$M_y \sin\psi$	⊥	$(M_y \sin\psi)\alpha'$		$+(M_y \sin\psi)\alpha'$		$-(M_y \sin^2\psi)\sigma'$		$+(M_y \sin\psi \cos\psi)\tau'$
			∥	$(M_y \sin\psi)\sigma''$						
M_z	⊥	$M_z \sin\psi \cos\psi$	⊥	$(M_z \sin\psi \cos\psi)\alpha'$	$+(M_z \sin^2\psi \cos\psi)\alpha'$		$+(M_z \sin\psi \cos^2\psi)\alpha'$		$+(M_z \sin\psi \cos\psi)\tau''$	
	∥	$M_z \sin^2\psi$	∥	$(M_z \sin\psi \cos\psi)\tau''$						
	∥	$M_z \sin^2\psi$	∥	$(M_z \sin^2\psi)\delta'$	$-(M_z \sin^2\psi \cos\psi)\delta'$		$+(M_z \sin^3\psi)\delta'$			
H	⊥	$H \sin\psi \cos\psi$	⊥	$(H \sin\psi \cos\psi)\beta'$				$-(H \sin\psi \cos^2\psi)\beta'$		$-(H \sin^2\psi \cos\psi)\beta'$
	∥	$H \sin^2\psi$	∥	$(H \sin^2\psi)\sigma'$				$-(H \sin^3\psi)\sigma$		$+(H \sin^2\psi \cos\psi)\tau$
			⊥	$(H \sin^2\psi)\alpha$		$+(H \sin^2\psi)\alpha$				
V	∥	$V \sin\psi$	∥	$(V \sin\psi)\tau'$					$+(V \sin\psi)\tau'$	
			⊥	$(V \sin\psi)\alpha''$	$+(V \sin^2\psi)\alpha''$		$+(V \sin\psi \cos\psi)\alpha''$			
W	⊥	$W \sin^2\psi$	⊥	$(W \sin^2\psi)\beta'$				$-(W \sin^2\psi \cos\psi)\beta'$		$-(W \sin^3\psi)\beta'$
	∥	$W \sin\psi \cos\psi$	∥	$(W \sin\psi \cos\psi)\tau$				$+(W \sin^2\psi \cos\psi)\tau$		$-(W \sin\psi \cos^2\psi)\tau$
			⊥	$(W \sin\psi \cos\psi)\alpha''$		$-(W \sin\psi \cos\psi)\alpha$				

54. Unit Horizontal Element.—Table 1 gives components of the forces and moments of a horizontal element resolved to the unit abutment surface, the deformations and rotations of the unit abutment surface, and the resulting movements of the unit horizontal element.

Net movements of the unit horizontal element, written in descending order of magnitude, are given by the following equations:

$$\Theta_x = M_x\left[\delta'\cos^3\psi + \alpha'\sin^2\psi\cos\psi\right] + V\left[\alpha''\sin\psi\cos\psi\right] + M_z\left[(\alpha'-\delta')\sin\psi\cos^2\psi\right] \tag{25}$$

$$\Theta_y^* = M_y\left[\alpha\cos\psi\right] - W\left[\alpha''\cos^2\psi\right] + H\left[\alpha''\sin\psi\cos\psi\right] \tag{26}$$

$$\Theta_z^* = M_z\left[\alpha'\cos^3\psi + \delta'\sin^2\psi\cos\psi\right] + V\left[\alpha''\cos^2\psi\right] + M_x\left[(\alpha'-\delta')\sin\psi\cos^2\psi\right] \tag{27}$$

$$\Delta x^* = -H\left[\beta'\cos^3\psi + \tau\sin^2\psi\cos\psi\right] - M_y\left[\tau''\sin\psi\cos\psi\right] - W\left[(\beta'-\tau)\sin\psi\cos^2\psi\right] \tag{28}$$

$$\Delta y = V\left[\tau'\cos\psi\right] + M_z\left[\tau''\cos^2\psi\right] + M_x\left[\tau''\sin\psi\cos\psi\right] \tag{29}$$

$$\Delta z = -W\left[\tau\cos^3\psi + \beta'\sin^2\psi\cos\psi\right] + M_y\left[\tau''\cos^2\psi\right] - H\left[(\beta'-\tau)\sin\psi\cos^2\psi\right] \tag{30}$$

---

*Indicates movement has opposite sign for right side of dam, see figure 13.

When the unit horizontal element is one of a contiguous series in a dam, certain equations of compatibility or continuity that must be satisfied definitely exclude the use of some of the terms or movements. Others may be eliminated because of their negligibility in the practical analysis of stresses and deflections in a dam.

Since a vertical adjustment is not made, θ_y and M_y are neglected. This excludes equation 26.

θ_x and Δz have a negligible effect on the behavior of a unit horizontal element and are neglected in the present methods of analysis. This excludes equations 25 and 30.

M_x in equations 27 and 29 and W in equation 28 have very little influence on the movement of a unit horizontal element and are neglected in the present methods of analysis.

With these eliminations, the equations are now,

$$\theta_z^* = M_z\left[\alpha' \cos^3\psi + \delta' \sin^2\psi \cos\psi\right] + V\left[\alpha'' \cos^2\psi\right] \tag{31}$$

$$\Delta x^* = -H\left[\beta' \cos^3\psi + \textcircled{\gamma'} \sin^2\psi \cos\psi\right] \tag{32}$$

$$\Delta y = V\left[\gamma' \cos\psi\right] + M_z\left[\gamma'' \cos^2\psi\right] \tag{33}$$

It is customary to require that β', δ' and $\textcircled{\gamma}$, see figure 13-b, of a unit differential area on one side of the dam be average values for the equivalent developed area of that side of the dam. If the dam site is approximately symmetrical about the maximum section, dimensions of the equivalent developed area for either or both sides of the dam are a and $\frac{b}{2}$. For this reason, ratios $\frac{b/2}{a}$ and $\frac{a}{b/2}$ are substituted for the ratio b/a in some cases in obtaining values from the curves in figures 5 to 9, inclusive. The ratios used are as follows:

For α', α'', γ', and γ'' use ratio $\frac{b}{a}$.

For β' and δ' use ratio $\frac{b/2}{a}$.

For $\textcircled{\gamma}$ use ratio $\frac{a}{b/2}$.

By substituting symbols for bracketed quantities, and using the identity $\alpha'' = \gamma''$, equations 31, 32, and 33 become

$$\theta_z^{*} = M_z \alpha + V \alpha_2 \tag{34}$$

$$\Delta x^{*} = -H\beta \tag{35}$$

$$\Delta y = V\tau + M_z \alpha_2 \tag{36}$$

in which

$$\alpha = \alpha' \cos^3 \psi + \delta' \sin^2 \psi \cos \psi$$

$$\alpha_2 = \alpha'' \cos^2 \psi$$

$$\beta = \beta' \cos^3 \psi + (\tau') \sin^2 \psi \cos \psi$$

$$\tau = \tau' \cos \psi$$

The application of these movements to the analysis of an arch is given in chapter V.

TABLE 2—MOVEMENTS OF VERTICAL ELEMENT DUE TO UNIT FORCES AND MOMENTS AT ABUTMENT OF ELEMENT. LEFT SIDE OF DAM

FORCE OR MOMENT	RESOLVED TO UNIT ABUTMENT SURFACE		DEFORMATION OR ROTATION OF UNIT ABUTMENT SURFACE		MOVEMENT OF UNIT ELEMENT *MOVEMENT HAS OPPOSITE SIGN FOR RIGHT SIDE OF DAM					
					θ_x	θ_y^{*}	θ_z^{*}	Δx^{*}	Δy	Δz
M_x	$\perp$	$M_x \sin\psi \cos\psi$	$\perp$	$(M_x \sin\psi \cos\psi)\alpha'$	$+(M_x \sin^2\psi \cos\psi)\alpha'$		$+(M_x \sin\psi \cos^2\psi)\alpha'$		$+(M_x \sin\psi \cos\psi)\gamma''$	
	$\parallel$	$M_x \cos^2\psi$	$\parallel$	$M_x \sin\psi \cos\psi\ \gamma''$						
	$\perp$		$\perp$	$(M_x \cos^2\psi)\delta'$	$+(M_x \cos^3\psi)\delta'$		$-(M_x \sin\psi \cos^2\psi)\delta'$			
M_y	$\perp$	$M_y \cos\psi$	$\perp$	$(M_y \cos\psi)\ (\alpha')$		$+(M_y \cos\psi)\ (\alpha')$				
			$\parallel$	$M_y \cos\psi\ (\gamma')$				$-(M_y \sin\psi \cos\psi)(\gamma')$		$+(M_y \cos^2\psi)(\gamma')$
M_z	$\perp$	$M_z \cos^2\psi$	$\perp$	$(M_z \cos^2\psi)\alpha'$	$+(M_z \sin\psi \cos^2\psi)\alpha'$		$+(M_z \cos^3\psi)\alpha'$			
			$\parallel$	$(M_z \cos^2\psi)\gamma''$					$+(M_z \cos^2\psi)\gamma''$	
	$\parallel$	$M_z \sin\psi \cos\psi$	$\parallel$	$(M_z \sin\psi \cos\psi)\delta'$	$-(M_z \sin\psi \cos^2\psi)\delta'$		$+(M_z \sin^2\psi \cos\psi)\delta'$			
H	$\perp$	$H \cos^2\psi$	$\perp$	$(H \cos^2\psi)\beta'$				$-(H \cos^3\psi)\beta'$		$-(H \sin\psi \cos^2\psi)\beta'$
	$\parallel$	$H \sin\psi \cos\psi$	$\parallel$	$(H \sin\psi \cos\psi)(\gamma')$				$-(H \sin^2\psi \cos\psi)(\tau')$		$+(H \sin\psi \cos^2\psi)(\tau')$
			$\perp$	$(H \sin\psi \cos\psi)(\alpha')$		$+(H \sin\psi \cos\psi)(\alpha')$				
V	$\parallel$	$V \cos\psi$	$\parallel$	$(V \cos\psi)\gamma'$					$+(V \cos\psi)\tau'$	
			$\perp$	$(V \cos\psi)\alpha''$	$+(V \sin\psi \cos\psi)\alpha''$		$+(V \cos^2\psi)\alpha''$			
W	$\perp$	$W \sin\psi \cos\psi$	$\perp$	$(W \sin\psi \cos\psi)\beta'$				$-(W \sin\psi \cos^2\psi)\beta'$		$-(W \sin^2\psi \cos\psi)\beta'$
	$\parallel$	$W \cos^2\psi$	$\parallel$	$(W \cos^2\psi)(\gamma')$				$+(W \sin\psi \cos^2\psi)(\gamma')$		$-(W \cos^3\psi)(\gamma')$
			$\perp$	$(W \cos^2\psi)(\alpha')$		$-(W \cos^2\psi)(\alpha')$				

55. Unit Vertical Element.—Table 2 gives components of forces and moments of a vertical element resolved to the unit abutment surface,

deformations and rotations of the unit abutment surface, and resulting movements of the unit vertical element.

Net movements of a unit vertical element, written in descending order of magnitude, are given by the following equations.

$$\Theta_x = M_x\left[\alpha' \sin^3\psi + \delta' \sin\psi \cos^2\psi\right] + V\left[\alpha'' \sin^2\psi\right] + M_z\left[(\alpha' - \delta')\sin^2\psi\cos\psi\right] \tag{37}$$

$$\Theta_y^* = M_y\left[\textcircled{\alpha'}\sin\psi\right] - W\left[\textcircled{\alpha''}\sin\psi\cos\psi\right] + H\left[\textcircled{\alpha''}\sin^2\psi\right] \tag{38}$$

$$\Theta_z^* = M_z\left[\delta'\sin^3\psi + \alpha'\sin\psi\cos^2\psi\right] + V\left[\alpha''\sin\psi\cos\psi\right] + M_x\left[(\alpha' - \delta')\sin^2\psi\cos\psi\right] \tag{39}$$

$$\Delta x^* = -H\left[\textcircled{r'}\sin^3\psi + \beta'\sin\psi\cos^2\psi\right] - M_y\left[\textcircled{r''}\sin^2\psi\right] - W\left[(\beta' - \textcircled{r'})\sin^2\psi\cos\psi\right] \tag{40}$$

$$\Delta y = V\left[r'\sin\psi\right] + M_x\left[r''\sin^2\psi\right] + M_z\left[r''\sin\psi\cos\psi\right] \tag{41}$$

$$\Delta z = -W\left[\beta'\sin^3\psi + \textcircled{r'}\sin\psi\cos^2\psi\right] + M_y\left[\textcircled{r''}\sin\psi\cos\psi\right] - H\left[(\beta' - \textcircled{r'})\sin^2\psi\cos\psi\right] \tag{42}$$

By fulfilling the same conditions of compatibility or continuity as those for a unit horizontal element and eliminating effects which are negligible, these equations are materially reduced.

As before, M_y and θ_y are neglected, thus excluding equation 38.

Δz had a negligible effect on the behavior of a unit vertical element and is neglected in present methods of analysis. This excludes equation 42.

W in equation 40, M_z in equations 37 and 41, and M_x and V in equation 39 have very little influence on a unit vertical element and are omitted in the present methods of analysis.

With these eliminations, the equations now are

$$\Theta_x = M_x\left[\alpha'\sin^3\psi + \delta'\sin\psi\cos^2\psi\right] + V\left[\alpha''\sin^2\psi\right] \tag{43}$$

$$\Theta_z^* = M_z\left[\delta'\sin^3\psi + \alpha'\sin\psi\cos^2\psi\right] \tag{44}$$

$$\Delta x^* = -H\left[\textcircled{r'}\sin^3\psi + \beta'\sin\psi\cos^2\psi\right] \tag{45}$$

$$\Delta y = V\left[r'\sin\psi\right] + M_x\left[r''\sin^2\psi\right] \tag{46}$$

Foundation deformations of a unit differential area are obtained as indicated in the discussion of a unit horizontal element.

By substituting symbols for bracketed quantities and using the identity $\alpha'' = \gamma''$, equations 43, 44, 45, and 46 become

$$\Theta_x = M_x \alpha + V \alpha_2 \tag{47}$$

$$\Theta_z^* = M_z \delta \tag{48}$$

$$\Delta x^* = -H \textcircled{r} \tag{49}$$

$$\Delta y = V r + M_x \alpha_2 \tag{50}$$

in which

$$\alpha = \alpha' \sin^3\psi + \delta' \sin\psi \cos^2\psi$$

$$\alpha_2 = \alpha'' \sin^2\psi$$

$$\delta = \delta' \sin^3\psi + \alpha' \sin\psi \cos^2\psi$$

$$\textcircled{r} = \textcircled{r'} \sin^3\psi + \beta' \sin\psi \cos^2\psi$$

$$r = r' \sin\psi$$

56. Application to Cantilevers.—In an arch dam each vertical cantilever element, except the one at the maximum section, rests on the same strip of foundation surface as the end of an arch. In this case radial shear at the base of a cantilever corresponds with radial shear at the abutment of the corresponding arch; tangential thrust of the cantilever corresponds with normal thrust of the arch; and twisting moment of the cantilever corresponds with bending moment of the arch. For this reason radial shears, tangential thrusts, and twisting moments of the cantilevers are applied to the arches as concentrated radial, tangential, and twist loads, respectively. Then radial, tangential, and angular movements at foundations of cantilevers are equal to corresponding movements of arch abutments.

Concentrated loads take care of all foundation cantilever movements except rotation in a vertical plane, θ_x. Therefore equation 47 is the only equation used in computing movements of unit loads for cantilevers resting on arch abutments.

.....DAM STUDY NO.....

ARCH ABUTMENT AND CANTILEVER FOUNDATION DEFORMATION FUNCTIONS..... ... . SIDE

μ = ... $\frac{b}{a}$ = ... $\alpha' = \frac{}{E_R T^2}$ $\alpha'' = \frac{}{E_R T}$ $\gamma' = \frac{}{E_R}$ $\frac{1}{E_R}$ = ...

$\frac{b/2}{a}$ = ... $\beta' = \frac{}{E_R}$ $\delta' = \frac{}{E_R T^2}$ $\frac{a}{b/2}$ = ... $\textcircled{\gamma} = \frac{}{E_R}$ By.....Date.... .....

EL.	T_A *	α'	α''	β'	γ'	$\textcircled{\gamma}$	δ'	ψ	SIN ψ	SIN$^2\psi$	SIN$^3\psi$	COS ψ	COS$^2\psi$	COS$^3\psi$	SINψCOS$^2\psi$	SIN$^2\psi$COSψ
				For all elevations.....	For all elevations.....	For all elevations.....										

ARCH

$\alpha = \alpha' \cos^3\psi + \delta' \sin^2\psi \cos\psi$ $\alpha_2 = \alpha'' \cos^2\psi$

$\beta = \beta' \cos^3\psi + \textcircled{\gamma} \sin^2\psi \cos\psi$ $\gamma = \gamma' \cos\psi$

EL.	α	β	γ	α_2

CANTILEVER

$\alpha = \frac{R\,axis}{r}(\alpha' \sin^3\psi + \delta' \sin\psi \cos^2\psi)$ $\alpha_2 = \frac{R\,axis}{r}(\alpha'' \sin^2\psi)$ $\gamma = \frac{R\,axis}{r}(\gamma' \sin\psi)$

$\textcircled{\gamma} = \frac{R\,axis}{r}(\textcircled{\gamma} \sin^3\psi + \beta' \sin\psi \cos^2\psi)$ $\delta = \frac{R\,axis}{r}(\delta' \sin^3\psi + \alpha' \sin\psi \cos^2\psi)$

CANT.	R axis/r	α	γ	δ	α_2	$\textcircled{\gamma}$

* T_A is thickness of dam at abutment.

FIGURE 14—COMPUTATION FORM FOR UNIT FOUNDATION MOVEMENTS

57. Computation Form.—A convenient computation form has been developed for calculating unit foundation movements by equations 34, 35, 36, 47, 48, 49, and 50, see figure 14.

For a symmetrical dam, abutment values need only be computed for the left half of the dam. If a dam is approximately symmetrical, abutment angles, ψ, are usually assumed the same for the left and right abutments of each arch. This simplifies the computations required for an analysis, and does not seriously affect the accuracy of results.

CHAPTER IV—ANALYSIS OF CANTILEVER ELEMENTS

INTRODUCTION

58. General.—This chapter describes methods now used by the Bureau of Reclamation in analyzing radial-side cantilevers of a concrete dam. The analysis includes evaluation of properties of the cantilever and its horizontal sections; evaluation of forces and moments due to weight of concrete, vertical component of water load, and unit radial, tangential, and twist loads; and computation of cantilever movements caused by these loads. Since a cantilever may be cracked by tension, methods of analysis are presented for an uncracked cantilever and for one with a reduced section due to cracking at either the upstream or downstream face. Unit loads described in chapter II are used in analyzing uncracked cantilevers.

The determination of loads resisted by cantilevers is described in chapter VII. The evaluation of stresses in uncracked cantilevers is given in chapter VIII. The evaluation of stresses in cantilevers cracked by tension is given in the latter part of this chapter. Convenient computation forms for use in analyzing cantilever elements are given at the end of this chapter.

59. Description of Cantilever Element.—The unit cantilever is composed of a series of horizontal differential elements, each bounded by the upstream and downstream faces of the dam and by vertical radial planes intersecting at the center of curvature for the elevation of the differential element, see figure 15-b.

The cantilever is an elastic unit, set on an elastic foundation. It supports concrete weight and resists external and internal forces and moments due to applied loads. As usually analyzed, the unit cantilever element has a thickness of one foot at the axis of the dam.

ANALYSIS OF UNCRACKED CANTILEVER

60. Notation.—The following symbols refer to the base of the cantilever or to the horizontal section under consideration.

$'$ indicates value for tailwater.

Subscripts: U = upstream face; D = downstream face; $axis$ = axis of dam; W = water; C = concrete.

T = thickness of dam or radial distance between faces.

R = radius or distance from center of curvature.

r = radius to center line.

lg = distance from upstream face to center of gravity.

$T\text{-}lg$ = distance from downstream face to center of gravity.

A = area.

I = moment of inertia of the horizontal cross section about a circumferential line through the center of gravity.

h = distance below water surface.

e = eccentricity of a resultant force from center of gravity, positive upstream.

W_C = weight of concrete or dead load above section.

M_C = moment of W_C about center of gravity.

W_W = weight of vertical water load above section.

M_W = moment of W_W about center of gravity.

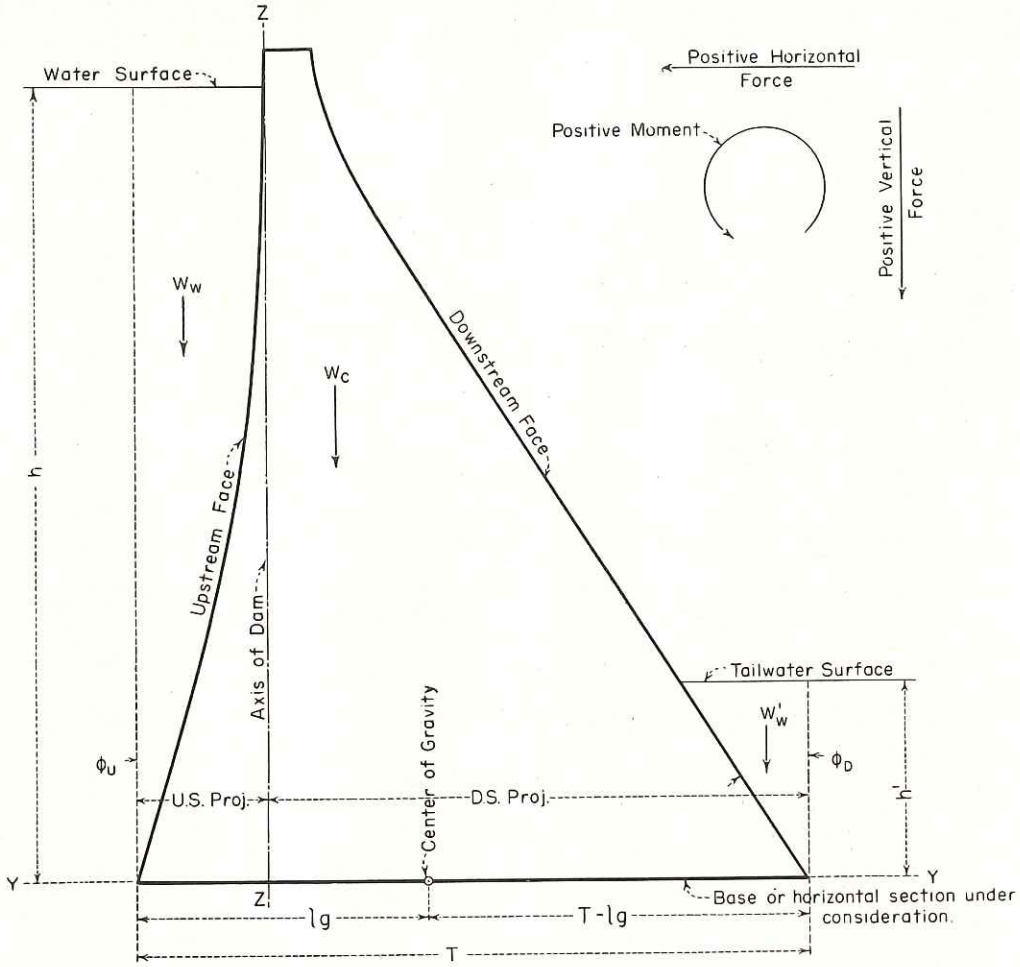
M = bending moment.

V = radial shear force.

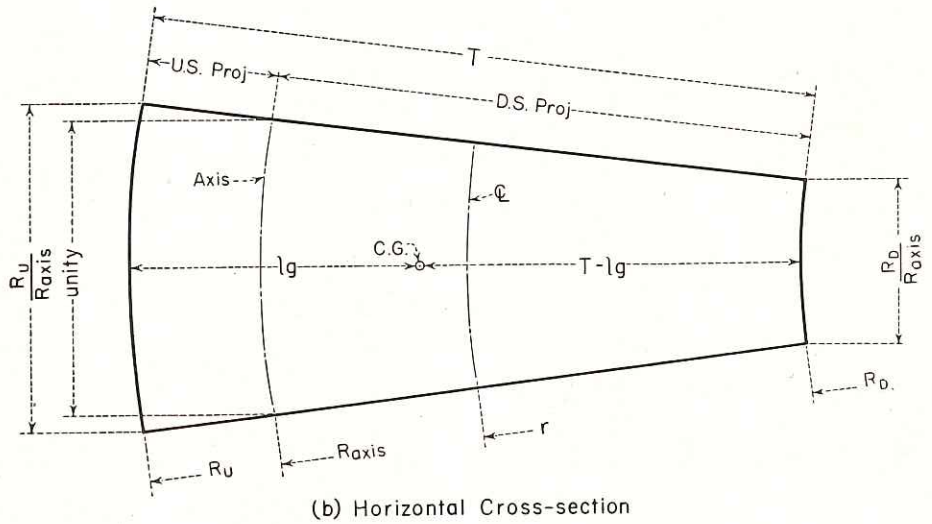
H = tangential shear force.

$\mathcal{M}$ = twisting moment in horizontal plane.

61. Evaluation of Properties.—Properties of horizontal sections of a cantilever are usually determined at elevations of sample arches. Additional sections may be used at intermediate elevations if more accuracy is required. After the thickness and radii at a section have been found, the area, A, moment of inertia, I, and distance from upstream face to center of gravity, lg, may be obtained from equations and curves in the following text.



(a) Vertical Cross-section



(b) Horizontal Cross-section

FIGURE 15—CROSS SECTION OF RADIAL-SIDE CANTILEVER

If the boundaries at the upstream and downstream faces in figure 15-b are assumed to be straight lines, the area of the section is

$$A = \left(\frac{\frac{R_U}{R_{axis}} + \frac{R_D}{R_{axis}}}{2} \right) T$$

Transposing and simplifying,

$$\frac{A}{T\left(\frac{R_U}{R_{axis}}\right)} = \frac{1}{2}\left(1 + \frac{R_D}{R_U}\right) \tag{51}$$

Likewise, the equation for the distance from the upstream face to the center of gravity is

$$lg = \frac{T}{3}\left(\frac{\frac{R_U}{R_{axis}} + 2\frac{R_D}{R_{axis}}}{\frac{R_U}{R_{axis}} + \frac{R_D}{R_{axis}}} \right)$$

Transposing and simplifying,

$$\frac{lg}{T} = \frac{1}{3}\left(\frac{1 + 2\frac{R_D}{R_U}}{1 + \frac{R_D}{R_U}} \right) \tag{52}$$

The moment of inertia of the section about the center of gravity is

$$I = \frac{T^3}{36}\left[\frac{\left(\frac{R_U}{R_{axis}}\right)^2 + 4\frac{R_U}{R_{axis}} \cdot \frac{R_D}{R_{axis}} + \left(\frac{R_D}{R_{axis}}\right)^2}{\frac{R_U}{R_{axis}} + \frac{R_D}{R_{axis}}} \right]$$

Transposing and simplifying,

$$\frac{I}{T^3 \frac{R_U}{R_{axis}}} = \frac{1}{36}\left[\frac{1 + 4\frac{R_D}{R_U} + \left(\frac{R_D}{R_U}\right)^2}{1 + \frac{R_D}{R_U}} \right] \tag{53}$$

The right sides of equations 51, 52, and 53 depend on the value of R_D/R_U only. Curves have been drawn for these values and are given in figure 16. Using these curves, values of A, lg, and I may be evaluated by multiplying quantities from the curves by the denominators of the fractions at the left sides of the equations.

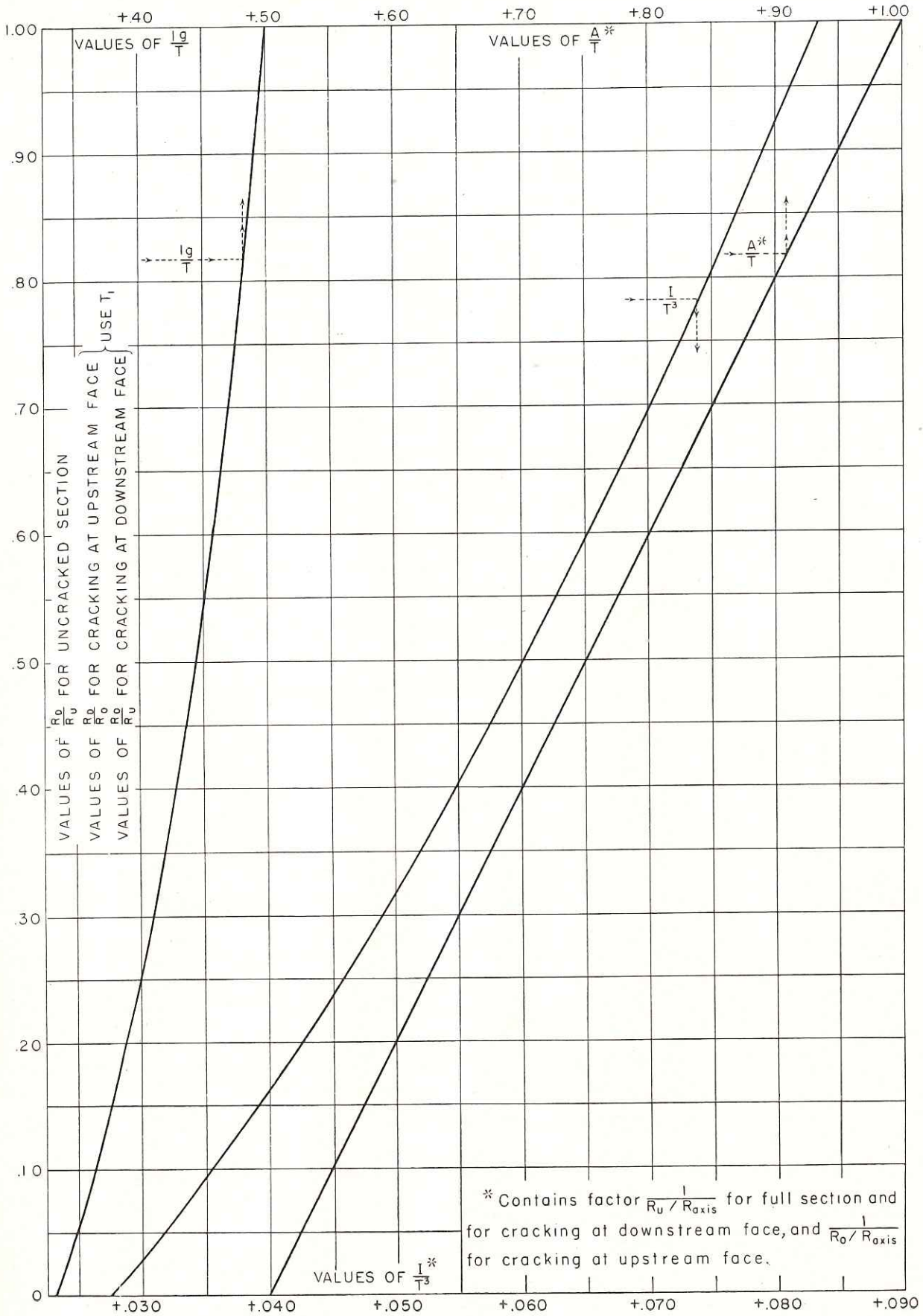


FIGURE 16—PROPERTIES OF RADIAL-SIDE CANTILEVER

62. Concrete and Vertical Water Loads.—A graphical method has been devised for determining weights and moments of horizontal slices of a radial-side cantilever due to concrete and vertical water load. The method consists of constructing a parallel-side cantilever equivalent to the given radial-side cantilever, from which the desired information is readily obtained by the use of a mechanical integrator. The construction of the equivalent cantilever, described in figure 17, depends on the fact

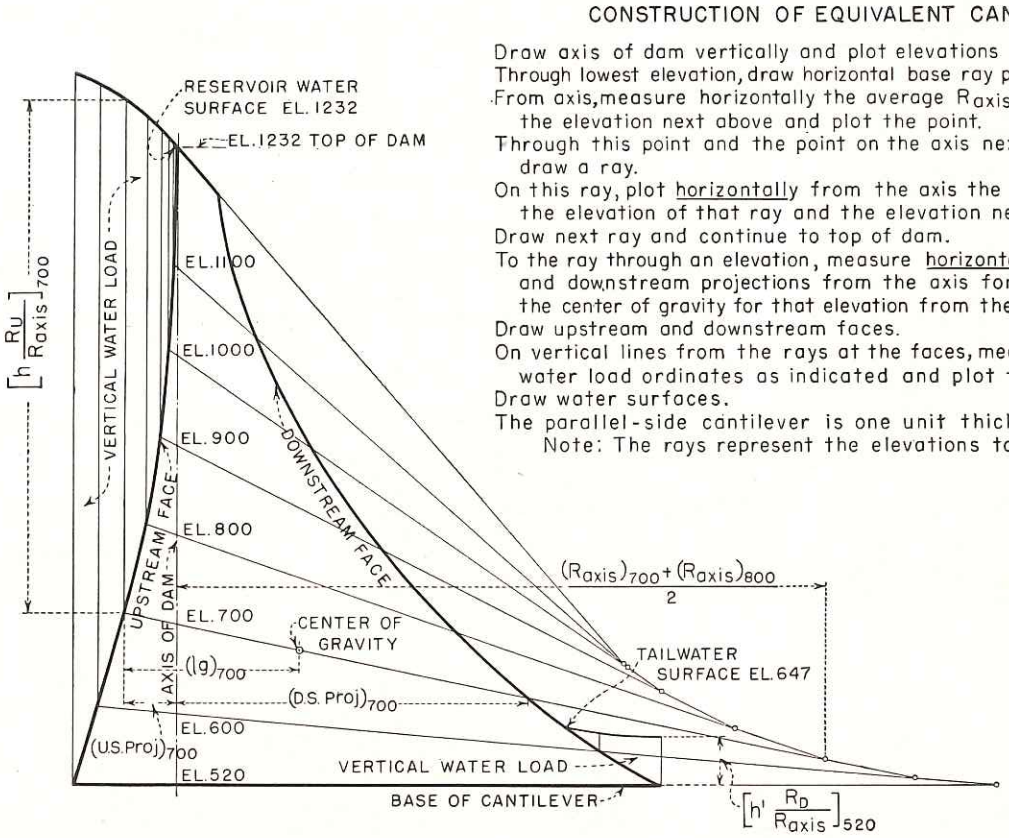
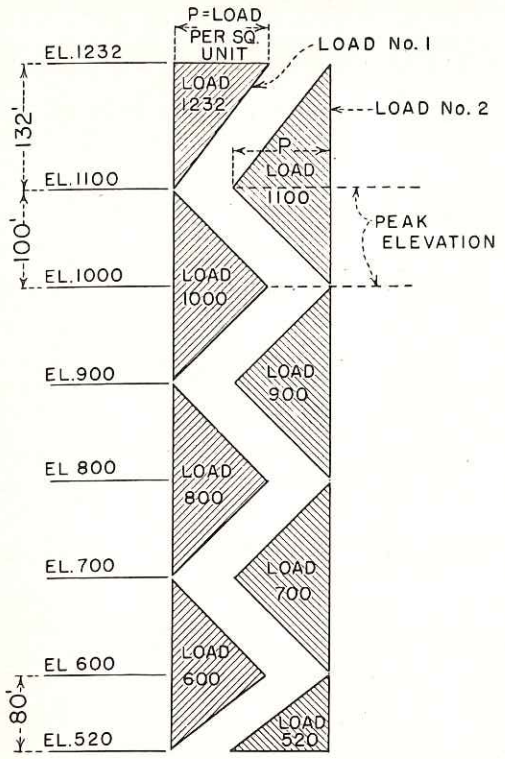

CONSTRUCTION OF EQUIVALENT CANTILEVER
Draw axis of dam vertically and plot elevations desired for analysis.
Through lowest elevation, draw horizontal base ray perpendicular to axis.
From axis, measure horizontally the average Raxis for the base and the elevation next above and plot the point.
Through this point and the point on the axis next above the base, draw a ray.
On this ray, plot horizontally from the axis the average Raxis for the elevation of that ray and the elevation next above.
Draw next ray and continue to top of dam.
To the ray through an elevation, measure horizontally the upstream and downstream projections from the axis for that elevation, and the center of gravity for that elevation from the upstream face.
Draw upstream and downstream faces.
On vertical lines from the rays at the faces, measure the vertical water load ordinates as indicated and plot the points.
Draw water surfaces.
The parallel-side cantilever is one unit thick normal to paper.
Note: The rays represent the elevations to be analyzed.
RESERVOIR WATER SURFACE EL. 1232
EL. 1232 TOP OF DAM
EL. 1100
EL. 1000
EL. 900
EL. 800
EL. 700
EL. 600
EL. 520
VERTICAL WATER LOAD
UPSTREAM FACE
DOWNSTREAM FACE
AXIS OF DAM
CENTER OF GRAVITY
TAILWATER SURFACE EL. 647
BASE OF CANTILEVER
(lg)700
(D.S. Proj)700
(U.S. Proj)700
(Raxis)700 + (Raxis)800 / 2
[h Ru/Raxis]700
[h' RD/Raxis]520

FIGURE 17—PARALLEL-SIDE CANTILEVER EQUIVALENT TO RADIAL-SIDE CANTILEVER

that, between any two consecutive horizontal sections, the volume of the cantilever, or vertical water load, and the radial position of the center of gravity of that volume are practically unchanged by the transformation.

63. Unit Triangular Loads.—Unit loads, triangular in shape, are used in analyzing uncracked cantilever elements. Basic unit loads, used to build up total radial, tangential, and twist loads, are shown in figure 18, together with illustrative calculations.

Each unit load is considered separately in the computations. Let ΔV represent the total portion of a triangular load above any given elevation. The value of ΔV is total shear at the given elevation, caused by the portion of the triangular load above that elevation, in the case of radial



ILLUSTRATIVE CALCULATIONS

	Load 1232 (No.1)	Load 1100 (No.2)
EL.1100	$\Delta V = \frac{132P}{2} = 66P$	$\Delta V = \frac{132P}{2} = 66P$
	$\Delta M = 66P\,(\frac{2}{3} \cdot 132) = 5{,}808P$	$\Delta M = 66P\,(\frac{1}{3} \cdot 132) = 2{,}904P$
EL.1000	$\Delta V = 66P$	$\Delta V = \frac{232P}{2} = 116P$
	$\Delta M = 66P\,(88+100) = 12{,}408P$	$\Delta M = 116P\,(110.66) = 12{,}837P$
EL.900	$\Delta V = 66P$	$\Delta V = 116P$
	$\Delta M = 66P\,(88+200) = 19{,}008P$	$\Delta M = 116P\,(100+110.66) = 24{,}437P$
EL.800	$\Delta V = 66P$	$\Delta V = 116P$
	$\Delta M = 66P\,(88+300) = 25{,}608P$	$\Delta M = 116P\,(200+110.66) = 36{,}037P$
EL.700	$\Delta V = 66P$	$\Delta V = 116P$
	$\Delta M = 66P\,(88+400) = 32{,}208P$	$\Delta M = 116P\,(300+110.66) = 47{,}637P$
EL.600	$\Delta V = 66P$	$\Delta V = 116P$
	$\Delta M = 66P\,(88+500) = 38{,}808P$	$\Delta M = 116P\,(400+110.66) = 59{,}237P$
EL.520	ETC.	ETC.

FIGURE 18—BASIC UNIT TRIANGULAR LOADS

and tangential loads; and total twisting couple at the elevation, caused by the portion of the triangular load above that elevation, in the case of twist loads. Let ΔM represent the moment of the portion of the triangular load above any given elevation, about the given elevation. Values of ΔM are used only in the case of radial loads. Then, in figure 18, ΔV is the area of the load diagram above an elevation, and ΔM, the product of ΔV and the distance of its centroid above an elevation.

For convenience, triangular loads may be designated *1, 2, 3,* and so forth, at the elevation of P, peak elevation, starting at the top. P is generally 1,000 pounds per square foot for radial and tangential loads, and 1,000 foot-pounds per square foot for twist loads.

64. Unit Radial Loads.—Unit triangular radial loads are horizontal radial forces applied at the upstream or downstream faces of the cantilever. Values of ΔV and ΔM for these radial loads are obtained by multiplying values of ΔV and ΔM of the basic unit load by R_U/R_{axis}, upstream face, or R_D/R_{axis}, downstream face, for the elevation at which the unit load peaks. These factors adjust the basic unit loads, one unit in width, to the width of the face where they are applied as unit radial loads. Multiplications of the corrected values of ΔV and ΔM by the trial load ordinates at the respective peak elevations, determined by trial load adjustments, give the shears and moments on the cantilever due to the loads applied.

Shears and moments at each elevation, due to unit loads above each elevation, are tabulated in columns for the different unit loads, as shown in figure 18. Values of the total shear and moment at each elevation, due to all loads above each elevation, are then obtained by adding quantities at each elevation and recording in a summation column at the right.

65. Unit Tangential Loads.—Unit tangential loads are tangential shear forces applied at the center line of the cantilever. Values of ΔH for a unit tangential load are obtained by multiplying values of ΔV of the basic unit load by r/R_{axis} for the elevation at which the basic unit load peaks. Values of tangential thrust, H, are obtained by multiplying values of ΔH by the trial load ordinates at the load peak elevations. The total tangential thrust at each elevation is then obtained by summation, and recorded in a column at the right, as before.

66. Unit Twist Loads.—Unit triangular twist loads are twisting moments in horizontal planes, applied at the center line of the cantilever. Values of $\Delta \mathcal{M}$ for a unit twist load are obtained by multiplying values of ΔV for the basic unit load by r/R_{axis} for the elevation at which the basic unit load peaks. The twisting moment, $\mathcal{M}$, is obtained by multiplying values of $\Delta \mathcal{M}$ by the trial load ordinates at the load peak elevations; and the total twisting moment at each elevation obtained by summation, as before.

MOVEMENTS OF UNCRACKED CANTILEVER

67. Notation.—Additional symbols used in considering cantilever movements are:

Δr = radial deflection of center line of cantilever.

Δs = tangential deflection of center line of cantilever.

θ = angular movement, in horizontal plane, of horizontal section of cantilever about the center line.

Subscripts: A = foundation; L = left part of dam, looking upstream; and R = right part of dam, looking upstream.

Σ = summation.

E_C = modulus of elasticity of concrete in direct stress.

G = modulus of elasticity of concrete in shear stress $= \dfrac{E_C}{2(1+\mu)}$.

K = ratio of detrusion due to actual shear distribution to detrusion due to equivalent shear distributed uniformly. K is usually assumed to be 1.25 for Δr and 1.00 for Δs.

Δz = increment of height between sections at which properties and forces are known.

ψ, α', β', γ', $\textcircled{\gamma}$, δ', and α'' = abutment functions as defined in chapter III.

α = angular movement of foundation in vertical radial plane due to one unit of bending moment, M_A; per unit length normal to plane of cantilever.

γ = radial movement of foundation due to one unit of radial shear force, V_A; per unit length normal to plane cantilever.

$\textcircled{\gamma}$ = tangential movement of foundation due to one unit of tangential shear force, H_A; per unit length normal to plane of cantilever.

δ = angular movement of foundation in horizontal plane due to one unit of twisting moment, $\frac{M}{M}_A$; per unit length normal to plane of cantilever.

α_2 = angular movement of foundation in radial plane due to one unit of radial shear force, V_A; per unit length normal to plane of cantilever; or radial movement of foundation due to one unit of bending moment, M_A; per unit length normal to plane of cantilever.

68. Foundation Movements.—In applying results obtained in chapter III to movements of cantilever elements, the following formulas are used:

$$\alpha = \frac{R_{axis}}{r}\left[\alpha' \sin^3\psi + \delta' \sin\psi \cos^2\psi\right] \tag{54}$$

$$r = \frac{R_{axis}}{r}\left[r' \sin\psi\right] \tag{55}$$

$$\textcircled{\gamma} = \frac{R_{axis}}{r}\left[\textcircled{\gamma'} \sin^3\psi + \beta' \sin\psi \cos^2\psi\right] \tag{56}$$

$$\delta = \frac{R_{axis}}{r}\left[\delta' \sin^3\psi + \alpha' \sin\psi \cos^2\psi\right] \tag{57}$$

$$\alpha_2 = \frac{R_{axis}}{r}\left[\alpha'' \sin^2\psi\right] \tag{58}$$

The factor $\frac{R_{axis}}{r}$ in the equations corrects the movements derived for a cantilever of unit width at the center line to the movements of a radial-side cantilever one unit wide at the axis. Foundation movements for a radial-side cantilever are assumed to be those for a parallel-side cantilever of the same cross sectional area.

69. Radial Deflections.—Radial cantilever deflections required for a trial load analysis are those caused by radial loads and vertical water loads. Radial deflections are produced by radial bending moments and radial shear forces. They are determined by the ordinary theory for cantilever beams with contributions from bending, shear, and foundation yielding included.

The radial deflection at any horizontal section due to bending moment is the integral of the slope of the center line from the base to the elevation of the given section. The slope of the center line at any horizontal section is the angular foundation rotation, normal to the foundation surface, plus the integral of the curvature of the center line from the base to the elevation of the given section. The curvature of the center line at any point is M/E_cI.

The radial deflection at any horizontal section due to radial shear forces is the foundation deformation plus the integral, from the base to that section, of the detrusion or change in horizontal position of the center line. The detrusion of the center line of a differential height is $(VK/AG)\Delta z$.

Actual calculations for radial deflections are performed by summating the quantities noted above, which are determined at sufficient horizontal sections to render the summation reasonably accurate. Operations followed in evaluating radial deflections may be expressed by the formula,

$$L \text{ and } R\Delta r = \sum \left(M_A \alpha + V_A \alpha_2 + \sum \frac{M}{E_c I} \Delta z \right) \Delta z + \left(V_A r^* + M_A \alpha_2^* + \sum \frac{VK}{AG} \Delta z \right) \quad (59)$$

in which terms designated by an asterisk apply only to the maximum cantilever or one that does not set on the end of an arch, see chapter III, section 56. In the other case, where the cantilever rests on an arch abutment, these terms are replaced by the equivalent movements of the arch abutment. In the application of the formula, the convention of signs to be used is given in figure 19, which shows directions of positive movements,

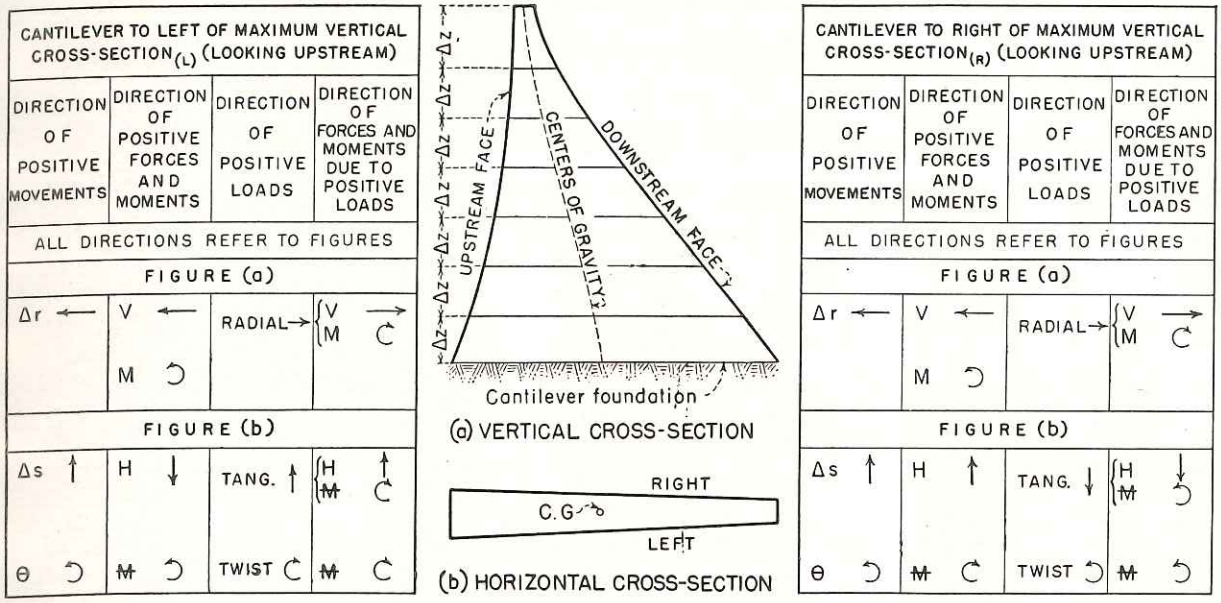


FIGURE 19—CONVENTION OF SIGNS, RADIAL-SIDE CANTILEVER

forces, moments, and loads, and directions of forces and moments due to positive loads. Figure 19 also gives the convention for tangential and twist loads and movements.

70. Tangential Deflections.—Tangential cantilever deflections are calculated for tangential loads. These loads, which represent tangential shear forces, produce deflections only by shear detrusions. Movements due to bending are omitted because tangential bending moments are negligible. Tangential deflections are determined by the method used for radial deflections due to radial shear forces. Expressed as formulas, they are

$$_L\Delta s = -H_A \textcircled{r}^* - \sum \frac{HK}{AG}\Delta z \qquad (60)$$

$$_R\Delta s = H_A \textcircled{r}^* + \sum \frac{HK}{AG}\Delta z \qquad (61)$$

in which terms designated by an asterisk apply only to the maximum cantilever or one that does not set on the abutment of an arch. In the alternate case they are replaced by the equivalent movements of the arch abutment. In applying the formulas to the evaluation of tangential movements, the value of K is taken as unity because tangential shear is assumed to be distributed uniformly at each horizontal section.

71. Angular Movements.—Angular movements or rotations of a cantilever are calculated for twist loads. These loads represent twisting

moments in horizontal planes, producing rotations about the cantilever center line. The angular movement at any horizontal section is the angular foundation rotation in a horizontal plane, plus the integral, from the base to that section, of the angular movements of the differential heights about the center line. Since the summation method is used to evaluate quantities to be integrated, and since the angular movement of a differential height is $(M/2GI)\Delta z$, formulas for the left and right sides of the dam may be written as

$$_L\Theta = M_A \delta^* + \sum \frac{M}{2GI} \Delta z \tag{62}$$

$$_R\Theta = -M_A \delta^* - \sum \frac{M}{2GI} \Delta z \tag{63}$$

in which the terms designated by an asterisk apply only to cantilevers that do not set on the abutment of an arch. In the other case they are replaced by the equivalent movements of the arch abutment.

Since cantilevers are units of a continuous structure, shears set up by twisting moments are assumed to act in tangential directions and to have a linear variation from the upstream face to the downstream face of the cantilever. With these assumptions, the equations correspond to formulas for twist in a continuous slab.

72. Secondary Movements.—Many secondary movements are produced by radial, tangential, and twist loads applied to the cantilevers in trial load adjustments. Most of these have have been investigated, and those having small effects are usually neglected. Only two secondary effects have been found large enough to consider in ordinary cases. They are radial deflections due to twist loads on the arch, which are discussed in chapter VII, and angular movements in horizontal planes due to tangential loads, which are discussed in the following paragraph.

Angular movements in horizontal planes exist because a tangential force, having an eccentricity from the center of gravity of each horizontal section below, causes twisting moments in those horizontal sections. In considering this effect, it is convenient to find the effect for each unit tangential load, and to establish the total effect by the use of these separate effects. It is first necessary to find the twisting moments due to each unit tangential load. These are obtained by multiplying the thrust of a unit load by the horizontal distance between the center line of the cantilever at the load peak elevation and the center of gravity of the section at which the moment is desired. After these twisting moments have been deter-

mined, angular movements due to unit tangential loads can be calculated by equations 62 and 63.

73. Tabulations.—Deflections due to the unit radial, tangential, and twist loads are tabulated for use in trial load adjustments. The tabulations are made for each cantilever and are usually arranged with all deflections for one type of load on one sheet. For example, radial deflections for all unit radial loads are tabulated on one sheet.

ANALYSIS OF CRACKED CANTILEVER

74. General Statement.—In a thin arch dam, radial loads carried by a cantilever may cause tensile stress on a horizontal section at either face. Although the cantilever is capable of withstanding a small amount of tension, it is deemed advisable to assume that such cantilevers are cracked to the point of zero stress. For this case, the resultant of the forces above a horizontal section is equal to the total stress on the uncracked area. Stresses are assumed to vary as a straight line from zero at the end of the crack to a maximum at the uncracked face.

If a horizontal crack occurs at the face of a cantilever below the water surface, the pressure of the water exerts an uplift or wedging force which may equal full water pressure near the face of the dam and may act with varying intensity over a large part of the area of the crack. This uplift pressure deflects the cantilever and causes cracking to progress farther toward the opposite face. However, the cantilever is restrained at some point of its movement by the resistance of the arches. The accurate inclusion of uplift effects in a trial load analysis is not usually feasible because the amount and variation of uplift pressure intensities is not definitely known and because the amount of cracking at each horizontal section would have to be determined by trial for each load adjustment. Although uplift effects are somewhat uncertain, equations and curves in the following sections include uplift forces and moments, so that they may be included when considered necessary.

75. Notation.—Additional symbols used in considering cracked cantilevers are given below. The symbols refer to the base of the cantilever or to the horizontal section under consideration.

e = eccentricity of a resultant force, measured from the center of gravity of the total horizontal section, positive upstream.

F = function for determining properties and functions of uncracked portion of section.

R_0 = radius to end of crack.

r_1 = radius to center line of uncracked portion of section.

lg_1 = distance from upstream edge of uncracked portion of section to center of gravity of that portion of section.

T_1 = radial thickness of uncracked portion of section.

A_1 = area of uncracked portion of section.

I_1 = moment of inertia of uncracked portion of section about a circumferential line through its center of gravity.

M_1 = resultant moment about center of gravity of uncracked portion of section.

U = uplift force on total horizontal section.

M_U = moment of U about center of gravity of total horizontal section.

σ_Z = vertical stress at uncracked face of cantilever.

76. Cracking at Upstream Face.—If a cantilever is cracked at the upstream face, it is necessary to determine how far the cracks extend radially along each horizontal section. Then the area, moment of inertia, resultant moment, and stress can be determined for the uncracked portion of each section.

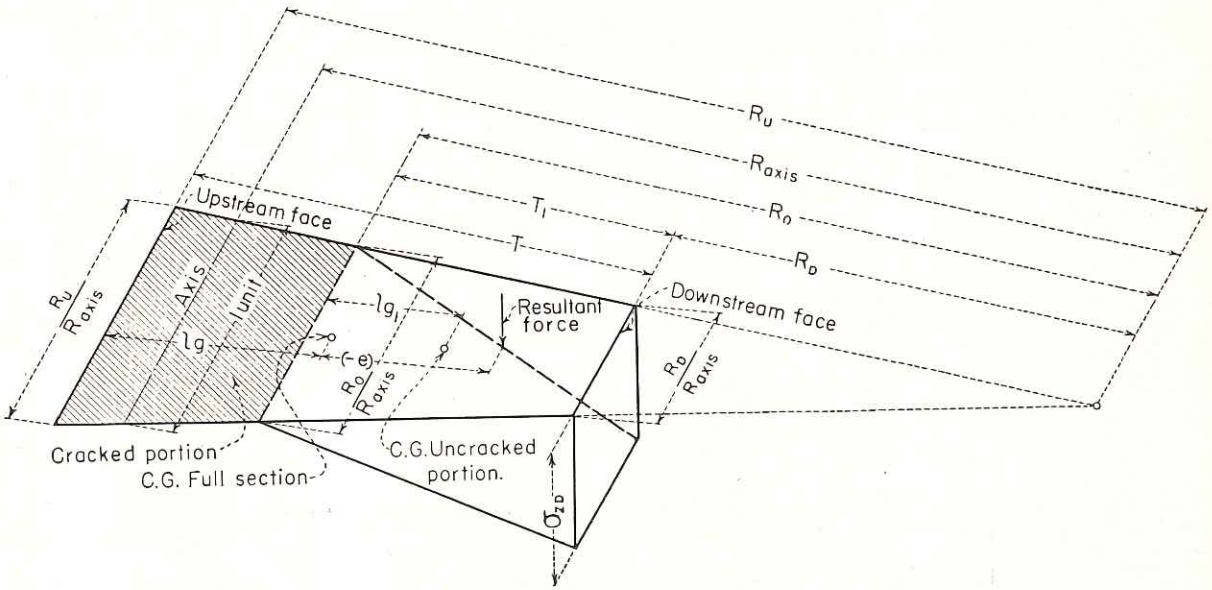


FIGURE 20—HORIZONTAL SECTION OF CANTILEVER CRACKED AT UPSTREAM FACE

From figure 20, which shows a horizontal section of a cantilever cracked at the upstream face, the volume of the indicated stress solid is

$$\Sigma W+U = \frac{T_1}{2}\sigma_{zD}\frac{R_O}{R_{axis}} - \frac{T_1}{3}\sigma_{zD}\left(\frac{R_O}{R_{axis}} - \frac{R_D}{R_{axis}}\right)$$

Simplifying,

$$\Sigma W+U = \sigma_{zD}\, T_1\left[\frac{R_O + 2R_D}{6R_{axis}}\right] \tag{64}$$

The moment of the stress solid about the center of gravity of the original section is

$$(\Sigma W+U)(-e) = \left[\left(\frac{T_1}{2}\sigma_{zD}\frac{R_O}{R_{axis}}\right)\left(T - lg - \frac{T_1}{3}\right)\right] - \left[\left(\frac{T_1}{3}\sigma_{zD}\right)\left(\frac{R_O}{R_{axis}} - \frac{R_D}{R_{axis}}\right)\left(T - lg - \frac{T_1}{4}\right)\right]$$

By substituting equation 64 for $\Sigma W + U$ and simplifying,

$$\frac{R_U - lg + e}{R_D} = \frac{1}{2}\left[\frac{1 + 2\frac{R_D}{R_O} + 3\left(\frac{R_D}{R_O}\right)^2}{\frac{R_D}{R_O} + 2\left(\frac{R_D}{R_O}\right)^2}\right] \tag{65}$$

in which the quantity $\dfrac{R_U - lg + e}{R_D}$ is designated by F. From this equation, it is evident that the value of F determines the ratio R_D/R_O for the uncracked portion of the horizontal section. A curve in figure 21, developed from equation 65, is used to evaluate the ratio R_D/R_O for given values of F. If R_D/R_O, as determined by the curve, is less than R_D/R_U, the section does not crack. Conversely, if R_D/R_O is greater than R_D/R_U, the section is cracked. If R_D/R_O is greater than unity, the section is completely cracked.

With the value of R_D/R_O determined, T_1 can be evaluated by the equation

$$T_1 = \left(\frac{R_O}{R_{axis}} - \frac{R_D}{R_{axis}}\right)R_{axis} \tag{66}$$

in which

$$\frac{R_O}{R_{axis}} = \frac{R_D}{R_{axis}} \div \frac{R_D}{R_O} \tag{67}$$

Equations for area, A_1, and moment of inertia, I_1, of the uncracked portion of the horizontal section are obtained by substituting A_1 for A; T_1 for T; R_O/R_{axis} for R_U/R_{axis}; and R_D/R_O for R_D/R_U, in equations 51 and 53. The revised formulas are

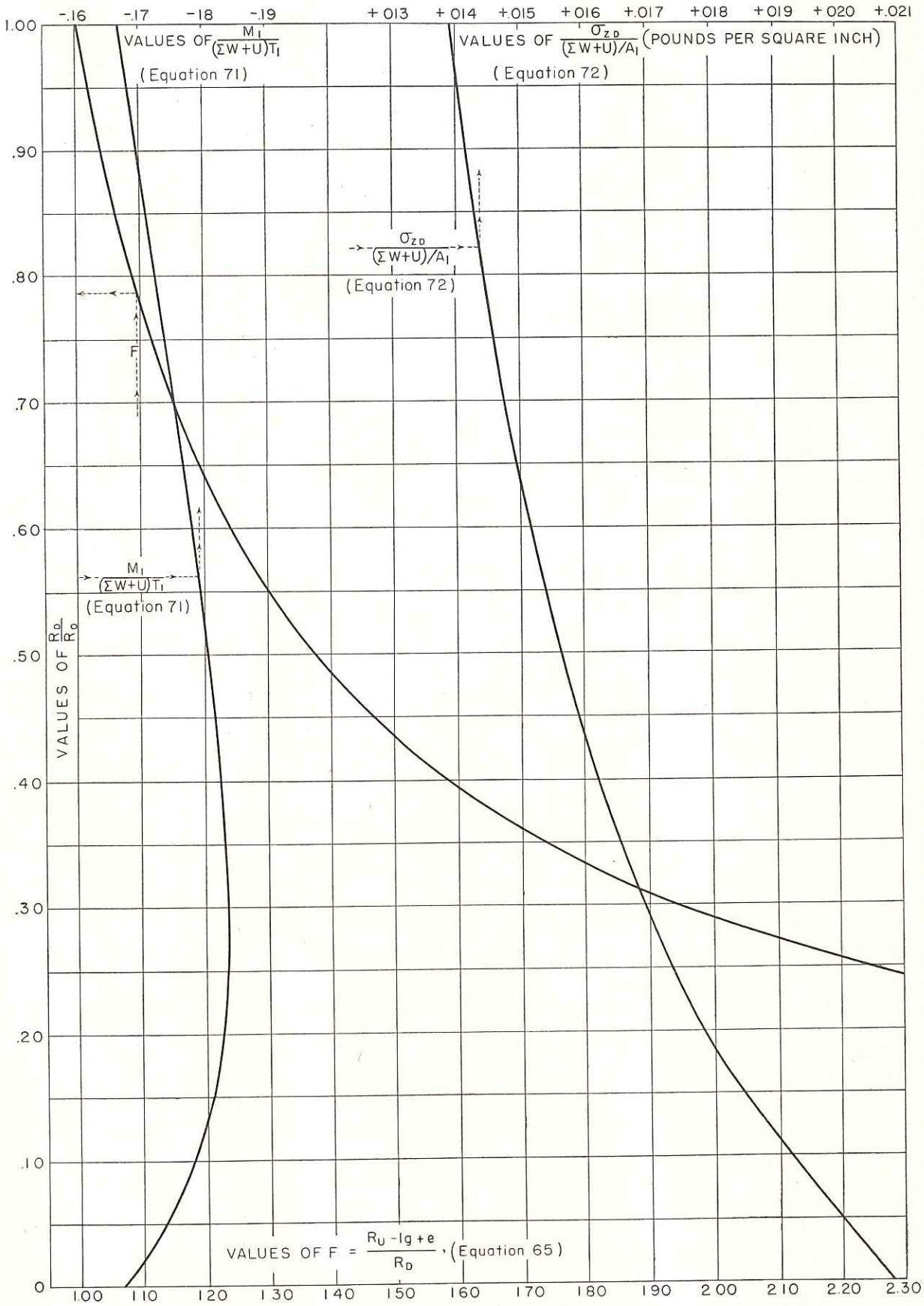


FIGURE 21—PROPERTIES OF CANTILEVER CRACKED AT UPSTREAM FACE

$$\frac{A_1}{T_1\left(\frac{R_0}{R_{axis}}\right)} = \frac{1}{2}\left(1+\frac{R_D}{R_0}\right) \tag{68}$$

$$\frac{I_1}{T_1^3\left(\frac{R_0}{R_{axis}}\right)} = \frac{1}{36}\left[\frac{1+4\frac{R_D}{R_0}+\left(\frac{R_D}{R_0}\right)^2}{1+\frac{R_D}{R_0}}\right] \tag{69}$$

Values of A_1 and I_1 are obtained from curves in figure 16, using proper multipliers and following the same procedure as indicated for the uncracked section.

The moment of the redistributed forces about the center of gravity of the uncracked portion of the horizontal section is determined from the stress equation at the end of the crack.

$$\frac{\Sigma W+U}{A_1} + M_1\frac{lg_1}{I_1} = 0 \tag{70}$$

If values of A_1 and I_1, obtained from equations 68 and 69, and the value of lg_1, obtained by proper symbolical substitution in equation 52, are substituted in equation 70, it reduces to

$$\frac{M_1}{(\Sigma W+U)T_1} = -\frac{1}{6}\left[\frac{1+4\frac{R_D}{R_0}+\left(\frac{R_D}{R_0}\right)^2}{1+3\frac{R_D}{R_0}+2\left(\frac{R_D}{R_0}\right)^2}\right] \tag{71}$$

M_1 is evaluated by multiplying values from the proper curve in figure 21, based on equation 71, by the quantity $(\Sigma W + U)T_1$.

The stress at the downstream face in pounds per square inch is determined by the equation,

$$\sigma_{ZD} = \frac{1}{144}\left[\frac{\Sigma W+U}{A_1} - M_1\frac{T_1-lg_1}{I_1}\right]$$

which reduces to

$$\frac{\sigma_{ZD}}{\frac{\Sigma W+U}{A_1}} = \frac{3}{144}\left(\frac{1+\frac{R_D}{R_0}}{1+2\frac{R_D}{R_0}}\right) \tag{72}$$

σ_{ZD} is evaluated from the curve given in figure 21.

77. Cracking at Downstream Face.—Equations and curves for a cantilever cracked at the downstream face are derived in the same manner as those for cracking at the upstream face. Figure 22 shows a horizontal

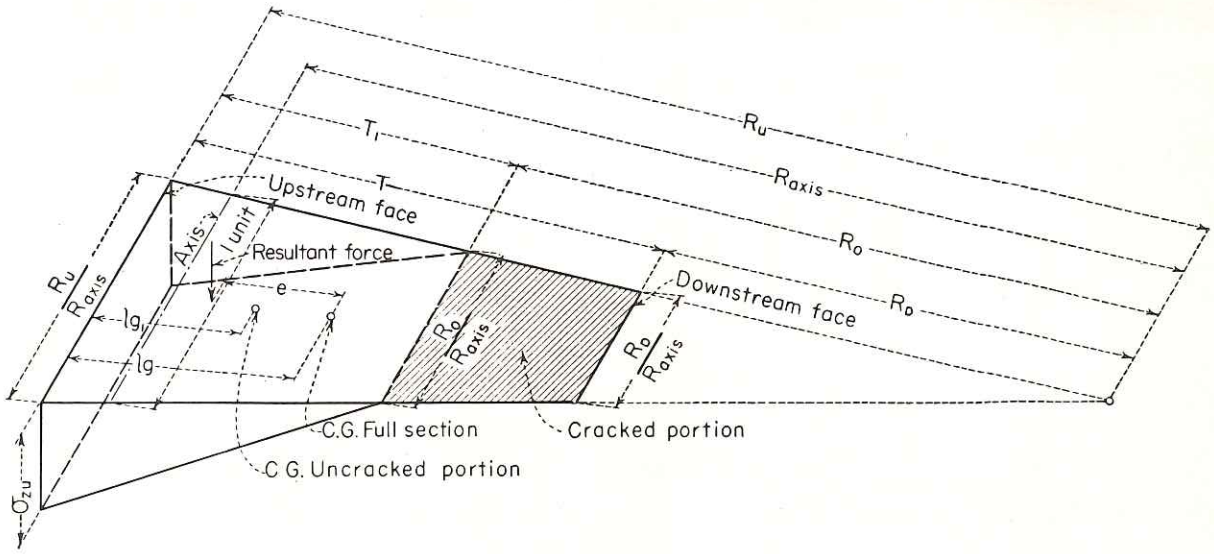


FIGURE 22—HORIZONTAL SECTION OF CANTILEVER CRACKED AT DOWNSTREAM FACE

section of a cantilever cracked at the downstream face. By considering forces, moments, and stresses in figure 22, the following equations may be derived:

$$F = \frac{lg-e}{R_U} = \frac{1}{2}\left[\frac{1-\left(\frac{R_o}{R_U}\right)^2}{\frac{R_o}{R_U}+2}\right] \tag{73}$$

$$T_1 = \left(\frac{R_U}{R_{axis}} - \frac{R_o}{R_{axis}}\right)R_{axis} \tag{74}$$

$$\frac{R_o}{R_{axis}} = \frac{R_U}{R_{axis}} \cdot \frac{R_o}{R_U} \tag{75}$$

$$\frac{A_1}{T_1\left(\frac{R_U}{R_{axis}}\right)} = \frac{1}{2}\left(1+\frac{R_o}{R_U}\right) \tag{76}$$

$$\frac{I_1}{T_1^3\left(\frac{R_U}{R_{axis}}\right)} = \frac{1}{36}\left[\frac{1+4\frac{R_o}{R_U}+\left(\frac{R_o}{R_U}\right)^2}{1+\frac{R_o}{R_U}}\right] \tag{77}$$

$$\frac{M_1}{(\Sigma W+U)T_1} = \frac{1}{6}\left[\frac{1+4\frac{R_o}{R_U}+\left(\frac{R_o}{R_U}\right)^2}{2+3\frac{R_o}{R_U}+\left(\frac{R_o}{R_U}\right)^2}\right] \tag{78}$$

$$\frac{\sigma_{ZU}}{\frac{\Sigma W+U}{A_1}} = \frac{3}{144}\left[\frac{1+\frac{R_O}{R_U}}{2+\frac{R_O}{R_U}}\right] \tag{79}$$

Curves for equations 73, 78, and 79 are given in figure 23. Values of A_1 and I_1 in equations 76 and 77 are determined from curves in figure 16, using R_O/R_U for R_D/R_U and multiplying by proper factors.

78. Deflections of Cracked Cantilever.—The cracking of a cantilever increases its flexibility. Consequently, effects of cracking must be included in calculations of deflections. This is accomplished by assuming that the cracked portion of the concrete offers no resistance to radial cantilever movements. To disregard cracked concrete in the deflection equation, A_1, I_1, and M_1 are substituted for A, I, and M in equation 59, transforming it to the following equation for a cracked cantilever,

$$_{L \text{ and } R}\Delta r = \sum\left(M_A\alpha + V_A\alpha_2 + \sum\frac{M_1}{E_C I_1}\Delta z\right)\Delta z + \left(V_A r^* + M_A\alpha_2^* + \sum\frac{VK}{A_1 G}\Delta z\right) \tag{80}$$

Terms indicated by an asterisk apply only to the maximum cantilever. Cantilever foundation movements to be used if the horizontal base is cracked are

$$\alpha = \frac{r}{r_1}\cdot\frac{T^2}{T_1^2}[\alpha]$$

$$r = \frac{r}{r_1}[r]$$

$$\alpha_2 = \frac{r}{r_1}\cdot\frac{T}{T_1}[\alpha_2]$$

Factors preceding brackets correct movements to the reduced thickness of the cantilever base. Factors within brackets are for total sections.

Since amounts of cracking and accompanying radial deflections are functions of total loads acting on a cantilever, the use of unit loads is impossible. Although it is necessary to calculate radial deflections for each trial load applied to the cantilever, the task of making the adjustments is not greatly complicated, since experience quickly shows the effects of various load changes.

Cracked cantilever movements to be used in trial load adjustments of radial deflections include effects of only those loads or forces capable of being distributed between the arch and cantilever elements. Loads and

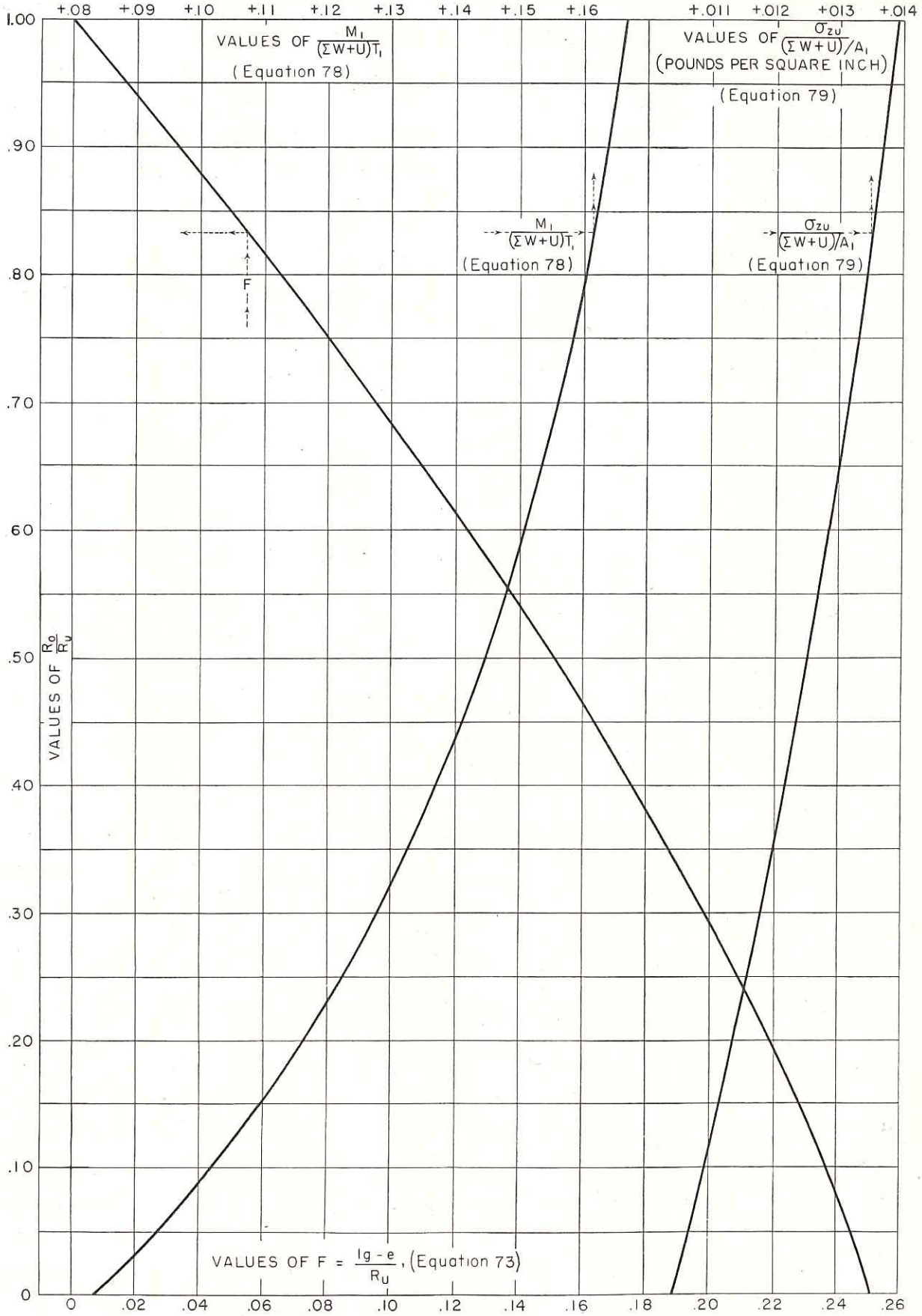


FIGURE 23—PROPERTIES OF CANTILEVER CRACKED AT DOWNSTREAM FACE

forces applied to a dam before grouting are resisted by the cantilevers only. Movements due to these loads and forces are not included in the adjustments. Following this general principle, the procedure for the ordinary case of a dam built in vertical sections consists of computing deflections due to concrete weight and subtracting these deflections from the total cracked cantilever movements. This gives correct cantilever deflections to be used in trial load adjustments.

It is evident that the major effect of cantilever cracking is to increase radial movements. The less important effects on tangential and angular movements are omitted in the present method of analysis.

COMPUTATION FORMS

79. General.—Figures 24 to 28, inclusive, show forms being used by the Bureau of Reclamation in analyzing cantilever elements of arch dams. Quantities indicated on the forms are shown symbolically, descriptively, or by both methods, depending on the space available or upon the adaptability for use. The computation sheets were developed for general use, and for this reason some of them are applicable to parallel-side cantilevers as well as to radial-side cantilevers. Figures 25 and 26 are of this type. Most of the forms shown include headings only.

80. Use of Forms.—Properties at horizontal cantilever sections are computed and tabulated on the form shown in figure 24. The following form, figure 25, is used in determining weights and moments of concrete and vertical water load. For example, for radial-side cantilevers, columns 1, 2, and 3 are used for tabulating integrator readings and their differences. Column 4 is used for tabulating areas and moments. The remaining column headings are self-explanatory.

Figure 26 is a general sheet for deflections of an uncracked cantilever. The form is arranged for calculating radial deflections due to bending moment and radial shear, as indicated at *a*. The left side of the sheet can be used to compute angular movements due to twist moments by substituting $2G$ for E, and omitting the last summation, as indicated at *b*. The right side can be used for calculating tangential movements due to tangential shear if V is replaced by the tangential shear force H, as indicated at *c*.

Computations for a cracked cantilever are made on forms shown in figures 27 and 28. The amount of cracking, properties of uncracked section, resultant moment, radial deflections, and stresses can be evaluated on these sheets.

On the form shown in figure 27 the function F is first calculated as indicated. For cracking at the upstream face, the ratio R_D/R_O is found from the curve in figure 21. A_1 and I_1, and the resultant moment M_1 are next determined from curves on figures 16 and 21. On the sheet shown in figure 28, radial deflections can now be computed by applying values determined on figure 27. Finally, the stress σ_Z, can be obtained from the curve on figure 21. If the cantilever is cracked at the downstream face, properties, moments, deflections, and stresses can be evaluated by a similar procedure, using curves in figures 16 and 23.

.....DAM. STUDY NO.....

RADIAL-SIDE CANTILEVER.....

DESCRIPTION OF CANTILEVER.....

By.....Date.....

EL.	Properties of Cantilever at Horizontal Sections																
	T	U.S. Proj.	D.S. Proj.	R_{axis}	R_U	r	R_D	$\frac{R_U}{R_{axis}}$	$\frac{R_D}{R_U}$	$\frac{A}{T\left(\frac{R_U}{R_{axis}}\right)}$	A	$\frac{lg}{T}$	lg	T-lg	$\frac{I}{T^3\left(\frac{R_U}{R_{axis}}\right)}$	I	$\frac{I}{I}$

FIGURE 24—COMPUTATION FORM FOR PROPERTIES OF HORIZONTAL SECTIONS

.....DAM.....SECTION STUDY NO.....

RADIAL SIDE CANTILEVER

FORCE.....CANTILEVER .....

Res. W.S. El.=.....T.W.S. El.=..... .....Unit Weight=. .....By.....Date.....

EL.	Integrator Readings			Areas and Moments	Force	Eccen. = e	Moment=.....
	1	2	3	4	=.....	C.G. of base	C.G. of base

FIGURE 25—COMPUTATION FORM FOR WEIGHTS AND MOMENTS OF VERTICAL LOADS

α = ... DAM STUDY NO. .....

α_2 = ...

RADIAL SIDE, CANTILEVER

γ = ... DEFLECTION..... LOAD NO..... CANTILEVER.....

ϕ = ... δ = ... $\frac{I}{E}$ = ... $\frac{K}{G} = \frac{}{G}$ = ... By ..... Date .....

EL.	Deflection due to						Deflection due to					Total Deflection
	Moment = M	$\frac{I}{EI}$	$\frac{M}{EI}$	$\frac{\Delta z}{2}$	$\Sigma\left(\frac{M}{EI}\right)\Delta z$	$\Sigma\Delta z\left(\Sigma\frac{M}{EI}\Delta z\right)$	Horizontal Force = V	$\frac{I}{A}$	$\frac{V}{A}$	$\left(\frac{K}{G}\right)\frac{\Delta z}{2}$	$\Sigma\left(\frac{V}{A}\right)\left(\frac{K}{G}\Delta z\right)$	
					Deflections due to Radial Loads (a)							
	Moment = ~~M~~	$\frac{I}{2GI}$	$\frac{~~M~~}{2GI}$	$\frac{\Delta z}{2}$	$\Sigma\left(\frac{~~M~~}{2GI}\right)\Delta z$							
	Angular Movements due to Twist Loads (b)						Tangential Force = H	$\frac{I}{A}$	$\frac{H}{A}$	$\left(\frac{K}{G}\right)\frac{\Delta z}{2}$	$\Sigma\left(\frac{H}{A}\right)\left(\frac{K}{G}\Delta z\right)$	
									Deflections due to Tangential Loads (c)			

FIGURE 26—COMPUTATION FORM FOR DEFLECTIONS OF UNCRACKED CANTILEVER

..... DAM STUDY NO.....

RADIAL-SIDE CRACKED CANTILEVER STRESS ANALYSIS

HORIZONTAL FORCES, RESULTANT MOMENTS, AND PROPERTIES. CANTILEVER.....

Trial Load No..... By..... Date . .

	Loads, Forces, Moments, and Eccentricities					Properties of Cracked Cantilever at Horizontal Sections									Resultant Moment	
Cracked at U.S. Face		V	M_P	$\Sigma M + M_U$	$e = \frac{\Sigma M + M_U}{\Sigma W + U}$	$\frac{R_D}{R_{axis}}$	$\frac{R_U - lg + e}{R_D}$	$\frac{R_D}{R_O}$	$\frac{R_D}{R_{axis}} \div \frac{R_D}{R_O}$	$\left(\frac{R_O}{R_{axis}} - \frac{R_D}{R_{axis}}\right) R_{axis}$	$\frac{A_1}{T_1 (R_O/R_{axis})}$		$\frac{I_1}{T_1^3 (R_O/R_{axis})}$		$\frac{M_1}{(\Sigma W + U) T_1}$	
Cracked at D.S. Face		V	M_P	$\Sigma M + M_U$	$e = \frac{\Sigma M + M_U}{\Sigma W + U}$	$\frac{R_U}{R_{axis}}$	$\frac{lg - e}{R_U}$	$\frac{R_O}{R_U}$	$\frac{R_U}{R_{axis}} \times \frac{R_O}{R_U}$	$\left(\frac{R_U}{R_{axis}} - \frac{R_O}{R_{axis}}\right) R_{axis}$	$\frac{A_1}{T_1 (R_U/R_{axis})}$		$\frac{I_1}{T_1^3 (R_U/R_{axis})}$		$\frac{M_1}{(\Sigma W + U) T_1}$	
EL.	Load factor	Horizontal force	Moment of horiz. force	Total bending moment	Eccentricity		F		R_O/R_{axis}	T_1		A_1		I_1		M_1

FIGURE 27—COMPUTATION FORM FOR FORCES, MOMENTS, AND PROPERTIES OF A CRACKED CANTILEVER

For crack at U.S. Face $\frac{r}{r_1} = \frac{r}{R_D + T_1/2}$

For crack at D.S. Face $\frac{r}{r_1} = \frac{r}{R_U - T_1/2}$

$\left(\frac{r}{r_1} \cdot \frac{T^2}{T_2^2}\right) \alpha =$ .....

$\left(\frac{r}{r_1} \cdot \frac{T}{T_1}\right) \alpha_2 =$ ..... $\left(\frac{r}{r_1}\right) \sigma =$ ..... $\frac{1}{E_C} =$ ..... $\frac{K}{G} = \frac{}{G} =$ .....

..... DAM STUDY NO.....

RADIAL-SIDE CRACKED CANTILEVER STRESS ANALYSIS

RADIAL DEFLECTIONS AND STRESSES. TRIAL LOAD NO.....CANTILEVER.....

By..... Date.....

EL.	Radial Deflection								Stresses at Uncracked Face Pounds per Square Inch				
	Due to Bending				Due to Shear			Total					
	$\frac{M_1}{E_C I_1}$	$\frac{\Delta z}{2}$	$\Sigma \left(\frac{M_1}{E_C I_1}\right) \Delta z$	$\Sigma \Delta z \left(\Sigma \frac{M_1}{E_C I_1} \Delta z\right)$	$\frac{V}{A_1}$	$\left(\frac{K}{G}\right) \frac{\Delta z}{2}$	$\Sigma \left(\frac{V}{A_1}\right) \left(\frac{K}{G} \Delta z\right)$		$\frac{\sigma_z}{(\Sigma W + U)/A_1}$	Vertical	Tan ϕ	Sec^2 ϕ	Parallel to Face

FIGURE 28—COMPUTATION FORM FOR RADIAL DEFLECTIONS AND STRESSES OF A CRACKED CANTILEVER

CHAPTER V—ANALYSIS OF ARCH ELEMENTS

INTRODUCTION

81. Preliminary Statement.—During the first use of the trial load method, arch elements were analyzed by the voussoir summation process.[4] Principles forming the basis for this process are given in various text books on concrete design, under the subject of arch bridges. Since formulas deduced in such books are generally simplified to include only bending effects, the equations were amplified to include effects of rib-shortening and shear along vertical radial sections. The use of the summation method required considerable time for the calculation of arch deflections. Therefore, the completion of a trial load analysis for an arch dam was a rather lengthy process.

A method of analyzing arch elements, based on the substitution of integrals for summations, was developed. This resulted in an appreciable saving of time, especially after effects of tangential shear and twist were included. A further saving in time was effected by computing tables of integrals needed in the calculations and tabulating functions for different types of unit loads which can be used in building up total loads carried by arch elements. Although the integral tables were prepared for circular arches of constant thickness, they may be adapted to the analysis of variable thickness arches.

This chapter presents theoretical considerations and formulas for analyzing general arch types, uniform thickness circular arches, variable thickness arches, and arches with fillets, together with brief discussions of other types of arches. Procedures followed in computations for various types of arches are given in the succeeding chapter on Calculation of Arch Data.

82. Arch Elements.—Horizontal sections of an arch dam are statically indeterminate elements with elastic abutments. A unit element is the volume of a dam intercepted by two horizontal planes one unit apart. For practical purposes and ease of computation, upstream and down-

[4]Howell, C. H. and Jaquith, A. C., "Analysis of Arch Dams by the Trial Load Method," Trans. Am. Soc. C. E., vol. 93, 1929, pp. 1191-1316.

stream faces of arch elements are assumed vertical. Arch elements resist radial forces applied at the upstream or downstream faces, tangential forces applied at the center lines, and horizontal and twisting moments applied at the center lines. The analysis of the elements consists of calculating movements and stresses due to these forces and moments. Movements are determined by the usual theory of flexure for curved beams with effects of rib-shortening, transverse shear, and yielding abutments included. Stresses are evaluated by equations for bending and direct stress. Equations for stresses are derived and discussed in chapter VIII, where a complete discussion is given for all kinds of stresses at any point in an arch dam.

83. Loads.—The various forces applied to arch elements during an analysis include radial, tangential, twist, and temperature loads. To build up these forces, a system of unit loads is used. Obviously, the unit load for temperature is a unit change in concrete temperature. For other applied loads, the units are uniform, triangular, or concentrated in their application. Unit loads may be symmetrical or nonsymmetrical. Symmetrical loads are the same on both left and right parts of the arch; nonsymmetrical loads are placed on one side of the arch only, the other side being unloaded. Sketches of unit patterns for radial, tangential, and twist loads are shown in figure 29.

Unit radial loads include a uniform load, number 1; triangular loads, numbers 2, 3, 4, and 5; and a concentrated load, all of which act in a radial direction and are positive when directed toward the arch center. The intensity of load, P, shown on figure 29, is usually 1,000 pounds per square foot at the upstream face. If necessary, the loads can be applied to the downstream face.

Tangential loads are applied tangentially along the arch center line. The uniform load, number 1, is positive in direction when applied from left to right for both parts of the arch. Triangular loads, numbers 2, 3, 4, and 5, and the concentrated load are positive in direction when applied from the abutment toward the crown in both parts of the arch. The value of the unit, P, is usually 1,000 pounds per square foot.

Twist loads are bending couples applied along the arch center line. The uniform load, number 1, is positive in direction of rotation when applied clockwise from left to right for both parts of the arch. Triangular loads, numbers 2, 3, 4, and 5, and the concentrated load are positive in direction of rotation when applied clockwise in the left part of the arch and counterclockwise in the right part of the arch. The value of the unit couple, P, is usually 1,000 foot-pounds per square foot.

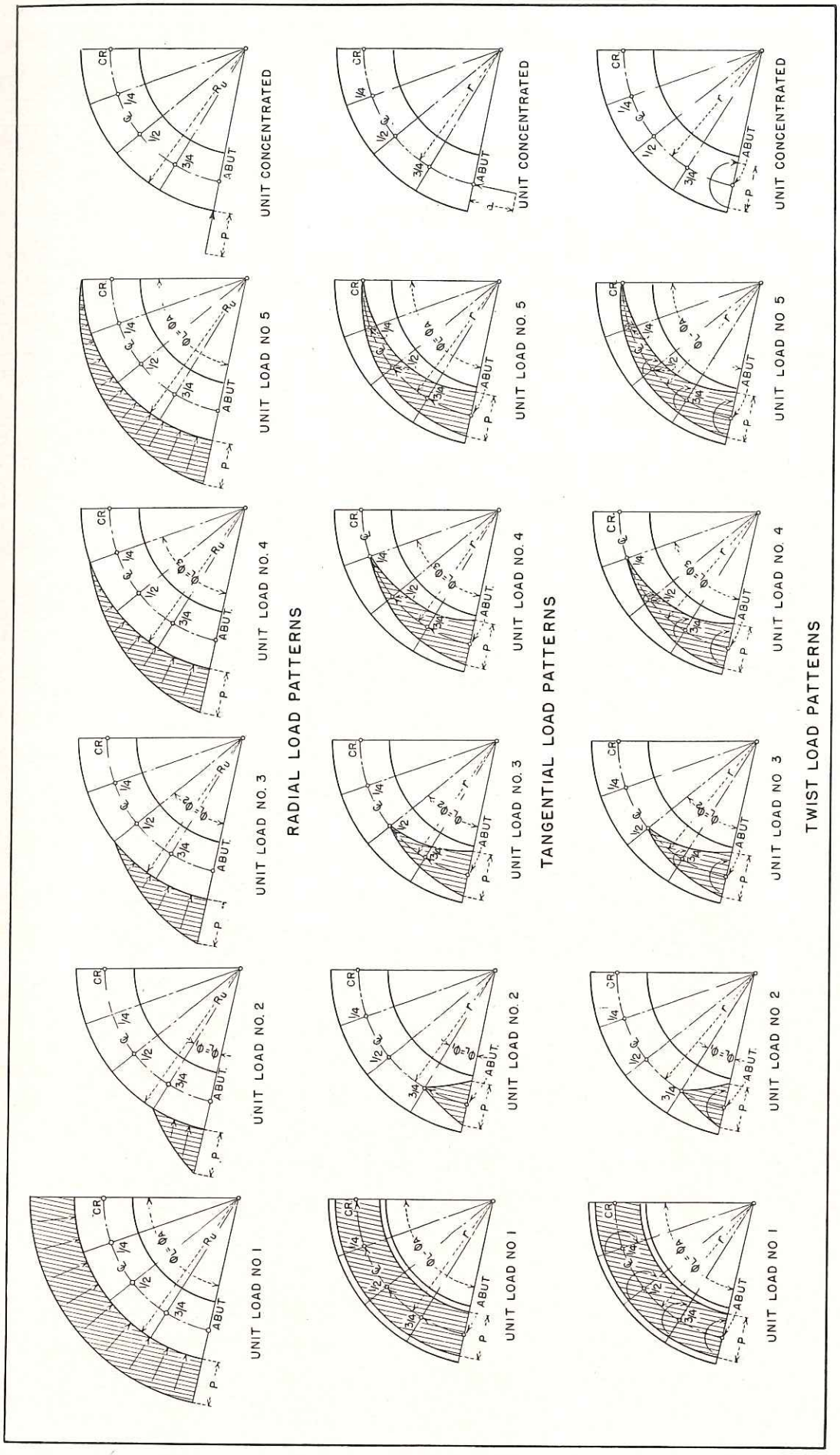


FIGURE 29—UNIT ARCH LOAD PATTERNS SHOWING POSITIVE APPLICATION

A temperature load is required in analyzing the effect of a change in mass concrete temperature after the dam has been grouted. It is not an adjustment load, such as radial, tangential, or twist loads. It is a definite effect that is assigned to the arches as an initial load. A positive load is one that denotes a rise in temperature. A convenient unit to use in computing temperature effects is one degree Fahrenheit.

84. Assumptions.—The following assumptions are made in analyzing arch elements:

1. The material in the arches is homogeneous and isotropic.
2. Hooke's law applies, and the proportional elastic limit is not exceeded.
3. A plane section before bending remains plane after bending.
4. Direct stresses have a linear variation from extrados to intrados.
5. The modulus of elasticity in direct stress is the same for tension and compression.
6. The ratio of the modulus of elasticity in direct stress to the shearing modulus of elasticity, E/G, equals $2(1 + \mu)$, in which μ is Poisson's ratio. To account for nonlinear distribution of shear stress, the value of K, ratio of detrusion caused by actual shear distribution to detrusion caused by an equivalent shear distributed uniformly, is assumed to be 1.25. With a Poisson's ratio of 0.2, the value of K/G is $(1.25 \times 2.4)/E$, which equals $3/E$. This relationship of moduli is included in the formulas and tabulated arch and load constants.
7. Temperature strains are proportional to temperature changes.
8. A temperature change is uniform throughout an arch.
9. Abutments of arch elements are radial.

GENERAL ARCH THEORY

85. General Statement.—The arch theory is developed for the general case of a nonsymmetrical arch with nonsymmetrical loads. Equations derived for this case are applicable to any type of arch with any type of loading. Later in the chapter the equations are modified for special use in analyzing uniform thickness and variable thickness circular arches.

In general, the method of solution consists of cutting the loaded arch at the crown and substituting a moment, thrust, and shear at the crown to replace the influence of the other side, see figure 30.

A load placed at any point on either part of the arch causes the crown point to be rotated and displaced in a horizontal plane. These movements

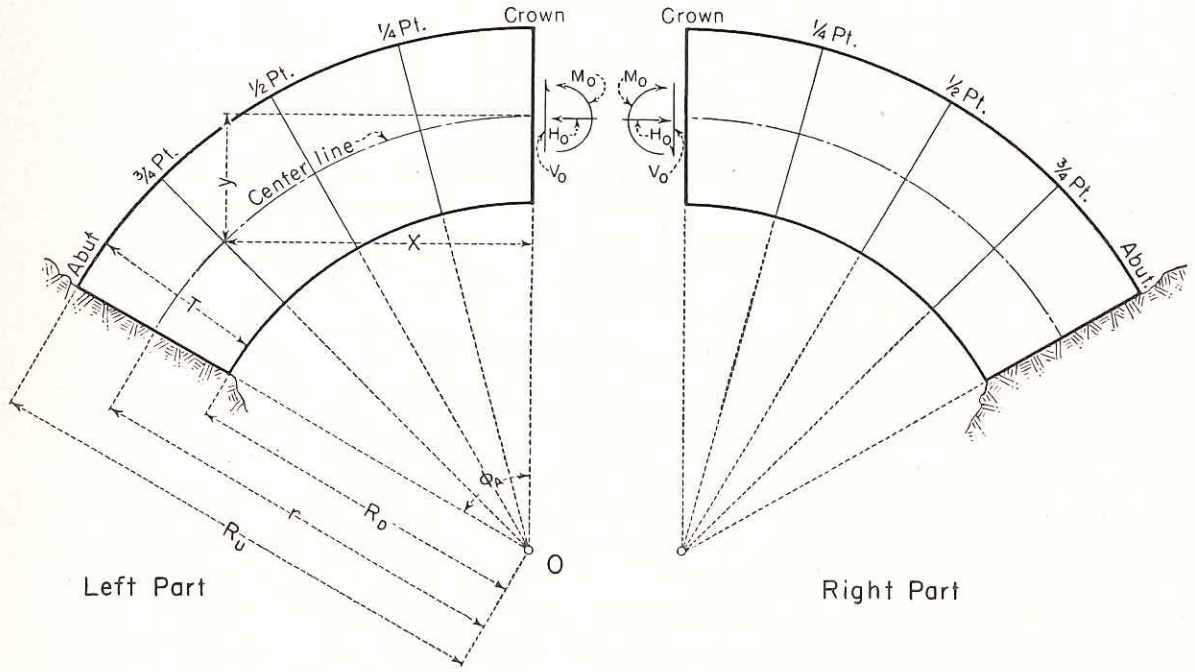


FIGURE 30—CIRCULAR ARCH CUT AT CROWN

are due partly to bending of the arch ring, partly to changes in length of the arch center line, and partly to detrusions caused by shearing stresses. Since the arch must remain continuous, the crown point of both parts of the arch must have the same rotation and displacement. By equating radial, tangential, and angular movements at the crown, three equations are obtained from which the unknown moment, thrust, and shear at the crown may be determined. These crown forces and moments are equal and opposite on the two parts of the arch, as shown in figure 30. In determining other forces and movements, each part of the arch may be considered as a curved cantilever beam. In the following discussions the left and right parts of the arch are the parts viewed when looking upstream.

86. Notation.—The following quantities are general and apply in all arch theory explained in this chapter, except as otherwise noted in later sections. Other quantities are introduced and defined wherever necessary in order to explain assumptions used in developing arch equations for different types of arches. Arch dimensions, angles, moments, forces, and movements are measured in the horizontal plane.

Crown = point where arch is assumed to be cut.

R_U = radius to upstream face.

R_D = radius to downstream face.

r = radius to center line of arch.

T = thickness of arch in a radial direction.

A = area of a radial arch section.

I = moment of inertia of a radial arch cross section about a vertical line through its center of gravity.

M, H, V = moment, thrust, and shear, respectively.

E_C = modulus of elasticity of concrete in tension and compression.

G_C = modulus of elasticity of concrete in shear.

μ = Poisson's ratio.

K = ratio of deflection caused by actual shear distribution to deflection caused by an equivalent total shear distributed uniformly.

c = coefficient of thermal expansion for concrete per degree Fahrenheit.

t = temperature change in degrees Fahrenheit, the plus sign indicating a rise in temperature.

α = angular movement of arch abutment due to unit moment at abutment.

β = tangential movement of arch abutment due to unit thrust at abutment.

γ = radial movement of arch abutment due to unit shear at abutment.

α_2 = angular movement of arch abutment due to unit shear at abutment; or radial movement of arch abutment due to unit moment at abutment.

θ = angular movement of arch center line.

Δr = radial deflection of arch center line.

Δs = tangential deflection of arch center line.

Subscripts.—Subscripts following a quantity such as M_0 or H_L, designate the following: $_0$ = crown of arch; $_A$ = abutment of arch; $_L$ = applied load on left part; $_R$ = applied load on right part. Initial subscripts preceding a quantity, such as $_LB_1$ or $_RD_1$, designate the following: $_L$ = left part of arch; $_R$ = right part of arch; $_A$ = abutment of arch.

87. Convention of Signs.—The convention of signs is shown in figure 31. Positive moments cause compression at the upstream face, positive thrusts cause compression, and positive shears cause positive moments on the section of the arch to the left, in the left part of the arch. In the right part of the arch a positive shear produces a positive moment in the section to the right, except V_O which acts as shown in figure 31. Directions of application of positive loads are given in section 83.

Positive moments, thrusts, and shears, M_L, H_L, V_L, or M_R, H_R, V_R, due to applied loads, are in the same direction as moments, thrusts, and shears of positive radial loads. Following this convention, moments, thrusts, and shears of all positive triangular and concentrated loads are positive, except

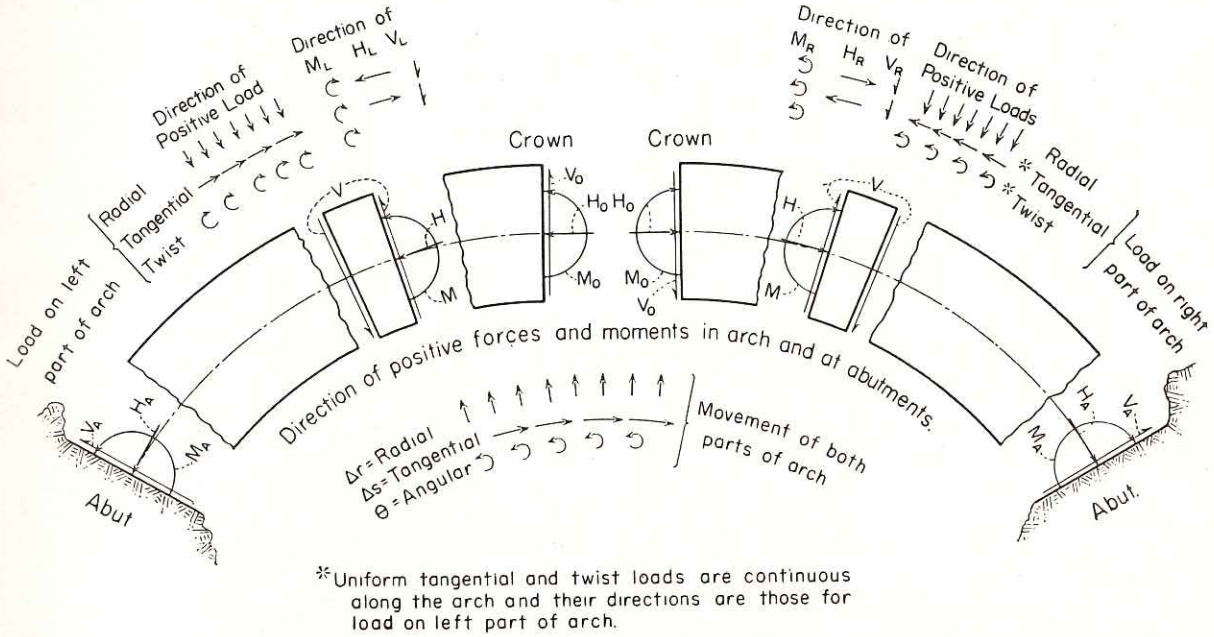


FIGURE 31—DIRECTION OF POSITIVE LOADS, FORCES, MOMENTS, AND MOVEMENTS

thrusts of tangential loads which are negative. Since the portion of the uniform tangential or twist load on the right part of the arch is applied in the same direction as the load on the left part, the M_R, H_R, and V_R for these loads change sign.

Positive radial deflections are upstream, positive tangential deflections are toward the right, and positive angular movements are counterclockwise. Sign conventions for abutment movements and loads are given in chapter III.

88. Movements of Arch at Crown.—Additional nomenclature used in deriving equations for movements of the arch at the crown, equations for solution of crown forces, section 89, and equations for moment, thrust, and shear at any arch point, except the crown, are shown in figure 32 and listed below.

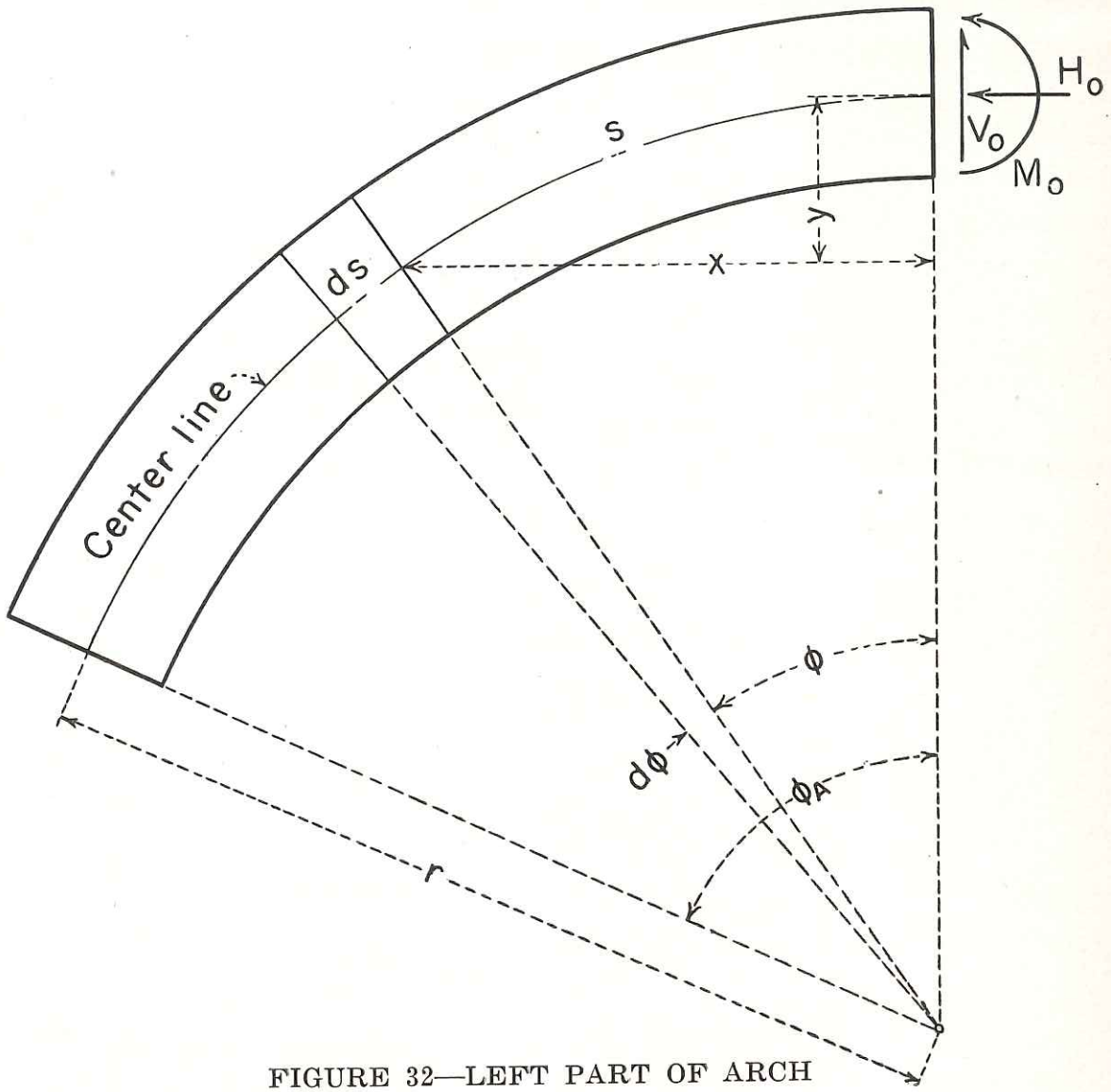


FIGURE 32—LEFT PART OF ARCH

s = length along center line of arch from crown to any arch point.
ϕ = angle from crown to any arch point.
ϕ_A = angle from crown of arch to abutment.
x, y = coordinates of any arch point with origin at crown.

Consider a differential element of length, *ds,* in the left part of the arch, as shown in figure 32. From mechanics, equations for arch movements at the crown, due to a moment, thrust, and shear acting on the differential elements, are:

$$d\theta_o = \frac{M ds}{E_c I} \tag{81}$$

$$d(\Delta r)_o = \frac{M x ds}{E_c I} - \frac{H}{E_c A} \sin\phi\, ds + \frac{K}{G_c}\frac{V}{A} \cos\phi\, ds + ct \sin\phi\, ds \tag{82}$$

$$d(\Delta s)_o = -\frac{M y ds}{E_c I} - \frac{H}{E_c A} \cos\phi\, ds - \frac{K}{G_c}\frac{V}{A} \sin\phi\, ds + ct \cos\phi\, ds \tag{83}$$

The first term in these equations gives the movement due to bending, the second term gives the movement due to rib-shortening, the third term gives the movement due to shear detrusion, and the last term gives the movement due to temperature. If the equations are integrated from the crown to the abutment, and rock abutment movements added, the following equations result:

$$ {}_L\theta_o = \int_o^s \frac{M\,ds}{E_c\,I} + M_A\,\alpha + V_A\,\alpha_2 \tag{84} $$

$$ {}_L\Delta r_o = \int_o^s \frac{Mx\,ds}{E_c\,I} - \int_o^s \frac{H}{E_c A}\sin\phi\,ds + \int_o^s \frac{K}{G_c}\frac{V}{A}\cos\phi\,ds $$
$$ + (\alpha x_A + \alpha_2\cos\phi_A)M_A - H_A\,\beta\sin\phi_A + (\tau\cos\phi_A + \alpha_2\,x_A)V_A + \int_o^s ct\sin\phi\,ds \tag{85} $$

$$ {}_L\Delta s_o = -\int_o^s \frac{My\,ds}{E_c\,I} - \int_o^s \frac{H}{E_c A}\cos\phi\,ds - \int_o^s \frac{K}{G_c}\frac{V}{A}\sin\phi\,ds $$
$$ -(\alpha y_A + \alpha_2\sin\phi_A)M_A - H_A\,\beta\cos\phi_A - (\tau\sin\phi_A + \alpha_2\,y_A)V_A + \int_o^s ct\cos\phi\,ds \tag{86} $$

By substituting,

$$ M = M_o + H_o y + V_o x - M_L $$

$$ H = H_o\cos\phi - V_o\sin\phi + H_L $$

$$ V = H_o\sin\phi + V_o\cos\phi - V_L $$

and using K/G_C equals $3/E_C$, as discussed in section 84, equations 84, 85, and 86 for the left part of the arch become

$$ {}_L\theta_o = M_o\left[\int_o^s \frac{ds}{E_c I} + \alpha\right] + H_o\left[\int_o^s \frac{y\,ds}{E_c I} + \alpha y_A + \alpha_2\sin\phi_A\right] + V_o\left[\int_o^s \frac{x\,ds}{E_c\,I} + \alpha x_A + \alpha_2\cos\phi_A\right] $$
$$ -\left[\int_o^s \frac{M_L\,ds}{E_c\,I} + {}_AM_L\,\alpha + {}_AV_L\,\alpha_2\right] \tag{87} $$

$$ {}_L\Delta r_o = M_o\left[\int_o^s \frac{x\,ds}{E_c I} + \alpha x_A + \alpha_2\cos\phi_A\right] + H_o\left[\int_o^s \frac{xy\,ds}{E_c I} - \int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A} + 3\int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A}\right. $$
$$ \left. + \alpha x_A y_A + \alpha_2 x_A\sin\phi_A - \beta\sin\phi_A\cos\phi_A + \tau\sin\phi_A\cos\phi_A + \alpha_2 y_A\cos\phi_A\right] $$
$$ + V_o\left[\int_o^s \frac{x^2\,ds}{E_c\,I} + \int_o^s \frac{\sin^2\phi\,ds}{E_c\,A} + 3\int_o^s \frac{\cos^2\phi\,ds}{E_c\,A} + \alpha x_A^2 + 2\,\alpha_2 x_A\cos\phi_A + \beta\sin^2\phi_A + \tau\cos^2\phi_A\right] $$
$$ -\left[\int_o^s \frac{M_L x\,ds}{E_c\,I} + \int_o^s \frac{H_L\sin\phi\,ds}{E_c\,A} + 3\int_o^s \frac{V_L\cos\phi\,ds}{E_c\,A} - \int_o^s ct\sin\phi\,ds + {}_AM_L(\alpha x_A + \alpha_2\cos\phi_A)\right. $$
$$ \left. + {}_AH_L\,\beta\sin\phi_A + {}_AV_L(\tau\cos\phi_A + \alpha_2 x_A)\right] \tag{88} $$

$$
\begin{aligned}
{}_L\Delta s_0 = -M_0\left[\int_0^s \frac{y\,ds}{E_c\,I} + \alpha y_A + \alpha_2 \sin\phi_A\right] - H_0\left[\int_0^s \frac{y^2 ds}{E_c\,I} + \int_0^s \frac{\cos^2\phi\,ds}{E_c\,A} + 3\int_0^s \frac{\sin^2\phi\,ds}{E_c\,A}\right. \\
\left. + \alpha y_A^2 + 2\alpha_2 y_A \sin\phi_A + \beta\cos^2\phi_A + \tau\sin^2\phi_A\right] - V_0\left[\int_0^s \frac{xy\,ds}{E_c\,I} - \int_0^s \frac{\sin\phi\cos\phi\,ds}{E_c\,A}\right. \\
\left. + 3\int_0^s \frac{\sin\phi\cos\phi\,ds}{E_c\,A} + \alpha x_A y_A + \alpha_2 y_A \cos\phi_A - \beta\sin\phi_A\cos\phi_A + \tau\sin\phi_A\cos\phi_A + \alpha_2 x_A \sin\phi_A\right] \\
+ \left[\int_0^s \frac{M_L\,y\,ds}{E_c\,I} - \int_0^s \frac{H_L\cos\phi\,ds}{E_c\,A} + 3\int_0^s \frac{V_L\sin\phi\,ds}{E_c\,A} + {}_AM_L(\alpha y_A + \alpha_2\sin\phi_A) - {}_AH_L\,\beta\cos\phi_A\right. \\
\left. + {}_AV_L(\tau\sin\phi_A + \alpha_2 y_A) + \int_0^s c\,t\cos\phi\,ds\right] \qquad (89)
\end{aligned}
$$

Inspection shows that some of the multipliers for M_0, H_0, and V_0 are duplicated in the equations. If symbols A_1, B_1, B_2, B_3, C_1, C_2, D_1, D_2, and D_3 are used for these multipliers and for the terms depending on loads, the equations may be written

$$ {}_L\theta_0 = {}_LA_1\,M_0 + {}_LB_1\,H_0 + {}_LC_1\,V_0 - {}_LD_1 \qquad (90) $$

$$ {}_L\Delta r_0 = {}_LC_1\,M_0 + {}_LB_2\,H_0 + {}_LC_2\,V_0 - {}_LD_2 \qquad (91) $$

$$ {}_L\Delta s_0 = -{}_LB_1\,M_0 - {}_LB_3\,H_0 - {}_LB_2\,V_0 + {}_LD_3 \qquad (92) $$

Equations for the right part, developed in the same manner, are

$$ {}_R\theta_0 = -{}_RA_1\,M_0 - {}_RB_1\,H_0 + {}_RC_1\,V_0 + {}_RD_1 \qquad (93) $$

$$ {}_R\Delta r_0 = {}_RC_1\,M_0 + {}_RB_2\,H_0 - {}_RC_2\,V_0 - {}_RD_2 \qquad (94) $$

$$ {}_R\Delta s_0 = {}_RB_1\,M_0 + {}_RB_3\,H_0 - {}_RB_2\,V_0 - {}_RD_3 \qquad (95) $$

89. Solution of Crown Forces.—Movements of each part of the arch, cut at the crown, may be determined by equations 90 to 95, inclusive. Using these equations and following the method outlined in section 85, the three general equations needed to solve the unknown crown forces, M_0, H_0, and V_0 are

$$ \left[{}_LA_1 + {}_RA_1\right]M_0 + \left[{}_LB_1 + {}_RB_1\right]H_0 + \left[{}_LC_1 - {}_RC_1\right]V_0 = \left[{}_LD_1 + {}_RD_1\right] \qquad (96) $$

$$ \left[{}_LC_1 - {}_RC_1\right]M_0 + \left[{}_LB_2 - {}_RB_2\right]H_0 + \left[{}_LC_2 + {}_RC_2\right]V_0 = \left[{}_LD_2 - {}_RD_2\right] \qquad (97) $$

$$ \left[{}_LB_1 + {}_RB_1\right]M_0 + \left[{}_LB_3 + {}_RB_3\right]H_0 + \left[{}_LB_2 - {}_RB_2\right]V_0 = \left[{}_LD_3 + {}_RD_3\right] \qquad (98) $$

By combining constants in the brackets so that $[{}_LA_1 + {}_RA_1] = A_1$, $[{}_LB_1 + {}_RB_1] = B_1$, and so forth, the equations reduce to

$$A_1 M_0 + B_1 H_0 + C_1 V_0 = D_1 \tag{99}$$

$$C_1 M_0 + B_2 H_0 + C_2 V_0 = D_2 \tag{100}$$

$$B_1 M_0 + B_3 H_0 + B_2 V_0 = D_3 \tag{101}$$

The various quantities included in the A_1, B_1, C_1, B_2, C_2, B_3, D_1, D_2, and D_3 terms in these equations are given in table 3. Quantities in the table are obtained from equations 87, 88, and 89 for the left part of the arch, and from similar equations for the right part.

By solving equations 99, 100, and 101 simultaneously, formulas for M_0, H_0, and V_0 are derived.

$$M_0 = \frac{I}{K}\left[D_1(B_3 C_2 - B_2^2) - D_3(B_1 C_2 - C_1 B_2) - D_2(B_3 C_1 - B_1 B_2)\right] \tag{102}$$

$$H_0 = \frac{I}{K}\left[-D_1(B_1 C_2 - B_2 C_1) + D_3(A_1 C_2 - C_1^2) + D_2(B_1 C_1 - A_1 B_2)\right] \tag{103}$$

$$V_0 = \frac{I}{K}\left[-D_1(B_3 C_1 - B_1 B_2) + D_3(B_1 C_1 - A_1 B_2) + D_2(A_1 B_3 - B_1^2)\right] \tag{104}$$

in which

$$K = A_1(B_3 C_2 - B_2^2) - B_1(B_1 C_2 - C_1 B_2) - C_1(B_3 C_1 - B_1 B_2) \tag{105}$$

For a symmetrical arch, these equations reduce to

$$M_0 = \frac{I}{K}\left[D_1 B_3 - D_3 B_1\right] \tag{106}$$

$$H_0 = \frac{I}{K}\left[-D_1 B_1 + D_3 A_1\right] \tag{107}$$

$$V_0 = \frac{D_2}{C_2} \tag{108}$$

in which

$$K = A_1 B_3 - B_1^2 \tag{109}$$

90. Arch Deflections.—In order to derive general equations for arch deflections at any arch point, the following meanings are assigned to terms used in equations 116 to 124, inclusive:

The origin is assumed to be at the point where the deflections are desired.

A_1			
RIGHT	Arch	$\int_0^s \frac{ds}{E_c I}$	
	Abut.	α	
LEFT	Arch	$\int_0^s \frac{ds}{E_c I}$	
	Abut.	α	
B_1			
RIGHT	Arch	$\int_0^s \frac{y\,ds}{E_c I}$	
	Abut.	αy_A	
		$\alpha_2 \sin \phi_A$	
LEFT	Arch	$\int_0^s \frac{y\,ds}{E_c I}$	
	Abut.	αy_A	
		$\alpha_2 \sin \phi_A$	
C_1			
RIGHT	Arch	$-\int_0^s \frac{x\,ds}{E_c I}$	
	Abut.	$-\alpha x_A$	
		$-\alpha_2 \cos \phi_A$	
LEFT	Arch	$\int_0^s \frac{x\,ds}{E_c I}$	
	Abut.	αx_A	
		$\alpha_2 \cos \phi_A$	
B_2			
RIGHT	Arch	$-\int_0^s \frac{xy\,ds}{E_c I}$	
		$\int_0^s \frac{\sin\phi \cos\phi\, ds}{E_c A}$	
		$-3\int_0^s \frac{\sin\phi \cos\phi\, ds}{E_c A}$	
	Abut.	$-\alpha x_A y_A$	
		$\beta \sin\phi_A \cos\phi_A$	
		$-\gamma \sin\phi_A \cos\phi_A$	
		$-\alpha_2 x_A \sin\phi_A$	
		$-\alpha_2 y_A \cos\phi_A$	
LEFT	Arch	$\int_0^s \frac{xy\,ds}{E_c I}$	
		$-\int_0^s \frac{\sin\phi \cos\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{\sin\phi \cos\phi\, ds}{E_c A}$	
	Abut.	$\alpha x_A y_A$	
		$-\beta \sin\phi_A \cos\phi_A$	
		$\gamma \sin\phi_A \cos\phi_A$	
		$\alpha_2 x_A \sin\phi_A$	
		$\alpha_2 y_A \cos\phi_A$	

C_2			
RIGHT	Arch	$\int_0^s \frac{x^2 ds}{E_c I}$	
		$\int_0^s \frac{\sin^2\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{\cos^2\phi\, ds}{E_c A}$	
	Abut.	αx_A^2	
		$\beta \sin^2\phi_A$	
		$\gamma \cos^2\phi_A$	
		$2\alpha_2 x_A \cos\phi_A$	
LEFT	Arch	$\int_0^s \frac{x^2 ds}{E_c I}$	
		$\int_0^s \frac{\sin^2\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{\cos^2\phi\, ds}{E_c A}$	
	Abut.	αx_A^2	
		$\beta \sin^2\phi_A$	
		$\gamma \cos^2\phi_A$	
		$2\alpha_2 x_A \cos\phi_A$	
B_3			
RIGHT	Arch	$\int_0^s \frac{y^2 ds}{E_c I}$	
		$\int_0^s \frac{\cos^2\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{\sin^2\phi\, ds}{E_c A}$	
	Abut.	αy_A^2	
		$\beta \cos^2\phi_A$	
		$\gamma \sin^2\phi_A$	
		$2\alpha_2 y_A \sin\phi_A$	
LEFT	Arch	$\int_0^s \frac{y^2 ds}{E_c I}$	
		$\int_0^s \frac{\cos^2\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{\sin^2\phi\, ds}{E_c A}$	
	Abut.	αy_A^2	
		$\beta \cos^2\phi_A$	
		$\gamma \sin^2\phi_A$	
		$2\alpha_2 y_A \sin\phi_A$	
D_1			
RIGHT	Arch	$\int_0^s \frac{M_R\, ds}{E_c I}$	
	Abut.	${}_A M_R \alpha$	
		${}_A V_R \alpha_2$	
LEFT	Arch	$\int_0^s \frac{M_L\, ds}{E_c I}$	
	Abut.	${}_A M_L \alpha$	
		${}_A V_L \alpha_2$	

D_2			
RIGHT	Arch	$-\int_0^s \frac{M_R x\, ds}{E_c I}$	
		$-\int_0^s \frac{H_R \sin\phi\, ds}{E_c A}$	
		$-3\int_0^s \frac{V_R \cos\phi\, ds}{E_c A}$	
	Abut.	$-{}_A M_R \alpha x_A$	
		$-{}_A H_R \beta \sin\phi_A$	
		$-{}_A V_R \gamma \cos\phi_A$	
		$-{}_A M_R \alpha_2 \cos\phi_A$	
		$-{}_A V_R \alpha_2 x_A$	
	Temp.	$ct y_A$	
LEFT	Arch	$\int_0^s \frac{M_L x\, ds}{E_c I}$	
		$\int_0^s \frac{H_L \sin\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{V_L \cos\phi\, ds}{E_c A}$	
	Abut.	${}_A M_L \alpha x_A$	
		${}_A H_L \beta \sin\phi_A$	
		${}_A V_L \gamma \cos\phi_A$	
		${}_A M_L \alpha_2 \cos\phi_A$	
		${}_A V_L \alpha_2 x_A$	
	Temp.	$-ct y_A$	
D_3			
RIGHT	Arch	$\int_0^s \frac{M_R y\, ds}{E_c I}$	
		$-\int_0^s \frac{H_R \cos\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{V_R \sin\phi\, ds}{E_c A}$	
	Abut.	${}_A M_R \alpha y_A$	
		$-{}_A H_R \beta \cos\phi_A$	
		${}_A V_R \gamma \sin\phi_A$	
		${}_A M_R \alpha_2 \sin\phi_A$	
		${}_A V_R \alpha_2 y_A$	
	Temp.	$ct x_A$	
LEFT	Arch	$\int_0^s \frac{M_L y\, ds}{E_c I}$	
		$-\int_0^s \frac{H_L \cos\phi\, ds}{E_c A}$	
		$3\int_0^s \frac{V_L \sin\phi\, ds}{E_c A}$	
	Abut.	${}_A M_L \alpha y_A$	
		$-{}_A H_L \beta \cos\phi_A$	
		${}_A V_L \gamma \sin\phi_A$	
		${}_A M_L \alpha_2 \sin\phi_A$	
		${}_A V_L \alpha_2 y_A$	
	Temp.	$ct x_A$	

TABLE 3—CONSTANTS FOR SOLUTION OF CROWN FORCES

M, H, V = moment, thrust, and shear at the point where deflections are desired. These values are computed from equations 110 to 112, inclusive, for the left part of the arch and equations 113 to 115 for the right part of the arch. In these equations, x, y, and ϕ are measured from the crown to the point where the deflections are desired, not from the point as defined below, which refers to equations 116 to 124, inclusive.

s = length along center line of arch from point where deflections are required.

ϕ = angle measured from arch point where deflections are desired to any point in the arch.

x, y = coordinates of any arch point with origin at the point at which deflections are desired.

ϕ_A = angle from arch point at which deflections are desired to abutment.

Arch deflections at any point are obtained by considering the portion of the arch between the point and the abutment as a curved cantilever beam. The desired arch movements are the sum of the movements due to the applied load between the point considered and the abutment, and the movements due to the moment, thrust, and shear acting at the point.

It is evident that the moment, thrust, and shear at the point at which deflections are desired must be determined before deflections can be evaluated. These are easily determined after crown values have been computed. Formulas for the left part of the arch are

$$M = M_0 + H_0 y + V_0 x - M_L \tag{110}$$

$$H = H_0 \cos\phi - V_0 \sin\phi + H_L \tag{111}$$

$$V = H_0 \sin\phi + V_0 \cos\phi - V_L \tag{112}$$

Similar formulas for the right part are

$$M = M_0 + H_0 y - V_0 x - M_R \tag{113}$$

$$H = H_0 \cos\phi + V_0 \sin\phi + H_R \tag{114}$$

$$V = H_0 \sin\phi - V_0 \cos\phi - V_R \tag{115}$$

Equations for arch deflections at any point are derived in the same way as equations 87, 88, and 89 for the crown. However, in this case total forces and moments acting on the abutments can be calculated by equations 110 to 115 after the crown values are known, which makes it possible to

consider effects of resulting abutment movements separately. Equations for deflections at any point in the left part of the arch are

$$ {}_L\theta = M\left[\int_o^s \frac{ds}{E_c I}\right] + H\left[\int_o^s \frac{y\,ds}{E_c I}\right] + V\left[\int_o^s \frac{x\,ds}{E_c I}\right] - \left[\int_o^s \frac{M_L\,ds}{E_c I}\right] + M_A\,\alpha + V_A\,\alpha_2 \tag{116} $$

$$ \begin{aligned} {}_L\Delta r = {} & M\left[\int_o^s \frac{x\,ds}{E_c I}\right] + H\left[\int_o^s \frac{xy\,ds}{E_c I} - \int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A} + 3\int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A}\right] + V\left[\int_o^s \frac{x^2 ds}{E_c I}\right. \\ & \left. + \int_o^s \frac{\sin^2\phi\,ds}{E_c A} + 3\int_o^s \frac{\cos^2\phi\,ds}{E_c A}\right] - \left[\int_o^s \frac{M_L x\,ds}{E_c I} + \int_o^s \frac{H_L \sin\phi\,ds}{E_c A} + 3\int_o^s \frac{V_L\cos\phi\,ds}{E_c A}\right. \\ & \left. - \int_o^s ct\sin\phi\,ds\right] + (M_A\,\alpha + V_A\,\alpha_2)x_A + (V_A\,\tau + M_A\,\alpha_2)\cos\phi_A - H_A\,\beta\sin\phi_A \end{aligned} \tag{117} $$

$$ \begin{aligned} {}_L\Delta s = {} & -M\left[\int_o^s \frac{y\,ds}{E_c I}\right] - H\left[\int_o^s \frac{y^2 ds}{E_c I} + \int_o^s \frac{\cos^2\phi\,ds}{E_c A} + 3\int_o^s \frac{\sin^2\phi\,ds}{E_c A}\right] - V\left[\int_o^s \frac{xy\,ds}{E_c I}\right. \\ & \left. - \int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A} + 3\int_o^s \frac{\sin\phi\cos\phi\,ds}{E_c A}\right] + \left[\int_o^s \frac{M_L\,y\,ds}{E_c I} - \int_o^s \frac{H_L\cos\phi\,ds}{E_c A}\right. \\ & \left. + 3\int_o^s \frac{V_L\sin\phi\,ds}{E_c A} + \int_o^s ct\cos\phi\,ds\right] - (M_A\,\alpha + V_A\,\alpha_2)y_A - (V_A\,\tau + M_A\,\alpha_2)\sin\phi_A - H_A\,\beta\cos\phi_A \end{aligned} \tag{118} $$

in which M_A, H_A, and V_A are the moment, thrust, and shear at the abutment.

By substituting symbols for integrals or groups of integrals in the brackets, the following equations may be written:

$$ {}_L\theta = A_1\,M + B_1\,H + C_1\,V - D_1 + M_A\,\alpha + V_A\,\alpha_2 \tag{119} $$

$$ {}_L\Delta r = C_1\,M + B_2\,H + C_2\,V - D_2 + (M_A\,\alpha + V_A\,\alpha_2)x_A - H_A\,\beta\sin\phi_A + (V_A\,\tau + M_A\,\alpha_2)\cos\phi_A \tag{120} $$

$$ {}_L\Delta s = -B_1\,M - B_3\,H - B_2\,V + D_3 - (M_A\,\alpha + V_A\,\alpha_2)y_A - (V_A\,\tau + M_A\,\alpha_2)\sin\phi_A - H_A\,\beta\cos\phi_A \tag{121} $$

Equations for deflections in the right part of the arch are

$$ {}_R\theta = -A_1\,M - B_1\,H - C_1\,V + D_1 - M_A\,\alpha - V_A\,\alpha_2 \tag{122} $$

$$ {}_R\Delta r = C_1\,M + B_2\,H + C_2\,V - D_2 + (M_A\,\alpha + V_A\,\alpha_2)x_A - H_A\,\beta\sin\phi_A + (V_A\,\tau + M_A\,\alpha_2)\cos\phi_A \tag{123} $$

$$ {}_R\Delta s = B_1\,M + B_3\,H + B_2\,V - D_3 + (M_A\,\alpha + V_A\,\alpha_2)y_A + (V_A\,\tau + M_A\,\alpha_2)\sin\phi_A + H_A\,\beta\cos\phi_A \tag{124} $$

The quantities A_1, B_1, B_2, B_3, C_1, and C_2 consist of integrals or groups of integrals which are functions of the arch and are therefore designated "arch constants." The quantities D_1, D_2, and D_3 consist of integrals or

combinations of integrals which are functions of both the arch and the load and are designated "load constants." Integrals for these constants also appear in quantities required for solution of crown forces. For this reason arch and load constants may be used for the solution of crown forces as well as for calculating arch deflections. The evaluation of constants for various types of arches, together with a general discussion concerning their use, is given in subsequent sections.

UNIFORM THICKNESS CIRCULAR ARCH

91. General Statement.—At this point in the development of arch theory it is convenient to consider the arch as circular and of uniform thickness, because arch and load constants can be evaluated for this special case. In derivations of arch and load constants in the following sections, the arch center line is used instead of the neutral axis. This introduces a small error, but simplifies the analysis. Using the notation in section 90, $I = T^3/12$, $s = r\phi$, and $ds = rd\phi$. Since T and E_C remain constant throughout the arch, and since $x = r \sin \phi$ and $y = r \operatorname{vers} \phi$, it is evident that the arch and load constants can be integrated.

92. Arch Constants.—Arch constants are deflections at an arch point due to a unit force or couple at the point.

A_1 is the angular movement at a point due to a unit moment at the point.

B_1 is the angular movement at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit moment at the point.

C_1 is the angular movement at a point due to a unit shear at the point; or, it is the radial deflection at a point due to a unit moment at the point.

B_2 is the radial deflection at a point due to a unit thrust at the point; or, it is the tangential deflection at a point due to a unit shear at the point.

C_2 is the radial deflection at a point due to a unit shear at the point.

B_3 is the tangential deflection at a point due to a unit thrust at the point.

The following formulas for arch constants are for any arch point and may be used for either the left or right parts of the arch.

$$A_1 = \int_o^s \frac{ds}{E_c I} = \frac{12r}{E_c T^3}\int_o^{\phi_A} d\phi = \frac{12r}{E_c T^3}\left[\phi_A\right] \tag{125}$$

$$B_1 = \int_o^s \frac{yds}{E_c I} = \frac{12r^2}{E_c T^3}\int_o^{\phi_A} \text{vers}\,\phi\, d\phi = \frac{12r^2}{E_c T^3}\left[\phi_A - \sin\phi_A\right] \tag{126}$$

$$C_1 = \int_o^s \frac{xds}{E_c I} = \frac{12r^2}{E_c T^3}\int_o^{\phi_A} \sin\phi\, d\phi = \frac{12r^2}{E_c T^3}\left[\text{vers}\,\phi_A\right] \tag{127}$$

$$B_2 = \int_o^s \frac{xyds}{E_c I} - \int_o^s \frac{\sin\phi\cos\phi\, ds}{E_c A} + 3\int_o^s \frac{\sin\phi\cos\phi\, ds}{E_c A}$$

$$= \frac{12r^3}{E_c T^3}\int_o^{\phi_A} \sin\phi\,\text{vers}\,\phi\, d\phi + \frac{2r}{E_c T}\int_o^{\phi_A} \sin\phi\cos\phi\, d\phi$$

$$= \frac{12r^3}{E_c T^3}\left[\text{vers}\,\phi_A - \frac{\sin^2\phi_A}{2}\right] + \frac{r}{E_c T}\left[\sin^2\phi_A\right] \tag{128}$$

$$C_2 = \int_o^s \frac{x^2 ds}{E_c I} + \int_o^s \frac{\sin^2\phi\, ds}{E_c A} + 3\int_o^s \frac{\cos^2\phi\, ds}{E_c A}$$

$$= \frac{12r^3}{E_c T^3}\int_o^{\phi_A} \sin^2\phi\, d\phi + \frac{r}{E_c T}\int_o^{\phi_A} \sin^2\phi\, d\phi + \frac{3r}{E_c T}\int_o^{\phi_A} \cos^2\phi\, d\phi$$

$$= \frac{12r^3}{E_c T^3}\left[\frac{\phi_A - \sin\phi_A\cos\phi_A}{2}\right] + \frac{r}{E_c T}\left[\frac{\phi_A - \sin\phi_A\cos\phi_A}{2} + 3\,\frac{\phi_A + \sin\phi_A\cos\phi_A}{2}\right] \tag{129}$$

$$B_3 = \int_o^s \frac{y^2 ds}{E_c I} + \int_o^s \frac{\cos^2\phi\, ds}{E_c A} + 3\int_o^s \frac{\sin^2\phi\, ds}{E_c A}$$

$$= \frac{12r^3}{E_c T^3}\int_o^{\phi_A} \text{vers}^2\phi\, d\phi + \frac{r}{E_c T}\int_o^{\phi_A} \cos^2\phi\, d\phi + \frac{3r}{E_c T}\int_o^{\phi_A} \sin^2\phi\, d\phi$$

$$= \frac{12r^3}{E_c T^3}\left[\phi_A - 2\sin\phi_A + \frac{\phi_A + \sin\phi_A\cos\phi_A}{2}\right] + \frac{r}{E_c T}\left[\frac{\phi_A + \sin\phi_A\cos\phi_A}{2} + 3\,\frac{\phi_A - \sin\phi_A\cos\phi_A}{2}\right] \tag{130}$$

Inspection of formulas 125 to 130 shows that quantities in brackets depend only on the arch angle. These quantities have been evaluated for angles between zero and 90 degrees and are given in chapter IX.

93. Load Constants.—Load constants are deflections at a point due to all loads between the point and the abutment.

D_1 is the angular movement at a point due to loads between the point and the abutment.

D_2 is the radial deflection at a point due to loads between the point and the abutment.

D_3 is the tangential deflection at a point due to loads between the point and the abutment.

Integrals for the D-terms for the left side of the arch are

$$D_1 = \int_o^s \frac{M_L\,ds}{E_c I} \tag{131}$$

$$D_2 = \int_o^s \frac{M_L x\,ds}{E_c I} + \int_o^s \frac{H_L \sin\phi\,ds}{E_c A} + 3\int_o^s \frac{V_L \cos\phi\,ds}{E_c A} - \int_o^s ct \sin\phi\,ds \tag{132}$$

$$D_3 = \int_o^s \frac{M_L y\,ds}{E_c I} - \int_o^s \frac{H_L \cos\phi\,ds}{E_c A} + 3\int_o^s \frac{V_L \sin\phi\,ds}{E_c A} + \int_o^s ct \cos\phi\,ds \tag{133}$$

These cannot be conveniently expressed until the terms M_L, H_L, and V_L for the left, or M_R, H_R, and V_R for right side of arch are evaluated. This is described in the following sections which deal with the various unit loads separately. Formulas for moment, thrust, shear, and D-terms are derived for the left part of the arch but may be used for the right part also.

It should be remembered that M_L, H_L, and V_L are the moment, thrust, and shear in the left part of the arch, due to load at the right of the point. For the purpose of evaluating D-term integrals and deriving equations for moment, thrust, and shear, M_L, H_L, and V_L or M_R, H_R, and V_R, at any arch point due to unit loads, the following additional notations are used, see figures 33-a and b.

${}_A\phi_1$—For uniform loads applied on the arch, ${}_A\phi_1$ is the angle from the point where deflections are desired to the abutment. For triangular loads, ${}_A\phi_1$ is the angle from the point where the loading begins to the abutment.

ϕ_0 = angle from point where deflections are desired, for arch points not under load, to beginning of external load.

ψ = angle from point where loading begins to any differential element of the arch under load.

ϕ_1 = angle from point where loading begins to any arch point under load.

P = intensity of applied load acting on the arch, as stated in section 83.

In the following equations for load constants, D_2 and D_3 values are divided into two terms. The first represents the effect of bending, and the second represents the effect of rib-shortening and shear detrusion.

94. Uniform Radial Load.—Moments, thrusts, and shears due to a uniform radial load at the upstream face, as shown in figure 33-a, may be computed by the following equations.

$$M_L = \int_0^{\phi_1} PR_U r \sin(\phi_1 - \psi)\, d\psi = PR_U r \operatorname{vers} \phi_1 \tag{134}$$

$$H_L = \int_0^{\phi_1} PR_U \sin(\phi_1 - \psi)\, d\psi = PR_U \operatorname{vers} \phi_1 \tag{135}$$

$$V_L = \int_0^{\phi_1} PR_U \cos(\phi_1 - \psi)\, d\psi = PR_U \sin \phi_1 \tag{136}$$

By substituting these values in equations 131, 132, and 133, the following formulas are obtained.

$$D_1 = \int_0^S \frac{M_L\, ds}{E_C I} = \frac{12 R_U r^2}{E_C T^3} P \left[\int_0^{A\phi_1} \operatorname{vers} \phi_1\, d\phi_1 \right] \tag{137}$$

$$D_{2,1^{st}\ term} = \int_0^S \frac{M_L\, x\, ds}{E_C I} = \frac{12 R_U r^3}{E_C T^3} P \left[\int_0^{A\phi_1} \sin \phi_1 \operatorname{vers} \phi_1\, d\phi_1 \right] \tag{138}$$

$$D_{2,\ 2^{nd}\ term} = \int_0^S \frac{H_L \sin \phi\, ds}{E_C A} + 3 \int_0^S \frac{V_L \cos \phi\, ds}{E_C A}$$

$$= \frac{R_U r}{E_C T} P \left[\int_0^{A\phi_1} \sin \phi_1 \operatorname{vers} \phi_1\, d\phi_1 + 3 \int_0^{A\phi_1} \sin \phi_1 \cos \phi_1\, d\phi_1 \right] \tag{139}$$

$$D_{3,\ 1^{st}\ term} = \int_0^S \frac{M_L\, y\, ds}{E_C I} = \frac{12 R_U r^3}{E_C T^3} P \left[\int_0^{A\phi_1} \operatorname{vers} \phi_1\, d\phi_1 - \int_0^{A\phi_1} \operatorname{vers} \phi_1 \cos \phi_1\, d\phi_1 \right] \tag{140}$$

$$D_{3,\ 2^{nd}\ term} = -\int_0^S \frac{H_L \cos \phi\, ds}{E_C A} + 3 \int_0^S \frac{V_L \sin \phi\, ds}{E_C A}$$

$$= \frac{R_U r}{E_C T} P \left[-\int_0^{A\phi_1} \operatorname{vers} \phi_1 \cos \phi_1\, d\phi_1 + 3 \int_0^{A\phi_1} \sin^2 \phi_1\, d\phi_1 \right] \tag{141}$$

The evaluation of these and succeeding integrals is discussed in chapter IX. It should be noted that the D_2 and D_3 temperature terms are omitted in equations 137 to 141, since the formulas are for uniform radial loads which do not include temperature effects.

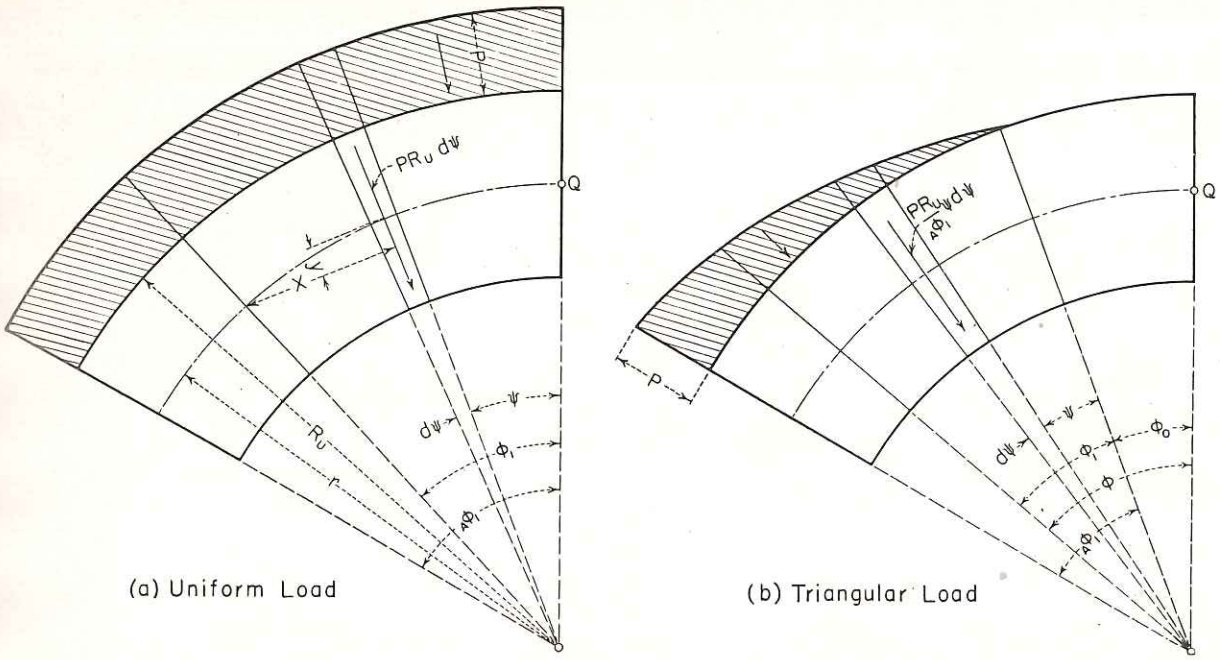


FIGURE 33—RADIAL ARCH LOADS

95. Triangular Radial Load.—Moments, thrusts, and shears due to a triangular radial load at the upstream face, as shown in figure 33-b, may be computed by the following equations.

$$M_L = \int_0^{\phi_1} \frac{PR_U r}{A\phi_1} \psi \sin(\phi_1 - \psi) d\psi = \frac{PR_U r}{A\phi_1}(\phi_1 - \sin\phi_1) \qquad (142)$$

$$H_L = \int_0^{\phi_1} \frac{PR_U}{A\phi_1} \psi \sin(\phi_1 - \psi) d\psi = \frac{PR_U}{A\phi_1}(\phi_1 - \sin\phi_1) \qquad (143)$$

$$V_L = \int_0^{\phi_1} \frac{PR_U}{A\phi_1} \psi \cos(\phi_1 - \psi) d\psi = \frac{PR_U}{A\phi_1}(\text{vers}\,\phi_1) \qquad (144)$$

Load constants for a triangular radial load are

$$D_1 = \int_0^s \frac{M_L ds}{E_C I} = \frac{12 R_U r^2}{E_C T^3} \cdot \frac{P}{A\phi_1}\left[\int_0^{A\phi_1} (\phi_1 - \sin\phi_1) d\phi_1\right] \qquad (145)$$

$$D_{2,\,1^{st}\,term} = \int_0^s \frac{M_L x ds}{E_C I} = \frac{12 R_U r^3}{E_C T^3} \cdot \frac{P}{A\phi_1}\left[\int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\sin\phi d\phi_1\right] \qquad (146)$$

$$D_{2,\,2^{nd}\,term} = \int_0^s \frac{H_L \sin\phi ds}{E_C A} + 3\int_0^s \frac{V_L \cos\phi ds}{E_C A}$$

$$= \frac{R_U r}{E_C T} \cdot \frac{P}{A\phi_1}\left[\int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\sin\phi d\phi_1 + 3\int_0^{A\phi_1} \text{vers}\,\phi_1 \cos\phi d\phi_1\right] \qquad (147)$$

$$D_3, 1^{st.}\,term = \int_0^s \frac{M_L y\,ds}{E_c I} = \frac{12 R_U r^3}{E_c T^3} \cdot \frac{P}{{}_A\phi_1}\left[\int_0^{{}_A\phi_1} (\phi_1 - \sin\phi_1)\,d\phi_1 - \int_0^{{}_A\phi_1} (\phi_1 - \sin\phi_1)\cos\phi\,d\phi_1\right] \qquad (148)$$

$$D_3, 2^{nd.}\,term = -\int_0^s \frac{H_L \cos\phi\,ds}{E_c A} + 3\int_0^s \frac{V_L \sin\phi\,ds}{E_c A}$$

$$= \frac{R_U r}{E_c T} \cdot \frac{P}{{}_A\phi_1}\left[-\int_0^{{}_A\phi_1} (\phi_1 - \sin\phi_1)\cos\phi\,d\phi_1 + 3\int_0^{{}_A\phi_1} \text{vers}\,\phi_1 \sin\phi\,d\phi_1\right] \qquad (149)$$

96. Uniform Tangential Load.—Moments, thrusts, and shears due to a uniform tangential load applied along the center line of an arch, as shown in figure 34-a, may be computed by the following equations.

$$M_L = \int_0^{\phi_1} Pr^2\,\text{vers}(\phi_1 - \psi)\,d\psi = Pr^2(\phi_1 - \sin\phi_1) \qquad (150)$$

$$H_L = -\int_0^{\phi_1} Pr\cos(\phi_1 - \psi)\,d\psi = -Pr\sin\phi_1 \qquad (151)$$

$$V_L = \int_0^{\phi_1} Pr\sin(\phi_1 - \psi)\,d\psi = Pr\,\text{vers}\,\phi_1 \qquad (152)$$

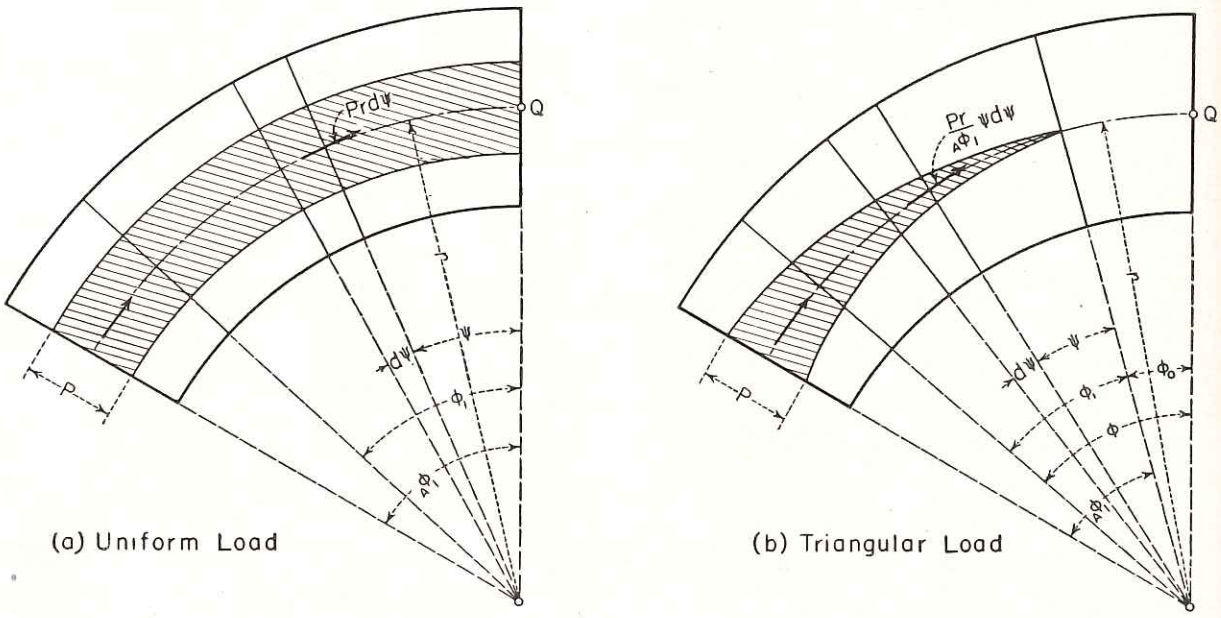


FIGURE 34—TANGENTIAL ARCH LOADS

Equations for load constants, derived by substituting equations for M_L, H_L, and V_L in equations 131, 132, and 133, are given below.

$$D_1 = \int_0^s \frac{M_L\,ds}{E_c I} = \frac{12 r^3}{E_c T^3} P\left[\int_0^{{}_A\phi_1} (\phi_1 - \sin\phi_1)\,d\phi_1\right] \qquad (153)$$

$$D_2,\ 1^{st}\ term = \int_0^s \frac{M_L x\,ds}{E_c I} = \frac{12r^4}{E_c T^3} P\left[\int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\sin\phi_1\, d\phi_1\right] \tag{154}$$

$$D_2,\ 2^{nd}\ term = \int_0^s \frac{H_L \sin\phi_1\, ds}{E_c A} + 3\int_0^s \frac{V_L \cos\phi_1\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} P\left[-\int_0^{A\phi_1} \sin\phi_1 \sin\phi_1\, d\phi_1 + 3\int_0^{A\phi_1} \text{vers}\,\phi_1 \cos\phi_1 d\phi_1\right] \tag{155}$$

$$D_3,\ 1^{st}\ term = \int_0^s \frac{M_L y\,ds}{E_c I} = \frac{12r^4}{E_c T^3} P\left[\int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\, d\phi_1 - \int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\cos\phi_1 d\phi_1\right] \tag{156}$$

$$D_3,\ 2^{nd}\ term = -\int_0^s \frac{H_L \cos\phi_1\, ds}{E_c A} + 3\int_0^s \frac{V_L \sin\phi_1\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} P\left[\int_0^{A\phi_1} \sin\phi_1 \cos\phi_1 d\phi_1 + 3\int_0^{A\phi_1} \text{vers}\,\phi_1 \sin\phi_1 d\phi_1\right] \tag{157}$$

97. Triangular Tangential Load.—Moments, thrusts, and shears due to a triangular tangential load, as shown in figure 34-b, are calculated by the following formulas:

$$M_L = \int_0^{\phi_1} \frac{Pr^2}{A\phi_1} \psi\, \text{vers}(\phi_1 - \psi)\, d\psi = \frac{Pr^2}{A\phi_1}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) \tag{158}$$

$$H_L = -\int_0^{\phi_1} \frac{Pr}{A\phi_1} \psi \cos(\phi_1 - \psi)\, d\psi = -\frac{Pr}{A\phi_1} \text{vers}\,\phi_1 \tag{159}$$

$$V_L = \int_0^{\phi_1} \frac{Pr}{A\phi_1} \psi \sin(\phi_1 - \psi)\, d\psi = \frac{Pr}{A\phi_1}(\phi_1 - \sin\phi_1) \tag{160}$$

Load constants for a triangular tangential load are

$$D_1 = \int_0^s \frac{M_L\, ds}{E_c I} = \frac{12r^3}{E_c T^3} \cdot \frac{P}{A\phi_1}\left[\int_0^{A\phi_1}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right) d\phi_1\right] \tag{161}$$

$$D_2,\ 1^{st}\ term = \int_0^s \frac{M_L x\,ds}{E_c I} = \frac{12r^4}{E_c T^3} \cdot \frac{P}{A\phi_1}\left[\int_0^{A\phi_1}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right)\sin\phi\, d\phi_1\right] \tag{162}$$

$$D_2,\ 2^{nd}\ term = \int_0^s \frac{H_L \sin\phi\, ds}{E_c A} + 3\int_0^s \frac{V_L \cos\phi\, ds}{E_c A}$$

$$= \frac{r^2}{E_c T} \cdot \frac{P}{A\phi_1}\left[-\int_0^{A\phi_1} \text{vers}\,\phi_1 \sin\phi\, d\phi_1 + 3\int_0^{A\phi_1} (\phi_1 - \sin\phi_1)\cos\phi\, d\phi_1\right] \tag{163}$$

$$D_{3,\,1^{st}.\,term} = \int_0^s \frac{M_L y\,ds}{E_c I} = \frac{12r^4}{E_c T^3}\cdot\frac{P}{{}_A\phi_1}\left[\int_0^{{}_A\phi_1}\left(\frac{\phi_1^2}{2}-\text{vers}\,\phi_1\right)d\phi_1-\int_0^{{}_A\phi_1}\left(\frac{\phi_1^2}{2}-\text{vers}\,\phi_1\right)\cos\phi\,d\phi_1\right] \quad (164)$$

$$D_{3,\,2^{nd}.\,term} = -\int_0^s \frac{H_L\cos\phi\,ds}{E_c A} + 3\int_0^s \frac{V_L\sin\phi\,ds}{E_c A}$$

$$= \frac{r^2}{E_c T}\cdot\frac{P}{{}_A\phi_1}\left[\int_0^{{}_A\phi_1}\text{vers}\,\phi_1\cos\phi\,d\phi_1+3\int_0^{{}_A\phi}(\phi_1-\sin\phi_1)\sin\phi\,d\phi_1\right] \quad (165)$$

98. Uniform Twist Load.—Twist loads are horizontal couples causing pure bending of the arches. Consequently, they do not produce thrusts or shears, and terms involving these quantities do not appear in the following formulas. The moment due to a uniform twist load applied along the center line, as shown in figure 35-a, is

$$M_L = \int_0^{\phi_1} Pr\,d\psi = Pr\phi_1 \quad (166)$$

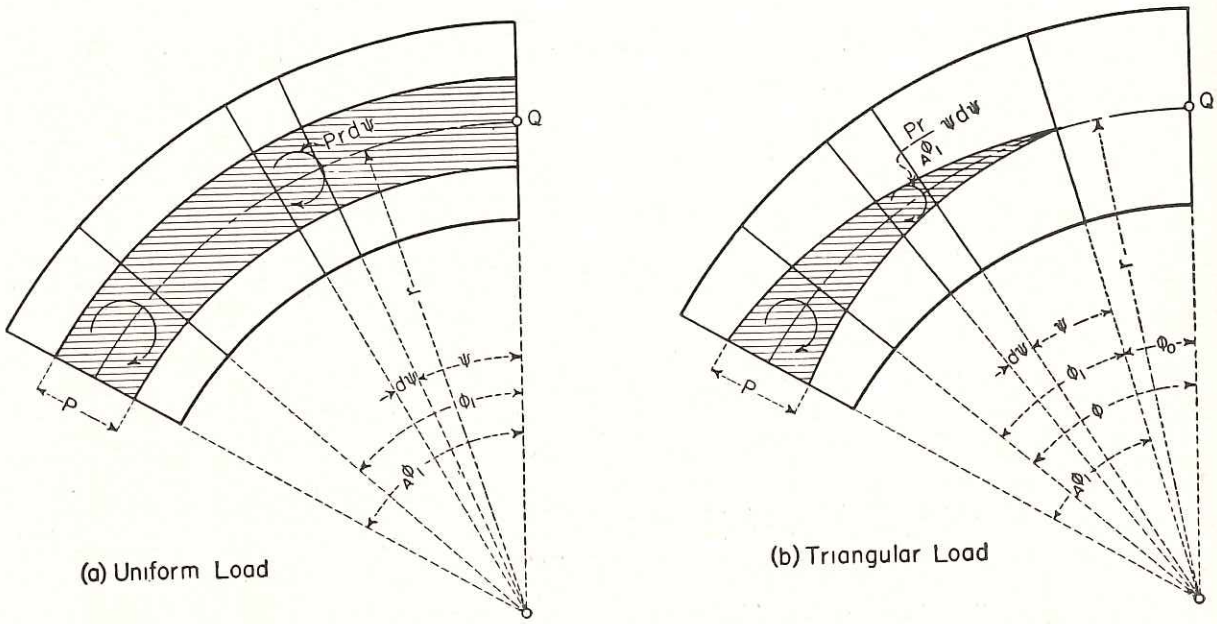


FIGURE 35—TWIST LOADS ON ARCH ELEMENTS

Load constants for a uniform twist load involving this internal moment are

$$D_1 = \int_0^s \frac{M_L\,ds}{E_c I} = \frac{12r^2}{E_c T^3}\,P\left[\int_0^{{}_A\phi_1}\phi_1 d\phi_1\right] \quad (167)$$

$$D_2 = \int_0^s \frac{M_L x\,ds}{E_c I} = \frac{12r^3}{E_c T^3}\,P\left[\int_0^{{}_A\phi_1}\phi_1\sin\phi_1 d\phi_1\right] \quad (168)$$

$$D_3 = \int_0^s \frac{M_L y\,ds}{E_c I} = \frac{12r^3}{E_c T^3}\,P\left[\int_0^{{}_A\phi_1}\phi_1 d\phi_1-\int_0^{{}_A\phi_1}\phi_1\cos\phi_1 d\phi_1\right] \quad (169)$$

99. Triangular Twist Load.—The moment due to a triangular twist load, as shown in figure 35-b, is obtained by the formula,

$$M_L = \int_0^{\phi_1} \frac{Pr}{A\phi_1} \psi \, d\psi = \frac{Pr}{A\phi_1} \frac{\phi_1^2}{2} \tag{170}$$

and load constants are

$$D_1 = \int_0^s \frac{M_L ds}{E_C I} = \frac{12r^2}{E_C T^3} \cdot \frac{P}{A\phi_1} \left[\int_0^{A\phi_1} \frac{\phi_1^2}{2} d\phi_1 \right] \tag{171}$$

$$D_2 = \int_0^s \frac{M_L x ds}{E_C I} = \frac{12r^3}{E_C T^3} \cdot \frac{P}{A\phi_1} \left[\int_0^{A\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1 \right] \tag{172}$$

$$D_3 = \int_0^s \frac{M_L y ds}{E_C I} = \frac{12r^3}{E_C T^3} \cdot \frac{P}{A\phi_1} \left[\int_0^{A\phi_1} \frac{\phi_1^2}{2} d\phi_1 - \int_0^{A\phi_1} \frac{\phi_1^2}{2} \cos\phi \, d\phi_1 \right] \tag{173}$$

100. Temperature Load.—Load constants for a uniform temperature change throughout an arch are

$$D_1 = 0 \tag{174}$$

$$D_2 = -\int_0^s c\,t \sin\phi_1 ds = -c\,t\,r \int_0^{A\phi_1} \sin\phi_1 d\phi_1 = -c\,t\,r \operatorname{vers}_A\phi_1 = -c\,t\,y_A \tag{175}$$

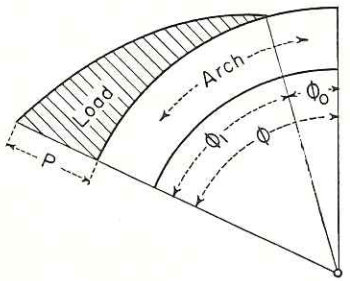
$$D_3 = \int_0^s c\,t \cos\phi_1 ds = c\,t\,r \int_0^{A\phi_1} \cos\phi_1 d\phi_1 = c\,t\,r \sin_A\phi_1 = c\,t\,x_A \tag{176}$$

101. Summary of Load Formulas.—Table 4 gives a summary of formulas for moments, thrusts, shears, and D-terms for all unit loads on a circular arch. Integrals involved in the formulas are given in the left part of the table and items for which they are used comprise the remainder of the table. Columns designated "item" give the formulas in their original form, as in equations 131, 132, and 133. Columns headed "designation" give the convenient D-term notation used for the load constants. The remaining "multiplier" columns give quantities by which the integrals must be multiplied to give desired arch movements. For example, under the headings "radial loads" and "triangular,"

$$D_1 = \int_0^s \frac{M_L ds}{E I} = \frac{PR_U r^2}{\phi_1 E I} \int_0^{\phi_1} (\phi_1 - \sin\phi_1) d\phi_1$$

which agrees with equation 145 previously derived. The term E in the formulas is the modulus of elasticity of concrete, E_C, the subscript c being omitted for convenience.

TRIGONOMETRIC PART	RADIAL LOADS						TANGENTIAL LOADS						TWIST LOADS					
	UNIFORM			TRIANGULAR			UNIFORM			TRIANGULAR			UNIFORM			TRIANGULAR		
	ITEM	DESIG-NATION	MULTI-PLIER	ITEM	DESIG-NATION	MULTI-PLIER	ITEM	DESIG-NATION	MULTI-PLIER	ITEM	DESIG-NATION	MULTI-PLIER	ITEM	DESIG-NATION	MULTI-PLIER	ITEM	DESIG-NATION	MULTI-PLIER
ϕ_1													M_L		Pr			
$\sin\phi_1$	V_L		PR_U				H_L		$-Pr$									
$\text{vers}\,\phi_1$	H_L		PR_U	V_L		$\frac{PR_U}{\phi_1}$	V_L		Pr	H_L		$-\frac{Pr}{\phi_1}$						
	M_L		$PR_U r$															
$\int_0^{\phi_1}\phi_1 d\phi_1$													$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{Pr^2}{EI}$	M_L		$\frac{Pr}{\phi_1}$
													$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Part	$\frac{Pr^3}{EI}$			
$\int_0^{\phi_1}\frac{\phi_1^2}{2} d\phi_1$																$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{Pr^2}{\phi_1 EI}$
																$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Part	$\frac{Pr^3}{\phi_1 EI}$
$\int_0^{\phi_1}\phi_1\sin\phi\, d\phi_1$													$\int_0^S \frac{M_L x ds}{EI}$	D_2	$\frac{Pr^3}{EI}$			
$\int_0^{\phi_1}\phi_1\cos\phi\, d\phi_1$													$\int_0^S \frac{M_L y ds}{EI}$	D_3 2nd Part	$-\frac{Pr^3}{EI}$			
$\int_0^{\phi_1}\frac{\phi_1^2}{2}\sin\phi\, d\phi_1$																$\int_0^S \frac{M_L x ds}{EI}$	D_2	$\frac{Pr^3}{\phi_1 EI}$
$\int_0^{\phi_1}\frac{\phi_1^2}{2}\cos\phi\, d\phi_1$																$\int_0^S \frac{M_L y ds}{EI}$	D_3 2nd Part	$-\frac{Pr^3}{\phi EI}$
$\int_0^{\phi_1}\sin\phi_1\sin\phi\, d\phi_1$	$\int_0^S \frac{V_L\sin\phi ds}{ET}$	D_3 2nd Term 2nd Part	$3\frac{PR_U r}{ET}$				$\int_0^S \frac{H_L\sin\phi ds}{ET}$	D_2 2nd Term 1st Part	$-\frac{Pr^2}{ET}$									
$\int_0^{\phi_1}\sin\phi_1\cos\phi\, d\phi_1$	$\int_0^S \frac{V_L\cos\phi ds}{ET}$	D_2 2nd Term 2nd Part	$3\frac{PR_U r}{ET}$				$\int_0^S \frac{H_L\cos\phi ds}{ET}$	D_3 2nd Term 1st Part	$\frac{Pr^2}{ET}$									
$\int_0^{\phi_1}\text{vers}\,\phi_1 d\phi_1$	$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{PR_U r^2}{EI}$	H_L		$\frac{PR_U}{\phi_1}$	M_L		Pr^2	V_L		$\frac{Pr}{\phi_1}$						
	$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 1st Part	$\frac{PR_U r^3}{EI}$	M_L		$\frac{PR_U r}{\phi_1}$												
$\int_0^{\phi_1}\text{vers}\,\phi_1\sin\phi\, d\phi_1$	$\int_0^S \frac{M_L x ds}{EI}$	D_2 1st Term	$\frac{PR_U r^3}{EI}$															
	$\int_0^S \frac{H_L\sin\phi ds}{ET}$	D_2 2nd Term 1st Part	$\frac{PR_U r}{ET}$	$\int_0^S \frac{V_L\sin\phi ds}{ET}$	D_3 2nd Term 2nd Part	$3\frac{PR_U r}{\phi_1 ET}$	$\int_0^S \frac{V_L\sin\phi ds}{ET}$	D_3 2nd Term 2nd Part	$3\frac{Pr^2}{ET}$	$\int_0^S \frac{H_L\sin\phi ds}{ET}$	D_2 2nd Term 1st Part	$-\frac{Pr^2}{\phi_1 ET}$							
$\int_0^{\phi_1}\text{vers}\,\phi_1\cos\phi\, d\phi_1$	$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 2nd Part	$-\frac{PR_U r^3}{EI}$															
	$\int_0^S \frac{H_L\cos\phi ds}{ET}$	D_3 2nd Term 1st Part	$-\frac{PR_U r}{ET}$	$\int_0^S \frac{V_L\cos\phi ds}{ET}$	D_2 2nd Term 2nd Part	$3\frac{PR_U r}{\phi_1 ET}$	$\int_0^S \frac{V_L\cos\phi ds}{ET}$	D_2 2nd Term 2nd Part	$3\frac{Pr^2}{ET}$	$\int_0^S \frac{H_L\cos\phi ds}{ET}$	D_3 2nd Term 1st Part	$\frac{Pr^2}{\phi_1 ET}$							
$\int_0^{\phi_1}\left[\phi_1-\sin\phi_1\right] d\phi_1$				$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{PR_U r^2}{\phi_1 EI}$	$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{Pr^3}{EI}$	M_L		$\frac{Pr^2}{\phi_1}$						
				$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 1st Part	$\frac{PR_U r^3}{\phi_1 EI}$	$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 1st Part	$\frac{Pr^4}{EI}$									
$\int_0^{\phi_1}\left[\phi_1-\sin\phi_1\right]\sin\phi\, d\phi_1$				$\int_0^S \frac{M_L x ds}{EI}$	D_2 1st Term	$\frac{PR_U r^3}{\phi_1 EI}$	$\int_0^S \frac{M_L x ds}{EI}$	D_2 1st Term	$\frac{Pr^4}{EI}$									
				$\int_0^S \frac{H_L\sin\phi ds}{ET}$	D_2 2nd Term 1st Part	$\frac{PR_U r}{\phi_1 ET}$				$\int_0^S \frac{V_L\sin\phi ds}{ET}$	D_3 2nd Term 2nd Part	$3\frac{Pr^2}{\phi_1 ET}$						
$\int_0^{\phi_1}\left[\phi_1-\sin\phi_1\right]\cos\phi\, d\phi_1$				$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 2nd Part	$-\frac{PR_U r^3}{\phi_1 EI}$	$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 2nd Part	$-\frac{Pr^4}{EI}$									
				$\int_0^S \frac{H_L\cos\phi ds}{ET}$	D_3 2nd Term 1st Part	$-\frac{PR_U r}{\phi_1 ET}$				$\int_0^S \frac{V_L\cos\phi ds}{ET}$	D_2 2nd Term 2nd Part	$3\frac{Pr^2}{\phi_1 ET}$						
$\int_0^{\phi_1}\left[\frac{\phi_1^2}{2}-\text{vers}\,\phi_1\right] d\phi_1$										$\int_0^S \frac{M_L ds}{EI}$	D_1	$\frac{Pr^3}{\phi_1 EI}$						
										$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 1st Part	$\frac{Pr^4}{\phi_1 EI}$							
$\int_0^{\phi_1}\left[\frac{\phi_1^2}{2}-\text{vers}\,\phi_1\right]\sin\phi\, d\phi_1$										$\int_0^S \frac{M_L x ds}{EI}$	D_2 1st Term	$\frac{Pr^4}{\phi_1 EI}$						
$\int_0^{\phi_1}\left[\frac{\phi_1^2}{2}-\text{vers}\,\phi_1\right]\cos\phi\, d\phi_1$										$\int_0^S \frac{M_L y ds}{EI}$	D_3 1st Term 2nd Part	$-\frac{Pr^4}{\phi_1 EI}$						



NOTATION

M_L = Moment
H_L = Thrust
V_L = Shear
For radial load, P = lb. per sq. ft.
For tangential load, P = lb. per sq. ft.
For twist load, P = ft.-lb. per sq. ft

NOTE

Formula = Trigonometric part x multiplier

TABLE 4—FORMULAS FOR CIRCULAR ARCH LOAD CONSTANTS

The evaluation of integrals in the column "trigonometric part" is discussed in chapter IX, which also gives tabulations of their numerical values for angles between 0° and 90°. Tables of numerical values for portions of D-terms that are functions of arch angles only are also given in chapter IX. These include 15 unit load patterns for all angles between 10° and 90°.

VARIABLE THICKNESS ARCH

102. General Statement.—Variable thickness arches discussed herein have a constant upstream radius, and variable center line and downstream radii as shown in figure 36. General arch formulas are used

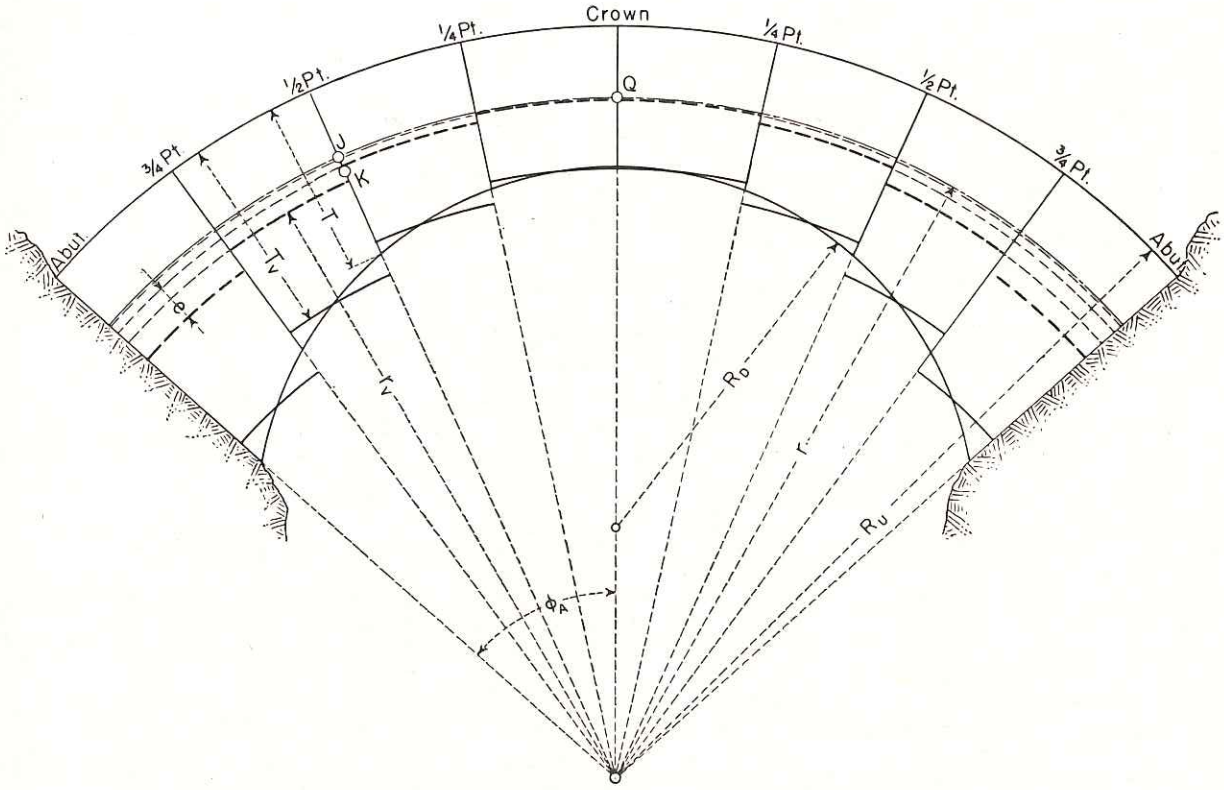


FIGURE 36—VARIABLE THICKNESS ARCH

in their analysis. By making certain revisions in various terms, as discussed subsequently, uniform thickness circular arch formulas are applicable to variable thickness arches.

Each side of the arch is divided into voussoirs by drawing lines radial to the upstream face with each voussoir subtended by a central angle equal to one-fourth the angle from the crown to the abutment. Each voussoir is considered as a segment of a circular arch whose thickness is equal to the average thickness of the voussoir. This makes it possible to use the tabulated arch and load constants for a uniform thickness arch.

Although this method is approximate, it is believed sufficiently accurate for a trial load analysis. The method was checked by comparing it with an accurate analysis of an arch in which the thickness varied as the secant of the central angle, the term $T_0 \sec \phi$ being integrated and substituted for T in the uniform thickness arch formulas. A relatively close agreement, within 2 percent, was found between the results of the two analyses. The approximate method is probably within the limits of accuracy of the trial load analysis for any variable thickness arch likely to be encountered in dam design. A discussion of the approximate method is given in the following section.

103. Notation.—The additional notation needed to discuss a variable thickness arch is given below.

T = actual thickness of arch at quarter points of the left or right part.

r_P = radius to arch center line at any quarter point, $= R_U - T/2$.

T_0 = actual thickness at crown.

r_0 = radius to center line at crown, $= R_U - T_0/2$.

T_V = average thickness of voussoir, radial to upstream face.

r_V = radius to center line of voussoir $= R_U - T_V/2$.

I_V = moment of inertia about center line of voussoir, $= T^3{}_V/12$.

e = eccentricity of voussoir center line from a circular arc drawn through the midpoint of the arch thickness at a quarter point, $= r_P - r_V$.

e' = eccentricity of voussoir center line with reference to the crown center line arc. This is the eccentricity of the applied tangential thrust with reference to the voussoir center line, $= r_0 - r_V$.

e_P = eccentricity of arch points, midpoints of arch thicknesses at quarter points, with reference to the crown center line arc, $= r_0 - r_P$.

Σ = summation of voussoir parts or quantities.

104. Arch Constants.—The method of computing arch constants is apparent from a brief examination of equations 125 to 130. Radius r and thickness T appear therein as constants. In the case of the variable thickness arch, these quantities are functions of ϕ and must be included within the integral sign. When the variable thickness arch is replaced by the approximation shown in figure 36, integrals of equations 125 to 130

are replaced by summations of successive integrals. Constants of integration due to abrupt changes in r and T appear and must be included in the computations. These integrals and integration constants may be written in terms of uniform thickness arch constants, a procedure which is followed in equations 177 to 182. The derivation of these equations from a physical standpoint is discussed below. In this discussion, constants of integration appear in terms of eccentricity of the arch center line.

The application of arch constants for a uniform thickness arch to the analysis of a variable thickness arch requires the determination of the effects of movements of an assumed uniform thickness voussoir on all arch points between it and the crown of the arch. The contribution of a voussoir to an arch constant at a point is the product of the multiplier for that voussoir, in terms of r_V, T_V, and I_V, and the increment of the uniform thickness tabular values for the angles from the point considered to the limits of the voussoir. It should be kept in mind that the tables of constants contain values of the integrals only, the multipliers being computed separately. The tabular values are commonly referred to as prime ($'$) values. In transferring voussoir movements to a distant arch point, the fact that the center lines of the voussoir are eccentric with respect to arch points makes it necessary to include terms involving eccentricities in the constants. For example, consider the contribution of the voussoir between 3/4 and 1/2 points to the B_1 term at the crown. By definition, B_1 is the angular movement at a point due to a unit thrust at the point. Applied to this case, the angular movement at Q is desired for a unit thrust at Q. The thrust at Q has an eccentricity, e, with respect to the voussoir considered. This results in a moment e which causes an additional angular movement at Q of $\frac{r_V}{E_C I_V} \Delta A_1' e$. Then the contribution of voussoir, 3/4 point to 1/2 point, to B_1 is:

$$B_1, \text{ 3/4 Pt. to 1/2 Pt.,} = \frac{r_V^2}{E_C I_V} (B_1', \text{ 3/4 Pt.} - B_1', \text{ 1/2 Pt.})$$

$$+ \frac{r_V}{E_C I_V} (A_1', \text{ 3/4 Pt.} - A_1', \text{ 1/2 Pt.}) e$$

The contributions from other voussoirs are determined similarly and B_1 is computed as shown in equation 178 below. Other terms involving e are determined in a similar manner.

The following equations for arch constants are for any point and represent the summation of the various voussoir contributions at the point considered.

$$A_1 = \sum \frac{r_V}{E_C\, I_V}\, \Delta A_1' \tag{177}$$

$$B_1 = \sum \frac{r_V^2}{E_C\, I_V}\, \Delta B_1' + \sum \frac{r_V}{E_C\, I_V}\, \Delta A_1'\, e \tag{178}$$

$$C_1 = \sum \frac{r_V^2}{E_C\, I_V}\, \Delta C_1' \tag{179}$$

$$B_2 = \sum \frac{r_V^3}{E_C\, I_V}\, \Delta B_2'\ 1^{st}\,term + \sum \frac{r_V}{E_C\, T_V}\, \Delta B_2'\ 2^{nd}\,term + \sum \frac{r_V^2}{E_C\, I_V}\, \Delta C_1'\, e \tag{180}$$

$$C_2 = \sum \frac{r_V^3}{E_C\, I_V}\, \Delta C_2'\ 1^{st}\,term + \sum \frac{r_V}{E_C\, T_V}\, \Delta C_2'\ 2^{nd}\,term \tag{181}$$

$$B_3 = \sum \frac{r_V^3}{E_C\, I_V}\, \Delta B_3'\ 1^{st}\,term + \sum \frac{r_V}{E_C\, T_V}\, \Delta B_3'\ 2^{nd}\,term + \sum \frac{r_V}{E_C\, I_V}\, \Delta A_1'\, e^2 + 2 \sum \frac{r_V^2}{E_C\, I_V}\, \Delta B_1' e \tag{182}$$

105. Load Constants.—Load constants are procured in much the same manner as arch constants. The contribution of a voussoir to a load constant at a point is the product of the multiplier for that voussoir, in terms of R_U, r_V, T_V, and I_V, and the increment of the uniform thickness tabular values for the angles from the point considered to the limits of the voussoir. Since uniform thickness tabular values include the effect of the load from the point to the rock abutment, fractional portions of tabular values must be used to obtain correct increments. Section 134 and table 9, chapter VI, show how voussoir D-terms are computed by using circular arch functions.

Equations for load constants in the following sections are for any point and represent the summation of the various voussoir contributions at the point. Separate formulas are given for radial, tangential, and twist loads because each type has different multipliers and different points of application.

106. Radial Load Constants.—Radial loads are applied normally at the upstream face. Equations for their load constants are,

$$D_1 = \sum \frac{r_V^2\, R_U}{E_C\, I_V}\, \Delta D_1' \tag{183}$$

$$D_2 = \sum \frac{r_V^3\, R_U}{E_C\, I_V}\, \Delta D_2'\ 1^{st}\,term + \sum \frac{r_V\, R_U}{E_C\, T_V}\, \Delta D_2'\ 2^{nd}\,term \tag{184}$$

$$D_3 = \sum \frac{r_v^3 R_U}{E_C I_V} \Delta D_3' \text{ 1st term} + \sum \frac{r_v R_U}{E_C T_V} \Delta D_3' \text{ 2nd term} + \sum \frac{r_v^2 R_U}{E_C I_V} \Delta D_1' e \qquad (185)$$

107. Tangential Load Constants.—Tangential loads act along the arch center line, theoretically, but for the purpose of simplification in this method of analyzing a variable thickness arch, they are assumed to be applied along the arc through the crown center line. Consequently, the eccentricity of this load application must be taken into account in computing D-terms. The moment of a tangential load at an arch point, as K in figure 36, is $H_L e'$ greater than the moment at point J which is on the arc through the crown center line. Terms involving this additional moment, $H_L e'$, are included in the following equations for tangential load constants.

$$D_1 = \sum \frac{r_v^3}{E_C I_V} \Delta D_1' + \sum -(H_L e') \frac{r_v}{E_C I_V} \Delta A_1' \qquad (186)$$

$$D_2 = \sum \frac{r_v^4}{E_C I_V} \Delta D_2' \text{ 1st term} + \sum \frac{r_v^2}{E_C T_V} \Delta D_2' \text{ 2nd term} + \sum -(H_L e') \frac{r_v^2}{E_C I_V} \Delta C_1' \qquad (187)$$

$$D_3 = \sum \frac{r_v^4}{E_C I_V} \Delta D_3' \text{ 1st term} + \sum \frac{r_v^2}{E_C T_V} \Delta D_3' \text{ 2nd term} + \sum -(H_L e') \frac{r_v}{E_C I_V} \Delta A_1' e$$
$$+ \sum -(H_L e') \frac{r_v^2}{E_C I_V} \Delta B_1' + \sum \frac{r_v^3}{E_C I_V} \Delta D_1' e \qquad (188)$$

108. Twist Load Constants.—Twist loads act along the center line of the arch, theoretically; but for purposes of evaluating D-terms, they are assumed to be applied along the center line arcs of the voussoirs. This assumption, which applies a discontinuous load, can only introduce a small and negligible error in the final results, since the total length of the voussoir center lines is very nearly equal to the length of the true center line of the arch. Application of twist load to the voussoir center line produces an eccentricity with reference to other arch points and causes an additional tangential movement. This factor is included in the equation for D_3. Equations for total D-terms for twist loads are,

$$D_1 = \sum \frac{r_v^2}{E_C I_V} \Delta D_1' \qquad (189)$$

$$D_2 = \sum \frac{r_v^3}{E_C I_V} \Delta D_2' \qquad (190)$$

$$D_3 = \sum \frac{r_V^3}{E_C I_V} \Delta D_3' + \sum \frac{r_V^2}{E_C I_V} \Delta D_1' e \qquad (191)$$

109. Application of Arch Formulas.—Besides the arch and load constants, revisions of several other quantities in circular arch formulas are necessary to make them applicable to variable thickness arches. The revisions occur in formulas for moments, thrusts, and shears due to unit loads, and in formulas for coordinates of arch points.

Formulas for M_L, H_L, and V_L are the same as for uniform thickness circular arches with the following revisions. The radius, r, in radial load formulas is the radius to the arch point, $R_U - T/2$; and the radius, r, in twist and tangential load formulas is the radius to the crown center line arc. Since the H_L of the tangential load is applied along the crown center line arc, the quantity, $H_L e_P$, must be combined with M_L quantity from the load formula to give the correct value for M_L at an arch point. The complete formula for M_L at an arch point is M_L, from the load formula, minus $H_L e_P$.

The coordinates x and y are changed from $x = r \sin \phi$ and $y = r \operatorname{vers} \phi$ to $x = r_P \sin \phi$ and $y = r_O - r_P \cos \phi$. These are used in the formulas for solving crown forces. In formulas 119 to 124 for arch deflections at any arch point, coordinates x_A and y_A for the abutment, with the arch points as orgins, are $x_A = r_A \sin \phi_A$ and $y_A = r_P - r_A \cos \phi_A$.

ARCH WITH INTRADOS FILLETS

110. General Statement.—If arch abutment stresses are much greater than crown stresses, they can be reduced by providing fillets at the ends of the intrados curves. This reduces abutment stresses because the arch thickness is increased and the resultant thrust is closer to the center line of the arch. If it is necessary to extend the fillet beyond the 1/2 point of the arch, the arch is more conveniently designed as a variable thickness arch. Short radius fillets should be avoided because of indefinite excavation limits. If excavation is extended further into the canyon walls than anticipated, short radius fillets may not intersect the rock abutments.

An arch with a fillet section, as shown in figure 37, is a special case of a variable thickness arch. In the analysis of such an arch, the left and right parts are each divided into two sections, a fillet section and a uniform thickness section. By making the fillet section and the uniform section subtend central angles of whole degrees, the uniform thickness arch data with certain revisions can be used for the analysis.

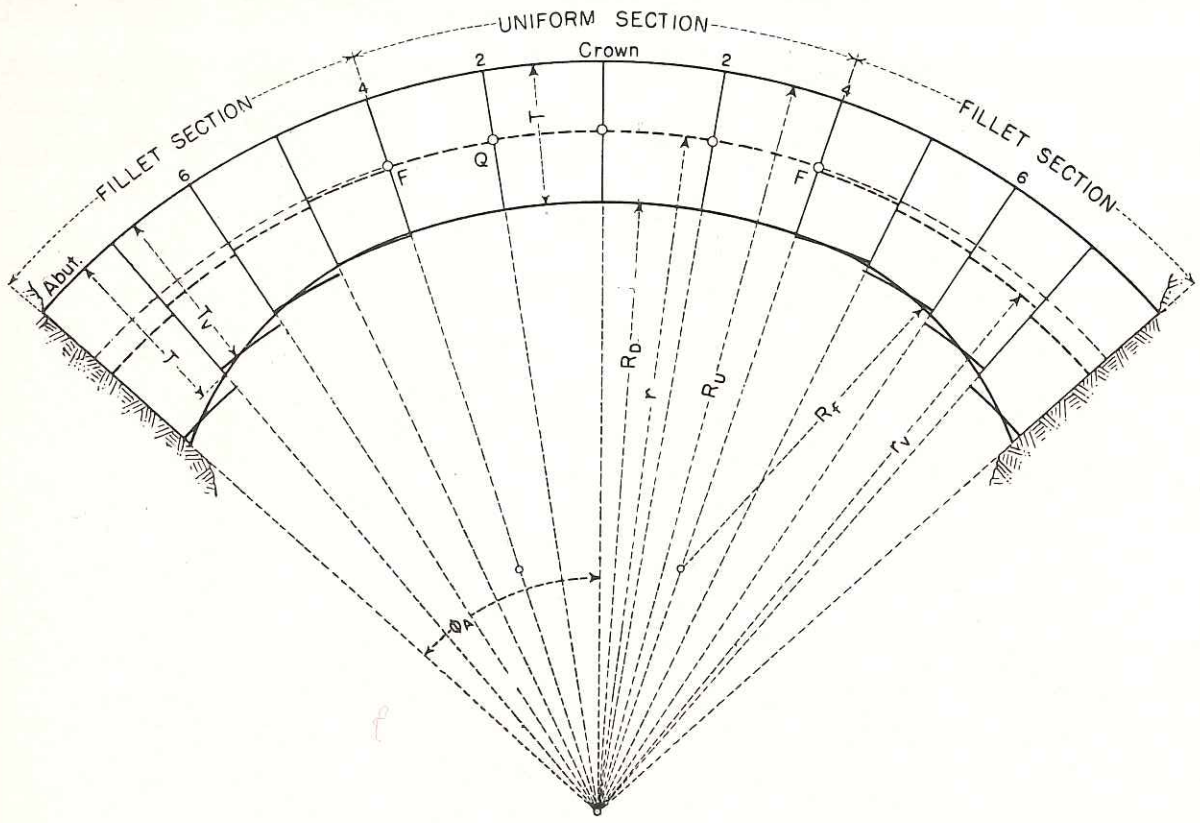


FIGURE 37—ARCH WITH INTRADOS FILLET

Arch constants are obtained in the same manner as for a variable thickness arch. Load constants are determined independently for the fillet section and the uniform thickness section. The transfer of load effects at the fillet crown, point *F*, figure 37, to points in the uniform thickness section introduces the influence of the fillet on that section.

111. Notation.—The notation for uniform and variable thickness arches applies to arches with fillets with the following additions:

F = crown of fillet section.

Q = arch point in uniform thickness section.

R_U = radius to upstream face and is the same for both sections.

r = radius to center line of uniform thickness section.

R_D = radius to downstream face of uniform thickness section.

R_f = radius to fillet at downstream face.

112. Arch Constants.—Arch constants are determined by the following procedure. The fillet section is divided into four voussoirs and the uniform section is included as an additional voussoir. Using these five voussoirs, equations 177 to 182 for variable thickness arch constants can be used for the fillet type of arch.

113. Load Constants.—Load constants at a point in the uniform thickness section, as Q in figure 37, are the sum of three quantities: first, the effect of load acting on the uniform section; second, the effect of forces and moments, due to load on the uniform section, acting on the fillet section; and, third, the effect of applied load between F and the abutment acting on the fillet section. The first quantity is evaluated in the same way as for uniform thickness arch D-terms. The second quantity is obtained by transferring the deflections at F, products of unit-load moments, thrusts, and shears, and the proper arch constants, to Q. The third quantity is determined by transferring to Q the D-terms at F due to the portion of the load on the fillet section.

In the following equations for D-terms at Q in the uniform thickness section, ($'$) values are values at F due to load on the fillet section; ($''$) values are values at Q due to load on the uniform section; M_L, H_L, and V_L refer to F; x and y are coordinates of F with Q as the origin; and ϕ is the angle from F to Q.

$$D_1 = \left[A_1' M_L - B_1' H_L + C_1' V_L\right] + D_1' + D_1'' \tag{192}$$

$$D_2 = \left[(A_1' x - B_1' \sin\phi + C_1' \cos\phi) M_L - (B_1' x - B_3' \sin\phi + B_2' \cos\phi) H_L + (C_1' x - B_2' \sin\phi + C_2' \cos\phi) V_L\right] + \left[D_1' x + D_2' \cos\phi - D_3' \sin\phi\right] + D_2'' \tag{193}$$

$$D_3 = \left[(A_1' y + B_1' \cos\phi + C_1' \sin\phi) M_L - (B_1' y + B_3' \cos\phi + B_2' \sin\phi) H_L + (C_1' y + B_2' \cos\phi + C_2' \sin\phi) V_L\right] + \left[D_1' y + D_2' \sin\phi + D_3' \cos\phi\right] + D_3'' \tag{194}$$

Quantities M_L, H_L, and V_L in these equations may be evaluated by equations for uniform thickness arches if Q is the crown point. If Q is between the fillet section and the crown of the arch, the effect of the load between Q and the crown must be eliminated. This has been done in the following equations which apply to radial, tangential, and twist loads. The subscripts F and Q preceding the terms refer to values at points F and Q, respectively, for the entire load between F or Q and the crown of the arch.

$$M_L \text{ at } F = {}_F M_L - {}_Q M_L + {}_Q H_L\, y - {}_Q V_L\, x \tag{195}$$

$$H_L \text{ at } F = {}_F H_L - {}_Q H_L \cos\phi - {}_Q V_L \sin\phi \tag{196}$$

$$V_L \text{ at } F = {}_F V_L + {}_Q H_L \sin\phi - {}_Q V_L \cos\phi \tag{197}$$

114. Application of Formulas.—Other revisions, in addition to those given for arch and load constants, occur in formulas for moments, thrusts, and shears due to applied loads, and in formulas for coordinates of arch points. These revisions are the same as for a variable thickness arch, given in section 109.

OTHER TYPES OF ARCHES

115. General Statement.—Ordinarily, the design of an arch dam can be made with circular arches of uniform thickness, variable thickness, or with downstream fillets. However, in some cases it is desirable to use circular arches of the three-centered uniform-thickness type or the poly-centered variable-thickness type. Noncircular types of arches are seldom used for arched concrete dams. Since such types of arches are seldom encountered, the discussions in the following sections are brief and limited to general considerations. Nevertheless, sufficient theory is given so that an analysis of each type can be made if necessary.

116. Three-centered Arch.—A three-centered arch of uniform thickness, as shown in figure 38, is used to make the line of thrust correspond more nearly with the center line. This makes it possible to reduce

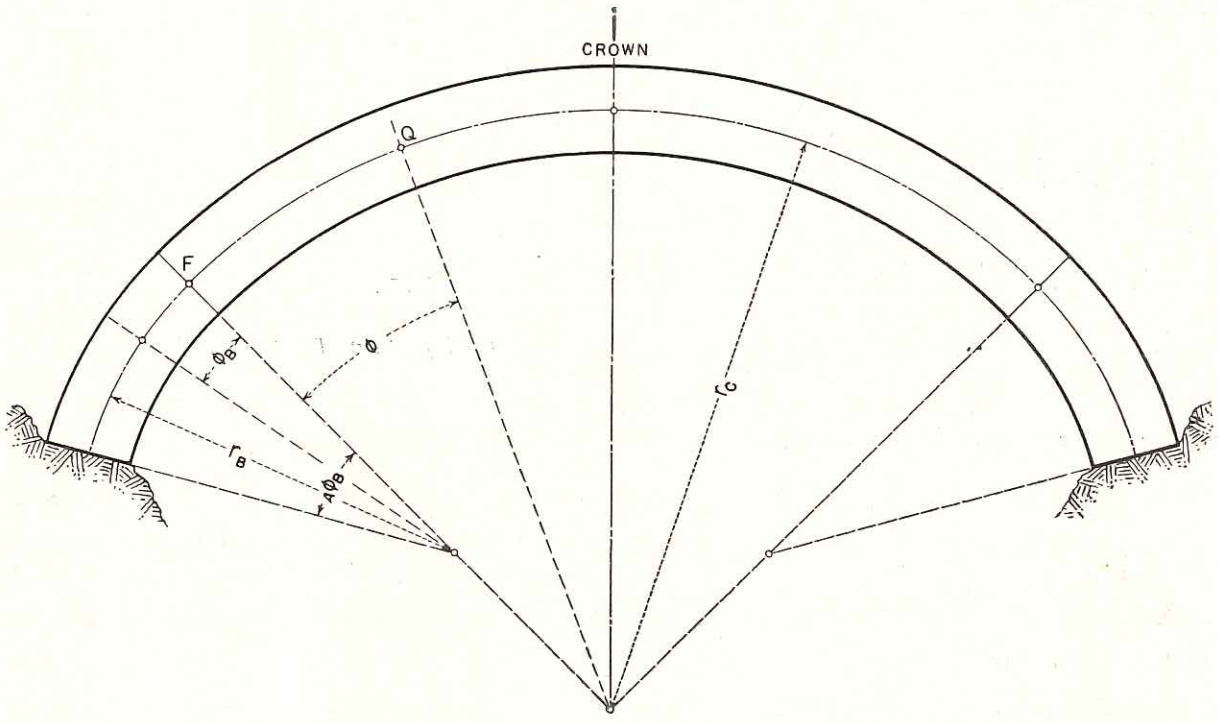


FIGURE 38—THREE-CENTERED ARCH OF UNIFORM THICKNESS

bending moments at the crown and abutments, and thereby eliminate the high tension stresses that would exist in a single-centered arch.

Formulas for a single-centered arch can be used for this case, but arch and load constants must be transferred from the crown of the abutment circular section, point F, to arch points in the central circular section. In the following equations for constants at a point Q in the central circular section, ($'$) values are values at F due to load on the abutment circular sections; ($''$) values are values at Q due to load on the central section; M_L, H_L, and V_L refer to point F; x and y are coordinates of point F with Q as the origin; and ϕ is the angle from F to Q.

$$A_1 = A_1' + A_1'' \tag{198}$$

$$B_1 = A_1' y + B_1' \cos\phi + C_1' \sin\phi + B_1'' \tag{199}$$

$$C_1 = A_1' x - B_1' \sin\phi + C_1' \cos\phi + C_1'' \tag{200}$$

$$B_2 = (A_1' y + B_1' \cos\phi + C_1' \sin\phi)x + (C_1' \cos\phi - B_1' \sin\phi)y + B_2'(\cos^2\phi - \sin^2\phi) + (C_2' - B_3')\sin\phi\cos\phi + B_2'' \tag{201}$$

$$B_3 = (A_1' y + 2B_1' \cos\phi + 2C_1' \sin\phi)y + 2B_2' \sin\phi\cos\phi + B_3' \cos^2\phi + C_2' \sin^2\phi + B_3'' \tag{202}$$

$$C_2 = (A_1' x - 2B_1' \sin\phi + 2C_1' \cos\phi)x - 2B_1' \sin\phi\cos\phi + B_3' \sin^2\phi + C_2' \cos^2\phi + C_2'' \tag{203}$$

$$D_1 = A_1' M_L - B_1' H_L + C_1' V_L + D_1' + D_1'' \tag{204}$$

$$D_2 = (A_1' x - B_1' \sin\phi + C_1' \cos\phi) M_L - (B_1' x - B_3' \sin\phi + B_2' \cos\phi) H_L + (C_1' x - B_2' \sin\phi + C_2' \cos\phi) V_L + (D_1' x + D_2' \cos\phi - D_3' \sin\phi) + D_2'' \tag{205}$$

$$D_3 = (A_1' y + B_1' \cos\phi + C_1' \sin\phi) M_L - (B_1' y + B_3' \cos\phi + B_2' \sin\phi) H_L + (C_1' y + B_2' \cos\phi + C_2' \sin\phi) V_L + (D_1' y + D_2' \sin\phi + D_3' \cos\phi) + D_3'' \tag{206}$$

The M_L, H_L, and V_L terms in these equations are determined in the same way as corresponding values for fillet arches at the fillet crown. Equations for these quantities are given in section 113, equations 195, 196, and 197.

In addition to equations for arch and load constants, formulas are needed for x and y values used in computing effect of abutment yield on points in the central circular section. These abutment coordinates, with the origin at any point Q in the central circular section, are as follows:

$$x = r_B \sin(\phi + {}_A\phi_B) + (r_C - r_B)\sin\phi \tag{207}$$

$$y = r_C - r_B \cos(\phi + {}_A\phi_B) - (r_C - r_B)\cos\phi \tag{208}$$

in which

r_C = radius to center line of central circular section.

r_B = radius to center line of abutment circular section.

ϕ = angle from any point Q to point F.

${}_A\phi_B$ = angle from point F to abutment.

It is also necessary to have equations for M_L, H_L, and V_L for points in the abutment circular section. These are developed by transferring the M_L, H_L, and V_L at F to points in the abutment circular section, giving the equations,

$$M_L = {}_BM_L + {}_FM_L - {}_FH_L y_B + {}_FV_L x_B \tag{209}$$

$$H_L = {}_BH_L + {}_FH_L \cos\phi_B + {}_FV_L \sin\phi_B \tag{210}$$

$$V_L = {}_BV_L - {}_FH_L \sin\phi_B + {}_FV_L \cos\phi_B \tag{211}$$

in which ${}_BM_L$, ${}_BH_L$, and ${}_BV_L$ are moments, thrusts, and shears of the load on the abutment circular section only; ${}_FM_L$, ${}_FH_L$, and ${}_FV_L$ refer to values at F; and x_B, y_B, and ϕ_B are coordinates from F to points in the abutment circular section.

117. Polycentered Arch.—The polycentered arch of variable thickness, shown in figure 39, offers a more complete means of reducing the undesirable tensile stresses that would exist in single-centered or three-centered types. However, it is seldom necessary to go to this refinement in the design of arch dams.

The method of analysis consists of dividing the central and abutment sections into four voussoirs, or less, and considering each section as a circular variable-thickness arch. Formulas for a variable-thickness arch can be applied to the two variable-thickness sections. Additional formulas needed to consider the two sections acting in conjunction with each other are the same as given for the three-centered arch of uniform thickness, except the coordinate equations 207 and 208, which must be revised to read

$$x = r_A \sin({}_A\phi_B + \phi) + ({}_CR_U - {}_BR_U)\sin\phi$$

$$y = r_C - r_A \cos({}_A\phi_B + \phi) - ({}_CR_U - {}_BR_U)\cos\phi$$

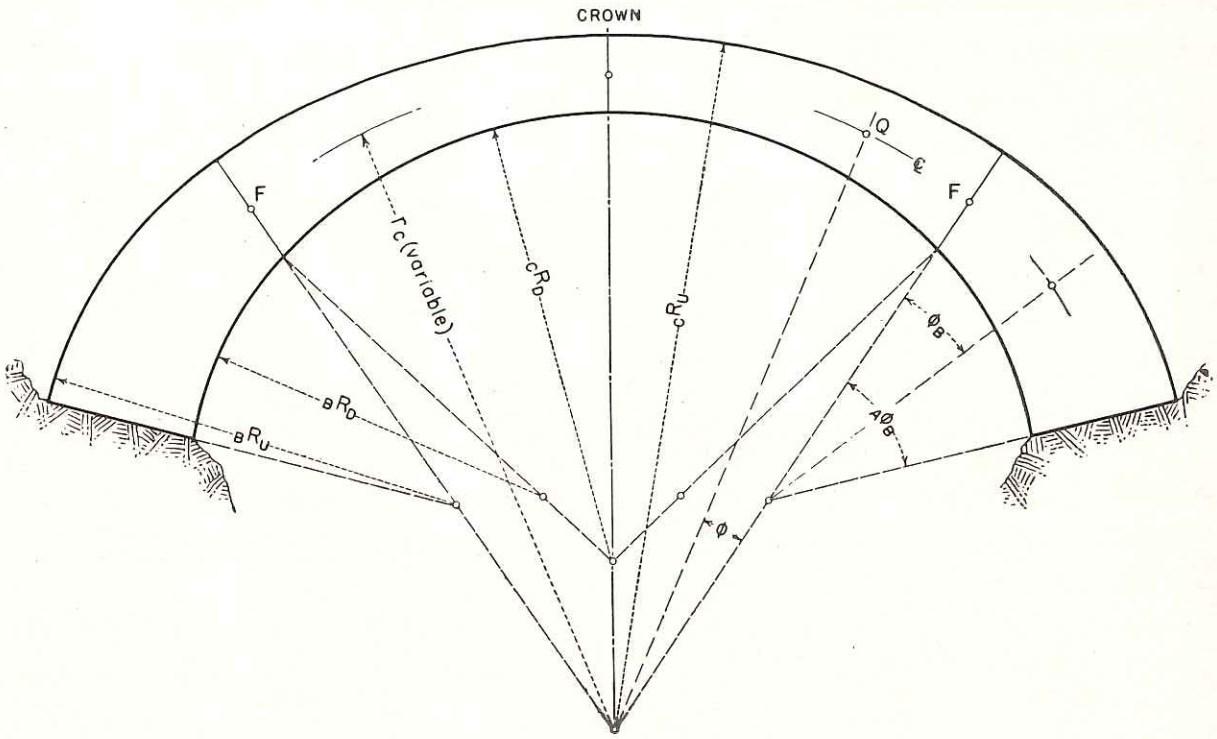


FIGURE 39—POLYCENTERED ARCH OF VARIABLE THICKNESS

where

$_cR_U$ = radius to upstream face in central section.

$_BR_U$ = radius to upstream face in abutment section.

r_C = radius to arch center line at any point Q in central section $= {}_cR_U - T/2$.

r_A = radius to arch center line at abutment $= {}_BR_U - T_A/2$.

Arch and load constants are given in equations 198 to 206, inclusive. Equations for moments, thrusts, and shears due to external load are the same as equations 209, 210, and 211. This method of analysis can also be used if either section is of uniform thickness.

118. Noncircular Arches.—Arches that do not have an extrados composed of circular segments can be analyzed by the voussoir summation method. Since tabulated arch and load constants cannot be used in the voussoir method, the summations for different trial loads require considerable time.

In the summation method integrals in general arch equations are replaced by mechanical summations of the various quantities calculated for the separate voussoirs, usually ten, into which each side of the arch is divided. Otherwise, equations for solution of crown forces and calculation of arch deflections are identical with those for the integration method.

119. Cracked Arches.—In the design of arched dams it is difficult to eliminate tension at the extrados at the abutments and at the intrados at the crown. If the customary assumption that concrete is incapable of withstanding tension is made, sections of the arch subjected to tension are disregarded in the analysis. In arch analyses based on this assumption, it has been found that deflections are not greatly different from those for an uncracked arch. Therefore, analyses of cracked arches have only a small influence on trial load adjustments of deflections. The small effect on the analysis leads to the omission of the use of cracked arches in all but special cases. Cracking of vertical cantilevers, however, does have an important effect on deflection adjustments. Consequently, cantilever cracking is usually considered in trial load analyses of arch dams where preliminary analyses indicate the occurrence of vertical tension stresses.

If it is decided that effects of cracking should be considered in analyzing arch elements, amounts of cracking, or portions of the arch to be neglected, are estimated by trial. Several trial analyses are usually necessary, since one trial does not definitely determine the largest interior arch which will be free from tension.

For the analysis of a cracked arch, the voussoir summation method described in the preceding section is used. In analyzing an interior arch, it should be remembered that radial loads are applied at the upstream face of the original arch and not at the upstream face of the interior arch.

CHAPTER VI—CALCULATION OF ARCH DATA

INTRODUCTION

120. Preliminary Statement.—This chapter discusses the application of computation forms in analyzing a uniform-thickness circular arch, a variable-thickness arch with a circular upstream face, and a circular arch with fillets at the downstream face. Calculations for other types of arches given in chapter V are not discussed herein. However, the forms illustrated can be used for other arch types by making a few changes and additions in tabular arrangements. Types of loads considered are symmetrical and nonsymmetrical unit radial, tangential, and twist loads on symmetrical or nonsymmetrical arches, and a temperature change load of + 1° F.

121. Quantities Required for Analysis.—Dimensions and physical properties, together with symbols, required in analyses of arch elements are given below.

E_C = modulus of elasticity of concrete in tension and compression.

E_R = modulus of elasticity of abutment rock in tension and compression.

T = radial thickness of arch. For variable thickness arches, T is required at the quarter points. For fillet arches, T is required at the quarter points of the fillet section.

T_A = arch thickness at abutment.

R_U = radius to upstream face of arch.

r = radius to center line of arch.

ϕ_A = angle from crown to abutment.

ϕ_L, ϕ_R = angles from reference plane or maximum cantilever section to left and right arch abutments, respectively. These are used as ϕ_A angles in analyzing nonsymmetrical arches. The value of ϕ_A for analyzing symmetrical arches is the average of ϕ_L and ϕ_R. Values of ϕ_A, or ϕ_L and ϕ_R, must be in whole degrees to utilize the tables in chapter IX.

ψ_L, ψ_R = angles between canyon walls and the vertical direction at the left and right arch abutments, respectively. They are measured in a vertical tangential plane. Unless ψ_L and ψ_R differ greatly, their average value can be used. This simplifies the computations and involves very little error.

a, b = short and long dimensions, respectively, of the equivalent rectangle of the developed foundation surface.

The above quantities are tabulated on a separate sheet for convenient use or reference. With values of E_R, T_A, ψ_L, ψ_R, a, and b, arch abutment constants α, β, γ, and α_2 can be determined on the computation form given in figure 14, chapter III.

COMPUTATION FORMS

122. General.—General computation forms consist of four sheets as shown in figures 40 to 43, inclusive. These are adaptable to the solution of any circular arch carrying symmetrical or nonsymmetrical unit loads. The forms are directly applicable to a nonsymmetrical circular arch with a nonsymmetrical loading. With a symmetrical arch or a symmetrical load, certain quantities for the left and right sides of the arch are eliminated or duplicated, obviating the use of certain portions of the forms and simplifying the computations. Analyses of variable thickness and fillet arches require supplementary computation forms for evaluating arch and load constants, as given later.

A comparison of the forms with the arch formulas in chapter V shows the purpose of the forms and the significance of the various quantities. The notation in the preceding chapter applies to the computation forms except that ϕ_A is the angle from the crown to the abutment and ϕ is the angle from the crown to the point where the deflection is desired. The following additional terms are also used on the forms:

ϕ_1 = angle from crown to quarter-point.

ϕ_2 = angle from crown to half-point.

ϕ_3 = angle from crown to three-quarter point.

123. Arch and Abutment Data.—The first sheet of the four general forms, figure 40, deals with arch constants, abutment constants, and functions needed in arch analysis. The procedure and sequence of computations for this sheet are as follows:

1. With values of r, T, R_U, and E_C compute the various factors involving these constants. The magnitude of the unit load, P, is

ARCH AND ABUTMENT CONSTANTS AND FUNCTIONS

LEFT SIDE

Arch Term	¾ Point (Use ϕ_1)	½ Point (Use ϕ_2)	¼ Point (Use ϕ_3)	Crown (Use ϕ_A)
A_1 Term	( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$
	A_1 =	A_1 =	A_1 =	A_1 =
B_1 Term	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$
	B_1 =	B_1 =	B_1 =	B_1 =
C_1 Term	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$
	C_1 =	C_1 =	C_1 =	C_1 =
B_2 1st Term	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
B_2 2nd Term	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
	B_2 =	B_2 =	B_2 =	B_2 =
C_2 1st Term	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
C_2 2nd Term	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
	C_2 =	C_2 =	C_2 =	C_2 =
B_3 1st Term	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
B_3 2nd Term	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
	B_3 =	B_3 =	B_3 =	B_3 =

For tabular arch terms see Table 10, Chapter IX

RIGHT SIDE

Crown (Use ϕ_A)	¼ Point (Use ϕ_3)	½ Point (Use ϕ_2)	¾ Point (Use ϕ_1)
( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$	( )$(\frac{12r}{E_cT^3})$
A_1 =	A_1 =	A_1 =	A_1 =
( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$
B_1 =	B_1 =	B_1 =	B_1 =
( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$	( )$(\frac{12r^2}{E_cT^3})$
C_1 =	C_1 =	C_1 =	C_1 =
( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
B_2 =	B_2 =	B_2 =	B_2 =
( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
C_2 =	C_2 =	C_2 =	C_2 =
( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$	( )$(\frac{12r^3}{E_cT^3})$
( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$	( )$(\frac{r}{E_cT})$
B_3 =	B_3 =	B_3 =	B_3 =

For trigonometric functions see Table 22, Chapter IX

‡ ϕ Point	Abut. (ϕ_A)	¾ Point (ϕ_3)	½ Point (ϕ_2)	¼ Point (ϕ_1)	Crown (ϕ_0)
ϕ					
SIN ϕ					
COS ϕ					
VERS ϕ					
$x = r$ SIN ϕ					
$y = r$ VERS ϕ					
*($\phi_A - \phi$) Point	Crown	¼ Point	½ Point	¾ Point	Abut.

Crown (ϕ_0)	¼ Point (ϕ_1)	½ Point (ϕ_2)	¾ Point (ϕ_3)	Abut. (ϕ_A)
Abut.	¾ Point	½ Point	¼ Point	Crown

Abutment Movement Functions		
	Left Abut.	Right Abut.
α		
β		
γ		
α_2		

Copied from comp. sheet, Figure 14, Chapter III

r = ... T = ... $\frac{1}{T}$ = ...

$\frac{1}{E_c}$ = ... R_U = ... P = ...

$\frac{r}{E_cT}$ = ... $\frac{r}{E_cT}R_U$ = ...

$\frac{12r}{E_cT^3}$ = ... $\frac{r^2}{E_cT}$ = ...

$\frac{12r^2}{E_cT^3}$ = ... $\frac{12r^2}{E_cT^3}R_U$ = ...

$\frac{12r^3}{E_cT^3}$ = ... $\frac{12r^3}{E_cT^3}R_U$ = ...

$\frac{12r^4}{E_cT^3}$ = ... c = ... t = ... °F

D-term multipliers	Radial Load	Tang. Load	Twist Load
D_1 Term	$\frac{12r^2}{E_cT^3}R_U$	$\frac{12r^3}{E_cT^3}$	$\frac{12r^2}{E_cT^3}$
D_2 1st Term	$\frac{12r^3}{E_cT^3}R_U$	$\frac{12r^4}{E_cT^3}$	$\frac{12r^3}{E_cT^3}$
D_2 2nd Term	$\frac{r}{E_cT}R_U$	$\frac{r^2}{E_cT}$	
D_3 1st Term	$\frac{12r^3}{E_cT^3}R_U$	$\frac{12r^4}{E_cT^3}$	$\frac{12r^3}{E_cT^3}$
D_3 2nd Term	$\frac{r}{E_cT}R_U$	$\frac{r^2}{E_cT}$	

Temperature load D-terms		
$D_1 = 0$	$D_2 = -cty$*	$D_3 = ctx$*

* Use x and y for point ($\phi_A - \phi$)

..... DAM

STUDY NO. .....

ARCH AT ELEVATION .....

COMPUTED BY ..... DATE .....

CHECKED BY ..... DATE .....

SHEET 1 OF 4

FIGURE 40—COMPUTATION FORM FOR ARCH AND ABUTMENT DATA

CONSTANTS FOR, AND SOLUTION OF CROWN FORCES

LEFT/RIGHT	Arch	A_1	=
	Abut.	α	=
	Arch	A_1	=
	Abut.	α	=
		A_1	=

RIGHT	Arch	B_1	=
	Abut.	αy_A	=
		$\alpha_2 \sin \phi_A$	=
LEFT	Arch	B_1	=
	Abut.	αy_A	=
		$\alpha_2 \sin \phi_A$	=
		B_1	=

RIGHT	Arch	$-C_1$	= −
	Abut.	$-\alpha x_A$	= −
		$-\alpha_2 \cos \phi_A$	= −
LEFT	Arch	C_1	=
	Abut.	αx_A	=
		$\alpha_2 \cos \phi_A$	=
		C_1	=

RIGHT	Arch	$-B_2$	= −
	Abut.	$-\alpha x_A y_A$	= −
		$\beta \sin \phi_A \cos \phi_A$	=
		$-\gamma \sin \phi_A \cos \phi_A$	= −
		$-\alpha_2 x_A \sin \phi_A$	= −
		$-\alpha_2 y_A \cos \phi_A$	= −
LEFT	Arch	B_2	=
	Abut.	$\alpha x_A y_A$	=
		$-\beta \sin \phi_A \cos \phi_A$	= −
		$\gamma \sin \phi_A \cos \phi_A$	=
		$\alpha_2 x_A \sin \phi_A$	=
		$\alpha_2 y_A \cos \phi_A$	=
		B_2	=

RIGHT	Arch	C_2	=
	Abut.	αx_A^2	=
		$\beta \sin^2 \phi_A$	=
		$\gamma \cos^2 \phi_A$	=
		$2\alpha_2 x_A \cos \phi_A$	=
LEFT	Arch	C_2	=
	Abut.	αx_A^2	=
		$\beta \sin^2 \phi_A$	=
		$\gamma \cos^2 \phi_A$	=
		$2\alpha_2 x_A \cos \phi_A$	=
		C_2	=

RIGHT	Arch	B_3	=
	Abut.	αy_A^2	=
		$\beta \cos^2 \phi_A$	=
		$\gamma \sin^2 \phi_A$	=
		$2\alpha_2 y_A \sin \phi_A$	=
LEFT	Arch	B_3	=
	Abut.	αy_A^2	=
		$\beta \cos^2 \phi_A$	=
		$\gamma \sin^2 \phi_A$	=
		$2\alpha_2 y_A \sin \phi_A$	=
		B_3	=

RIGHT	Arch	D_1	=
	Abut.	${}_A M_R \alpha$	=
		${}_A V_R \alpha_2$	=
LEFT	Arch	D_1	=
	Abut.	${}_A M_L \alpha$	=
		${}_A V_L \alpha_2$	=
		D_1	=

RIGHT	Arch	$-D_2$	= −
	Abut.	$-{}_A M_R \alpha x_A$	= −
		$-{}_A H_R \beta \sin \phi_A$	= −
		$-{}_A V_R \gamma \cos \phi_A$	= −
		$-{}_A M_R \alpha_2 \cos \phi_A$	= −
		$-{}_A V_R \alpha_2 x_A$	= −
LEFT	Arch	D_2	=
	Abut.	${}_A M_L \alpha x_A$	=
		${}_A H_L \beta \sin \phi_A$	=
		${}_A V_L \gamma \cos \phi_A$	=
		${}_A M_L \alpha_2 \cos_A$	=
		${}_A V_L \alpha_2 x_A$	=
		D_2	=

RIGHT	Arch	D_3	=
	Abut.	${}_A M_R \alpha y_A$	=
		$-{}_A H_R \beta \cos \phi_A$	= −
		${}_A V_R \gamma \sin \phi_A$	=
		${}_A M_R \alpha_2 \sin \phi_A$	=
		${}_A V_R \alpha_2 y_A$	=
LEFT	Arch	D_3	=
	Abut.	${}_A M_L \alpha y_A$	=
		$-{}_A H_L \beta \cos \phi_A$	= −
		${}_A V_L \gamma \sin \phi_A$	=
		${}_A M_L \alpha_2 \sin \phi_A$	=
		${}_A V_L \alpha_2 y_A$	=
		D_3	=

EQUATIONS OF EQUILIBRIUM

$$A_1 M_0 + B_1 H_0 + C_1 V_0 = D_1$$
$$B_1 M_0 + B_3 H_0 + B_2 V_0 = D_3$$
$$C_1 M_0 + B_2 H_0 + C_2 V_0 = D_2$$

SOLUTION FOR CROWN FORCES

$K = A_1(B_3C_2 - B_2^2) - B_1(B_1C_2 - C_1B_2) - C_1(B_3C_1 - B_1B_2)$

= .....

= ..... $\frac{1}{K}$ = ...

$M_0 = [D_1(B_3C_2 - B_2^2) - D_3(B_1C_2 - C_1B_2) - D_2(B_3C_1 - B_1B_2)]\frac{1}{K}$

= .....

$\times \frac{1}{K}$ = ...

$H_0 = [-D_1(B_1C_2 - B_2C_1) + D_3(A_1C_2 - C_1^2) + D_2(B_1C_1 - A_1B_2)]\frac{1}{K}$

= .....

$\times \frac{1}{K}$ = ...

$V_0 = [-D_1(B_3C_1 - B_1B_2) + D_3(B_1C_1 - A_1B_2) + D_2(A_1B_3 - B_1^2)]\frac{1}{K}$

= .....

$\times \frac{1}{K}$ = ...

NOTE

These crown forces are acting on the left side of the arch, assumed to be cut at the crown.

..... DAM

STUDY NO. .....

ARCH AT ELEVATION .....

..... LOAD NO. ..... SYMMETRICAL / NON-SYMMETRICAL

COMPUTED BY ..... DATE .....

CHECKED BY ..... DATE .....

SHEET 2 OF 4

For arch parts of constants use crown terms.
For load on one side of arch only, use only the D-term for that side.

FIGURE 41—COMPUTATION FORM FOR CROWN CONSTANTS AND FORCES

	MOMENTS					THRUSTS				SHEARS			
	For M_{L-R}, H_{L-R}, and V_{L-R} see External Load Formulas in Tables 5, 6, and 7, Chapter VI												
RIGHT SIDE	Point	M_R	$H_0 y^{\ddagger}$	$V_0 x^{\ddagger}$	$M = M_0 + H_0 y - V_0 x - M_R$	H_R	$H_0 \cos\phi^{\ddagger}$	$V_0 \sin\phi^{\ddagger}$	$H = H_0 \cos\phi + V_0 \sin\phi + H_R$	V_R	$H_0 \sin\phi^{\ddagger}$	$V_0 \cos\phi^{\ddagger}$	$V = H_0 \sin\phi - V_0 \cos\phi - V_R$
	Abut. ϕ_A												
	$\frac{3}{4}$ ϕ_3												
	$\frac{1}{2}$ ϕ_2												
	$\frac{1}{4}$ ϕ_1												
	CROWN ϕ_0												$\mp$
LEFT SIDE	$\frac{1}{4}$ ϕ_1												
	$\frac{1}{2}$ ϕ_2												
	$\frac{3}{4}$ ϕ_3												
	Abut. ϕ_A												
	Point	M_L	$H_0 y^{\ddagger}$	$V_0 x^{\ddagger}$	$M = M_0 + H_0 y + V_0 x - M_L$	H_L	$H_0 \cos\phi^{\ddagger}$	$V_0 \sin\phi^{\ddagger}$	$H = H_0 \cos\phi - V_0 \sin\phi + H_L$	V_L	$H_0 \sin\phi^{\ddagger}$	$V_0 \cos\phi^{\ddagger}$	$V = H_0 \sin\phi + V_0 \cos\phi - V_L$

‡ Use coordinates and trigonometric functions for points ϕ, comp. sheet 1 of 4, Figure 40

RADIAL DEFLECTIONS

Signs of D_2 below operate on sign of D-Term as calculated

	ABUTMENT MOVEMENTS			¾ Point	½ Point	¼ Point	Crown
RIGHT SIDE	$M_A\alpha + V_A\alpha_2 = \Theta$ = ...	Arch	$-D_2$ =	–	–	–	–
			$C_1 M$ =				
	$-H_A\beta = \Delta s$ = ...		$B_2 H$ =				
			$C_2 V$ =				
			Sub-total =				
	$V_A\gamma + M_A\alpha_2 = \Delta r$ = ...	Abut.	$(M_A\alpha + V_A\alpha_2)x^*$ =				
			$(-H_A\beta)\sin\phi^*$ =				
			$(V_A\gamma + M_A\alpha_2)\cos\phi^*$ =				
			Total Δr =				
LEFT SIDE	$M_A\alpha + V_A\alpha_2 = \Theta$ = ...	Arch	$-D_2$ =	–	–	–	–
			$C_1 M$ =				
	$-H_A\beta = \Delta s$ = ...		$B_2 H$ =				
			$C_2 V$ =				
			Sub-total =				
	$V_A\gamma + M_A\alpha_2 = \Delta r$ = ...	Abut.	$(M_A\alpha + V_A\alpha_2)x^*$ =				
			$(-H_A\beta)\sin\phi^*$ =				
			$(V_A\gamma + M_A\alpha_2)\cos\phi^*$ =				
			Total Δr =				

* Use coordinates and trigonometric functions for points $(\phi_A - \phi)$, comp. sheet 1 of 4, Figure 40

D-TERM		LEFT SIDE	RIGHT SIDE
From Tables 17 to 21 inclusive, Chapter IX			
Cr.	D_1	( )( )	( )( )
Cr.	D_3 1 ST TERM	( )( )	( )( )
	D_3 2 ND TERM	( )( )	( )( )
Cr.	D_2 1 ST TERM	( )( )	( )( )
	D_2 2 ND TERM	( )( )	( )( )
¼	D_2 1 ST TERM	( )( )	( )( )
	D_2 2 ND TERM	( )( )	( )( )
½	D_2 1 ST TERM	( )( )	( )( )
	D_2 2 ND TERM	( )( )	( )( )
¾	D_2 1 ST TERM	( )( )	( )( )
	D_2 2 ND TERM	( )( )	( )( )

For multipliers see sheet 1 of 4

For multipliers see sheet 1 of 4

.....DAM

STUDY NO.....

ARCH AT ELEVATION.....

.....LOAD NO..... SYMMETRICAL NON-SYMMETRICAL

COMPUTED BY.....DATE.....

CHECKED BY.....DATE.....

SHEET 3 OF 4

FIGURE 42—COMPUTATION FORM FOR MOMENTS, THRUSTS, SHEARS, AND RADIAL DEFLECTIONS

TANGENTIAL DEFLECTIONS

	ABUTMENT MOVEMENTS			3/4 Point	1/2 Point	1/4 Point	Crown
RIGHT SIDE	$M_A\alpha + V_A\alpha_2 = \theta$ = ...	Arch	$-D_3$ =	−	−	−	−
			B_1M =				
			B_3H =				
			B_2V =				
			Sub-total =				
	$V_A\sigma + M_A\alpha_2 = \Delta r$ = ...	Abut.	$(M_A\alpha + V_A\alpha_2)y^*$ =				
			$(V_A\sigma + M_A\alpha_2)\sin\phi^*$ =				
	$H_A\beta = \Delta s$ = ...		$(H_A\beta)\cos\phi^*$ =				
			Total Δs =				
LEFT SIDE	$-M_A\alpha - V_A\alpha_2 = \theta$ = ...	Arch	D_3 =	+	+	+	+
			$-B_1M$ =				
			$-B_3H$ =				
			$-B_2V$ =				
			Sub-total =				
	$-V_A\sigma - M_A\alpha_2 = \Delta r$ = ...	Abut.	$(-M_A\alpha - V_A\alpha_2)y^*$ =				
			$(-V_A\sigma - M_A\alpha_2)\sin\phi^*$ =				
	$-H_A\beta = \Delta s$ = ...		$(-H_A\beta)\cos\phi^*$ =				
			Total Δs =				

* Use coordinates and trigonometric functions for points $(\phi_A - \phi)$ comp. sheet 1 of 4, Figure 40

D-TERM		LEFT SIDE	RIGHT SIDE
Cr.	D_3 1 ST TERM	( )( )	( )( )
	D_3 2 ND TERM	( )( )	( )( )
1/4	D_3 1 ST TERM	( )( )	( )( )
	D_3 2 ND TERM	( )( )	( )( )
1/2	D_3 1 ST TERM	( )( )	( )( )
	D_3 2 ND TERM	( )( )	( )( )
3/4	D_3 1 ST TERM	( )( )	( )( )
	D_3 2 ND TERM	( )( )	( )( )
Cr.	D_1	( )( )	( )( )
1/4	D_1	( )( )	( )( )
1/2	D_1	( )( )	( )( )
3/4	D_1	( )( )	( )( )

For multipliers see sheet 1 of 4

For multipliers see sheet 1 of 4

For tabular D-terms see tables 17 to 21, inclusive, Chapter IX

ANGULAR MOVEMENTS

	ABUTMENT MOVEMENTS			3/4 Point	1/2 Point	1/4 Point	Crown
RIGHT SIDE	$-M_A\alpha - V_A\alpha_2 = \theta$ = ...	Arch	D =	+	+	+	+
			$-A_1M$ =				
			$-B_1H$ =				
			$-C_1V$ =				
			Sub-total =				
		Ab.	$(-M_A\alpha - V_A\alpha_2)$ =				
			Total θ =				
LEFT SIDE	$M_A\alpha + V_A\alpha_2 = \theta$ = ...	Arch	$-D_1$ =	−	−	−	−
			A_1M =				
			B_1H =				
			C_1V =				
			Sub-total =				
		Ab.	$(M_A\alpha + V_A\alpha_2)$ =				
			Total θ =				

..... DAM

STUDY NO: .....

ARCH AT ELEVATION .....

..... LOAD NO. ..... SYMMETRICAL / NON-SYMMETRICAL

COMPUTED BY ..... DATE .....

CHECKED BY ..... DATE .....

SHEET 4 OF 4

FIGURE 43—COMPUTATION FORM FOR TANGENTIAL AND ANGULAR MOVEMENTS

usually 1,000. The coefficient of expansion of concrete, c, for 1° F. change of temperature may be found in text books on concrete; and the magnitude of the temperature change, t, is usually taken as + 1° F.

2. The value of ϕ_A and the quarter values thereof, in degrees, are tabulated on the line designated ϕ. Trigonometric functions of angles are secured from tables in chapter IX.

3. Arch constants, A_1, B_1, and C_1, are each the product of a tabular value secured from table 10, chapter IX, and a multiplier which is evaluated at the bottom of the computation sheet. Arch constants, B_2, C_2, and B_3, are each the sum of two products as indicated. Each product is made up of a tabular value and a multiplier, which are obtained in the same way as A_1, B_1, and C_1.

4. Abutment movement functions are copied from the form given in figure 14, chapter III.

124. Crown Constants and Forces.—Figure 41, sheet 2 of the set, deals with the evaluation of crown constants and their use in obtaining the moment, thrust, and shear at the crown. A comparison of the form with table 3, chapter V, shows that if integrals in the table are replaced by their respective symbols, the constants are the same. Equations for the solution of crown forces are given in chapter V. The procedure is briefly discussed below.

1. Copy arch parts of A_1, B_1, C_1, B_2, C_2, and B_3 from the crown point on the preceding form, sheet 1, and compute abutment parts from the values on the same sheet. Total values are the sum of the arch and abutment parts.

2. The arch part of D_1, D_2, or D_3 is the product of a tabular term and its multiplier, or the sum of two such products. These products are secured from their component parts, for the crown point, listed at the right central portion of sheet 3, see subsequent section for explanation. Abutment parts are composed of products of values found on sheets 1 and 3. The ${}_AM_L$, ${}_AH_L$, and ${}_AV_L$, or the ${}_AM_R$, ${}_AH_R$, and ${}_AV_R$ values for these terms are from sheet 3. Total values of D-terms are sums of arch and abutment parts. For concentrated loads, total D-terms include only the effects of yielding abutments. Temperature D-terms are evaluated by formulas given on sheet 1. These are total D-terms because temperature changes are assumed to have no direct effect on abutment yielding.

3. After values of M_O, H_O, and V_O have been computed by the equations on the right side of the sheet, they should be checked by substituting them in the equations of equilibrium.

125. Moments, Thrusts, Shears, and Radial Deflections.—Moments, thrusts, and shears at arch points are computed in the upper part of sheet 3, figure 42. Radial deflections are computed in the lower part of the sheet. Equations for moments, thrusts, and shears are the same as equations 110 to 115, inclusive, given in chapter V. Computations of radial deflections are based on equations 120 and 123 in chapter V. The procedure is as follows:

1. Before sheet 2 can be completed, it is necessary to enter on sheet 3 tabular values of D-terms for the crown point and values of M_L, H_L, and V_L for the point, Abut. ϕ_A. However, at the time these values are determined, it is expedient to fill in values for other points between the crown and abutment. Formulas for M_L, H_L, and V_L for uniform and triangular radial, tangential, and twist loads are given in tables 5, 6, and 7. Radial, tangential, and twist concentrated loads are, respectively, a shear ${}_AV_L = P$, a thrust ${}_AH_L = P$, and a moment ${}_AM_L = P$, P being negative in the case of thrust.

2. Tabular D-term constants at the right end of the sheet are copied from tables 17 to 21, inclusive, chapter IX. Products of the tabular values and their proper multipliers are D-terms or D-constants used in computing deflections. Crown D-terms are used on sheet 2 in computing crown forces.

3. With M_O, H_O, and V_O, computed on sheet 2, and coordinates and trigonometric functions for arch points, computed on sheet 1, the remaining columns across the top of the sheet can be computed, giving values of M, H, and V at arch points.

4. All quantities for radial deflections are now calculated. The portion of the deflection at a point due to the elastic movement of the arch is the algebraic sum of the D-terms on sheet 3 and the arch constants, from sheet 1, multiplied by the moments, thrusts, and shears on sheet 3. The portion of the deflection due to elastic deformation of the yielding abutment is obtained by multiplying M_A, H_A, and V_A by the proper abutment constants, α, α_2, β, and γ on sheet 1, transferred to arch points by using functions of the angle $\phi_A - \phi$ on sheet 1.

126. Tangential and Angular Movements.—The computation form for tangential and angular movements, sheet 4, is shown in figure 43. The indicated calculations for tangential deflections are based on equations 121 and 124, and calculations for angular movements are based on equations 119 and 122, chapter V. The procedure is as follows:

1. Tabular D-term constants at the upper right corner of the sheet are copied from tables 17 to 21, inclusive, chapter IX.

2. Total tangential deflections and total angular movements are computed by performing the indicated operations, which are similar to those outlined for computing radial deflections.

127. Use of Forms.—Since deflection summations are sometimes multiplied by large factors and since the number of significant figures in the summations is often reduced by algebraic additions, the use of seven significant figures is necessary to provide sufficient accuracy. Suggestions regarding general computation forms are given in the following paragraphs.

1. Computations on sheet 1 should be independently checked, because a slight discrepancy may produce errors of considerable magnitude in subsequent calculations.

2. Total values of A_1, B_1, C_1, B_2, C_2, and B_3 on sheet 2 should be checked. Since these values can be used for any unit load, they are computed only once for each arch element. Total values of load constants D_1, D_2, and D_3 should also be verified.

3. On the upper part of sheet 3, only the total values of M, H, and V at the crown and abutment points should be calculated at first. With these and other values already calculated, radial deflections on sheet 3 and tangential deflections and angular movements on sheet 4 may be determined at the crown of the arch and checked as follows:

Tangential deflections and angular movements for symmetrical loads are zero.

Radial deflections, tangential deflections, and angular movements for the two sides of the arch are equal for nonsymmetrical loads.

If crown movements satisfy these conditions, calculations of quantities for other arch points are resumed and carried to completion.

4. Inspection of plotted deflection curves may indicate appreciable errors at the arch quarter-points, as discussed in section 137.

UNIFORM THICKNESS CIRCULAR ARCH

128. General.—The analysis of a uniform-thickness circular arch is made on the four general computation forms previously discussed. In an analysis of this type the arch is usually divided at the center or crown point, making the arch symmetrical if the abutment angles, ψ, are the same for the left and right sides. A division of the element at any other point makes the arch nonsymmetrical.

The symmetry of the dam about the maximum vertical section determines the applicability of symmetrical or nonsymmetrical loads. If the dam is symmetrical about the maximum section, it is obvious that both arches and loads must be symmetrical. Consequently, only one side of the dam need be analyzed. If the dam is nonsymmetrical the unbalanced loadings required on the left and right parts of the arches necessitate the use of nonsymmetrical loads and sometimes nonsymmetrical arches.

In computations for a unit load on a circular arch, it is necessary to evaluate moments, thrusts, and shears at the arch quarter-points. Although general formulas for these terms are given in chapter V, it is convenient to have separate formulas for each quarter-point. Tabulations of such formulas for M_L, H_L, and V_L, which apply also for M_R, H_R, and V_R, are given in tables 5, 6, and 7, for unit radial, tangential, and twist loads, respectively. Numerical values of the various quantities in the formulas can be obtained from sheet 1 of the general computation forms and from table 22, chapter IX.

129. Symmetrical Arch with Symmetrical Loads.—As stated in the last section, a symmetrical arch with symmetrical loads requires the analysis of only one half of the arch, the analysis of the other half being identical. For convenience, the left side of the arch is the one usually analyzed.

All unit loads discussed in chapter V are applicable to the analysis of an arch of this type, with the exception of uniform tangential and twist loads. These two types of loads are not symmetrical about the crown due to their direction of application on the right side of the arch, see section 83, chapter V. Incidentally, these two loads are unnecessary in the trial load analysis of a symmetrical dam.

If the analysis of a symmetrical arch with a symmetrical load is made using both parts of the arch, it is evident that load and arch functions

TABLE 5—FORMULAS FOR MOMENTS, THRUSTS, AND SHEARS OF RADIAL LOADS

	POINT	M_L	H_L	V_L
UNIT LOAD NO. 1	1/4	$PR_U r \text{ vers } \phi_1$	$PR_U \text{ vers } \phi_1$	$PR_U \sin \phi_1$
	1/2	$PR_U r \text{ vers } \phi_2$	$PR_U \text{ vers } \phi_2$	$PR_U \sin \phi_2$
	3/4	$PR_U r \text{ vers } \phi_3$	$PR_U \text{ vers } \phi_3$	$PR_U \sin \phi_3$
	ABUT.	$PR_U r \text{ vers } \phi_A$	$PR_U \text{ vers } \phi_A$	$PR_U \sin \phi_A$
UNIT LOAD NO. 2	1/4			
	1/2			
	3/4			
	ABUT.	$\frac{PR_U r}{\phi_1}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_1}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_1} \text{ vers } \phi_1$
UNIT LOAD NO. 3	1/4			
	1/2			
	3/4	$\frac{PR_U r}{\phi_2}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_2}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_2} \text{ vers } \phi_1$
	ABUT.	$\frac{PR_U r}{\phi_2}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_2}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_2} \text{ vers } \phi_2$
UNIT LOAD NO. 4	1/4			
	1/2	$\frac{PR_U r}{\phi_3}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_3}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_3} \text{ vers } \phi_1$
	3/4	$\frac{PR_U r}{\phi_3}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_3}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_3} \text{ vers } \phi_2$
	ABUT.	$\frac{PR_U r}{\phi_3}(\phi_3 - \sin \phi_3)$	$\frac{PR_U}{\phi_3}(\phi_3 - \sin \phi_3)$	$\frac{PR_U}{\phi_3} \text{ vers } \phi_3$
UNIT LOAD NO. 5	1/4	$\frac{PR_U r}{\phi_A}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_A}(\phi_1 - \sin \phi_1)$	$\frac{PR_U}{\phi_A} \text{ vers } \phi_1$
	1/2	$\frac{PR_U r}{\phi_A}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_A}(\phi_2 - \sin \phi_2)$	$\frac{PR_U}{\phi_A} \text{ vers } \phi_2$
	3/4	$\frac{PR_U r}{\phi_A}(\phi_3 - \sin \phi_3)$	$\frac{PR_U}{\phi_A}(\phi_3 - \sin \phi_3)$	$\frac{PR_U}{\phi_A} \text{ vers } \phi_3$
	ABUT.	$\frac{PR_U r}{\phi_A}(\phi_A - \sin \phi_A)$	$\frac{PR_U}{\phi_A}(\phi_A - \sin \phi_A)$	$\frac{PR_U}{\phi_A} \text{ vers } \phi_A$

See Table 22, Chapter IX for tabulation of trigonometric functions.

TABLE 6—FORMULAS FOR MOMENTS, THRUSTS, AND SHEARS OF TANGENTIAL LOADS

	POINT	M_L	H_L	V_L
UNIT LOAD NO. 1	1/4	$Pr^2(\phi_1 - \sin\phi_1)$	$-Pr \sin\phi_1$	Pr vers ϕ_1
	1/2	$Pr^2(\phi_2 - \sin\phi_2)$	$-Pr \sin\phi_2$	Pr vers ϕ_2
	3/4	$Pr^2(\phi_3 - \sin\phi_3)$	$-Pr \sin\phi_3$	Pr vers ϕ_3
	ABUT.	$Pr^2(\phi_A - \sin\phi_A)$	$-Pr \sin\phi_A$	Pr vers ϕ_A
UNIT LOAD NO. 2	1/4			
	1/2			
	3/4			
	ABUT	$\frac{Pr^2}{\phi_1}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right)$	$-\frac{Pr}{\phi_1}$ vers ϕ_1	$\frac{Pr}{\phi_1}(\phi_1 - \sin\phi_1)$
UNIT LOAD NO. 3	1/4			
	1/2			
	3/4	$\frac{Pr^2}{\phi_2}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right)$	$-\frac{Pr}{\phi_2}$ vers ϕ_1	$\frac{Pr}{\phi_2}(\phi_1 - \sin\phi_1)$
	ABUT.	$\frac{Pr^2}{\phi_2}\left(\frac{\phi_2^2}{2} - \text{vers}\,\phi_2\right)$	$-\frac{Pr}{\phi_2}$ vers ϕ_2	$\frac{Pr}{\phi_2}(\phi_2 - \sin\phi_2)$
UNIT LOAD NO. 4	1/4			
	1/2	$\frac{Pr^2}{\phi_3}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right)$	$-\frac{Pr}{\phi_3}$ vers ϕ_1	$\frac{Pr}{\phi_3}(\phi_1 - \sin\phi_1)$
	3/4	$\frac{Pr^2}{\phi_3}\left(\frac{\phi_2^2}{2} - \text{vers}\,\phi_2\right)$	$-\frac{Pr}{\phi_3}$ vers ϕ_2	$\frac{Pr}{\phi_3}(\phi_2 - \sin\phi_2)$
	ABUT.	$\frac{Pr^2}{\phi_3}\left(\frac{\phi_3^2}{2} - \text{vers}\,\phi_3\right)$	$-\frac{Pr}{\phi_3}$ vers ϕ_3	$\frac{Pr}{\phi_3}(\phi_3 - \sin\phi_3)$
UNIT LOAD NO. 5	1/4	$\frac{Pr^2}{\phi_A}\left(\frac{\phi_1^2}{2} - \text{vers}\,\phi_1\right)$	$-\frac{Pr}{\phi_A}$ vers ϕ_1	$\frac{Pr}{\phi_A}(\phi_1 - \sin\phi_1)$
	1/2	$\frac{Pr^2}{\phi_A}\left(\frac{\phi_2^2}{2} - \text{vers}\,\phi_2\right)$	$-\frac{Pr}{\phi_A}$ vers ϕ_2	$\frac{Pr}{\phi_A}(\phi_2 - \sin\phi_2)$
	3/4	$\frac{Pr^2}{\phi_A}\left(\frac{\phi_3^2}{2} - \text{vers}\,\phi_3\right)$	$-\frac{Pr}{\phi_A}$ vers ϕ_3	$\frac{Pr}{\phi_A}(\phi_3 - \sin\phi_3)$
	ABUT.	$\frac{Pr^2}{\phi_A}\left(\frac{\phi_A^2}{2} - \text{vers}\,\phi_A\right)$	$-\frac{Pr}{\phi_A}$ vers ϕ_A	$\frac{Pr}{\phi_A}(\phi_A - \sin\phi_A)$

See Table 22, Chapter IX for tabulation of trigonometric functions.

TABLE 7—FORMULAS FOR MOMENTS, THRUSTS, AND SHEARS OF TWIST LOADS

	POINT	M_L	H_L	V_L
UNIT LOAD NO. 1	1/4	$Pr\,\phi_1$	0	0
	1/2	$Pr\,\phi_2$	0	0
	3/4	$Pr\,\phi_3$	0	0
	ABUT.	$Pr\,\phi_A$	0	0
UNIT LOAD NO. 2	1/4			
	1/2			
	3/4			
	ABUT.	$\frac{Pr}{\phi_1}\,\frac{\phi_1^2}{2}$	0	0
UNIT LOAD NO. 3	1/4			
	1/2			
	3/4	$\frac{Pr}{\phi_2}\,\frac{\phi_1^2}{2}$	0	0
	ABUT	$\frac{Pr}{\phi_2}\,\frac{\phi_2^2}{2}$	0	0
UNIT LOAD NO. 4	1/4			
	1/2	$\frac{Pr}{\phi_3}\,\frac{\phi_1^2}{2}$	0	0
	3/4	$\frac{Pr}{\phi_3}\,\frac{\phi_2^2}{2}$	0	0
	ABUT	$\frac{Pr}{\phi_3}\,\frac{\phi_3^2}{2}$	0	0
UNIT LOAD NO. 5	1/4	$\frac{Pr}{\phi_A}\,\frac{\phi_1^2}{2}$	0	0
	1/2	$\frac{Pr}{\phi_A}\,\frac{\phi_2^2}{2}$	0	0
	3/4	$\frac{Pr}{\phi_A}\,\frac{\phi_3^2}{2}$	0	0
	ABUT.	$\frac{Pr}{\phi_A}\,\frac{\phi_A^2}{2}$	0	0

See Table 22, Chapter IX for tabulation of trigonometric functions.

and constants are identical for both parts of the arch. For this reason the following omissions may be made on the general computation forms:

Sheet 1.—All arch constants, trigonometric functions, coordinates, and abutment movement functions for the right side of the arch, because they are the same as on the left side.

C_2 for the crown point of the left side of the arch, see omissions on sheets 2 and 3.

Sheet 2.—All computations for C_1, B_2, and D_2, because the summations are zero.

All quantities for C_2, because in the third equation of equilibrium, with C_1, B_2, and D_2 equal to zero, C_2 is not needed.

The right side values for A_1, B_1, B_3, D_1, and D_3, because a common factor 2 cancels in the solution for crown forces if values for both sides are used.

Sheet 3.—Moments, thrusts, shears, D-terms, and radial deflections for the right side of the arch.

Columns for $V_0 x$, $V_0 \sin \phi$, and $V_0 \cos \phi$ for the left side of the arch, because V_0 is zero.

Sheet 4.—D-terms, tangential deflections, and angular movements for the right side of the arch.

The omissions on sheet 2 reduce the equations of equilibrium to

$$A_1 M_0 + B_1 H_0 = D_1 \tag{212}$$

$$B_1 M_0 + B_3 H_0 = D_3 \tag{213}$$

whose simultaneous solution gives

$$K = A_1 B_3 - B_1^2 \tag{214}$$

$$M_0 = (D_1 B_3 - D_3 B_1)\frac{1}{K} \tag{215}$$

$$H_0 = (-D_1 B_1 + D_3 A_1)\frac{1}{K} \tag{216}$$

$$V_0 = 0 \tag{217}$$

for the solution of crown forces. The total constants A_1, B_1, and B_3 on this sheet are applicable to any symmetrical load on the arch under consideration and need not be recomputed.

130. Symmetrical Arch with Nonsymmetrical Loads.—The analysis of a nonsymmetrical dam usually requires nonsymmetrical unit triangular and concentrated loads. Analyses are made for loads placed on the left side of the arch. With a symmetrical arch, their numerical results for corresponding points can be used for loads placed on the right side of the arch if the changes of sign indicated in table 8 are made. For example, the tangential deflection at the left 1/4 point due to a load on

TABLE 8—ALGEBRAIC SIGNS OF QUANTITIES FOR LOADS ON RIGHT SIDE OF ARCH RELATIVE TO QUANTITIES FOR SAME LOADS ON LEFT SIDE

Quantity	Radial Loads	Tangential Loads	Twist Loads
Radial deflections	Same	Same	Same
Tangential deflections	Opposite	Opposite	Opposite
Angular movements	Opposite	Opposite	Opposite
Resultant M, H, and V	Same	Same	Same

the right side of the arch is numerically equal, but of opposite sign, to the tangential deflection at the right 1/4 point due to a load on the left side of the arch.

If the uniform radial load and the temperature change load are worked in conjunction with nonsymmetrical loads, it is more convenient to load both halves of the arch and use the total arch and load constants on sheet 2. These constants include values for both sides of the arch and are double the values obtained in section 129. The advantage of this operation is that one set of arch constants can be used for the computation of these loads and the nonsymmetrical loads.

In using computation forms for the analysis of nonsymmetrical triangular and concentrated loads on symmetrical arches, many computations are omitted. These are briefly listed as follows:

Sheet 1.—All arch constants, trigonometric functions, coordinates, and abutment movement functions for the right side of the arch, because they are the same as on the left side.

Sheet 2.—All computations for C_1 and B_2, because the summations are zero.

All values for D_1, D_2, and D_3 for the right side of the arch, because the load is absent from that side of the arch.

Sheet 3.—D-terms and M_R, H_R, and V_R for the right side of the arch, because the load is absent from that side of the arch.

Sheet 4.—D-terms for the right side of the arch, because the load is absent from that side of the arch.

The omissions on sheet 2 reduce the equations of equilibrium to

$$A_1 M_0 + B_1 H_0 = D_1 \tag{218}$$

$$B_1 M_0 + B_3 H_0 = D_3 \tag{219}$$

$$C_2 V_0 = D_2 \tag{220}$$

the simultaneous solution of which gives

$$K = A_1 B_3 - B_1^2 \tag{221}$$

$$M_0 = (D_1 B_3 - D_3 B_1)\frac{1}{K} \tag{222}$$

$$H_0 = (-D_1 B_1 + D_3 A_1)\frac{1}{K} \tag{223}$$

$$V_0 = \frac{D_2}{C_2} \tag{224}$$

for the solution of crown forces.

It will be noted that V_0, the resultant crown shear, has both a positive and a negative sign in the table at the top of sheet 3. The positive sign is used for computing deflections on the left half of the arch, as shown on sheet 2, and the negative sign for computing deflections on the right half.

A nonsymmetrical analysis requires uniform tangential and twist loads so that loadings may be applied at the crown of the arch. To be continuous, these loads must act in the same direction on both sides of the arch. From section 83, chapter V, it is evident that this amounts to placing a negative uniform load on the right side of the arch. This requires the reversal of algebraic sign of the following quantities for the portion of the uniform load on the right side of the arch: M_R, H_R, and V_R; all right-side parts of the total D_1, D_2, and D_3 terms used for the solution of crown forces; and the D_1, D_2, and D_3 terms used in computing deflections on the right side of the arch. These changes cause the total D_1 and D_3 terms and the values of M_0 and H_0 on sheet 2 to become zero.

131. Nonsymmetrical Arch.—A nonsymmetrical arch of uniform thickness is encountered only if it is desirable to have the crown of the

arch coincide with the maximum cantilever section, or if abutment yielding conditions are different on the two sides of the canyon. Otherwise the crown is placed at the middle and the arch is analyzed as symmetrical.

A nonsymmetrical arch will obviously have nonsymmetrical loads, requiring separate calculation of loads on both sides of the arch. The general computation forms with no omissions are used for their analysis.

VARIABLE THICKNESS ARCH

132. General.—The previously described computation forms are used for analyzing a variable thickness arch with a constant upstream radius. Before the forms can be used, however, arch and load constants must be evaluated by the supplementary calculations explained in chapter V and discussed in the following sections. These constants are inserted on the general forms and the remainder of the analysis completed as described for a uniform thickness arch, making the changes discussed in section 135.

Eccentricities and multipliers for arch and load constants are calculated on the form shown in figure 44, after the thickness and center-line radius at the arch quarter-points have been determined.

133. Arch Constants.—The evaluation of arch constants by equations 177 to 182 in chapter V is explained in the text preceding those equations. A computation form on which calculations for arch and load constants can be conveniently made is shown in figure 45. For convenience, all summations involving Δ values of any constant are worked on one sheet, and those summations which are parts of other constants are transferred to the proper sheets for combination with other principal parts.

134. Load Constants.—The equations used to evaluate load constants are numbers 183 to 191 in chapter V. Equations 183 to 185 are for radial loads, 186 to 188 for tangential loads, and 189 to 191 for twist loads. Quantities involved in these equations are defined and explained in chapter V.

Calculations are made in the same way as for arch constants, except that prime values cannot be obtained directly from tables in chapter IX. However, with the assistance of table 9, a prime value which is the effect at the point under consideration due to load between that point and a limit of a voussoir, may be obtained from tables 17 to 21, chapter IX.

.....DAM STUDY NO.....

ARCH AT EL.....

COMPUTED BY..... DATE.....

CHECKED BY..... DATE.....

E_C =..... $\frac{1}{E_C}$ =.....

R_U =.....

$r = R_U - \frac{T}{2}$ $I_V = \frac{T_V^3}{12}$

$r_V = R_U - \frac{T_V}{2}$

$e_{0,1,2,3} = r_P - r_V = e$, for transfer of arch and load constants.

$e_0 = e'$, for tangential load eccentricity.

e_P = eccentricity of arch point with reference to arc of center line through crown.

POINT	ϕ	VOUSSOIR	AT POINT		FOR VOUSSOIR		VOUSSOIR TO POINT				e_P
			T	r_P	T_V	r_V	e_0	e_1	e_2	e_3	
0 – (Cr.)											
		0 to 1									
1 – (1/4)											
		1 to 2									
2 – (1/2)											
		2 to 3									
3 – (3/4)											
		3 to 4									
4 – (Abut.)											

VOUSSOIR	$\frac{1}{E_C T_V}$	$\frac{r_V}{E_C T_V}$	$\frac{r_V^2}{E_C T_V}$	$\frac{r_V}{E_C I_V}$	$\frac{r_V^2}{E_C I_V}$	$\frac{r_V^3}{E_C I_V}$	$\frac{r_V^4}{E_C I_V}$	$\frac{r_V R_U}{E_C T_V}$	$\frac{r_V^2 R_U}{E_C I_V}$	$\frac{r_V^3 R_U}{E_C I_V}$
0 to 1										
1 to 2										
2 to 3										
3 to 4										

FIGURE 44—COMPUTATION FORM FOR ECCENTRICITIES AND MULTIPLIERS FOR ARCH AND LOAD CONSTANTS

.....DAM STUDY NO.....

ARCH AT EL.....

COMPUTED BY.....

DATE.....

PT.	ϕ							
CROWN								
1/4								
1/2								
3/4								

FIGURE 45—COMPUTATION FORM FOR ARCH OR LOAD CONSTANTS FOR VARIABLE-THICKNESS ARCHES

TABLE 9—CIRCULAR ARCH D-TERMS USED FOR COMPUTING VOUSSOIR D-TERMS

PT.	ϕ	D VALUE AT ARCH POINT FOR LOAD PATTERN FROM ARCH POINT THROUGH ANGLE ϕ				
		NO. 1 LOAD	NO. 2 LOAD	NO. 3 LOAD	NO. 4 LOAD	NO. 5 LOAD
CROWN	0	0	0	0	0	0
	ϕ_1	D, Load No. 1, 3/4 Pt.	0	0	0	1/4 D, Load No. 2, 3/4 Pt.
	ϕ_2	D, Load No. 1, 1/2 Pt.	0	0	1/3 D, Load No. 2, 1/2 Pt.	1/2 D, Load No. 3, 1/2 Pt.
	ϕ_3	D, Load No. 1, 1/4 Pt.	0	1/2 D, Load No. 2, 1/4 Pt.	2/3 D, Load No. 3, 1/4 Pt.	3/4 D, Load No. 4, 1/4 Pt
	ϕ_A	D, Load No. 1, Cr.	D, Load No. 2, Cr.	D, Load No. 3, Cr.	D, Load No. 4, Cr.	D, Load No. 5, Cr.
1/4	ϕ_1	0	0	0	0	0
	ϕ_2	D, Load No. 1, 3/4 Pt.	0	0	1/3 D, Load No. 2, 3/4 Pt.	1/2 D, Load No. 3, 3/4 Pt.
	ϕ_3	D, Load No. 1, 1/2 Pt.	0	1/2 D, Load No. 2, 1/2 Pt.	2/3 D, Load No. 3, 1/2 Pt.	3/4 D, Load No. 4, 1/2 Pt.
	ϕ_A	D, Load No. 1, 1/4 Pt.	D, Load No. 2, 1/4 Pt.	D, Load No. 3, 1/4 Pt.	D, Load No. 4, 1/4 Pt.	D, Load No. 5, 1/4 Pt.
1/2	ϕ_2	0	0	0	0	0
	ϕ_3	D, Load No. 1, 3/4 Pt.	0	1/2 D, Load No. 2, 3/4 Pt.	2/3 D, Load No. 3, 3/4 Pt.	3/4 D, Load No. 4, 3/4 Pt.
	ϕ_A	D, Load No. 1, 1/2 Pt.	D, Load No. 2, 1/2 Pt.	D, Load No. 3, 1/2 Pt.	D, Load No. 4, 1/2 Pt.	D, Load No. 5, 1/2 Pt.
3/4	ϕ_3	0	0	0	0	0
	ϕ_A	D, Load No. 1, 3/4 Pt.	D, Load No. 2, 3/4 Pt.	D, Load No. 3, 3/4 Pt.	D, Load No. 4, 3/4 Pt.	D, Load No. 5, 3/4 Pt.

Table 9 gives fractions of circular arch D-terms to use for prime values. Their differences give voussoir D-terms, or $\Delta D'$ values.

The formulas appearing in table 9 are best explained by reference to figure 46, which illustrates the method followed in determining formulas

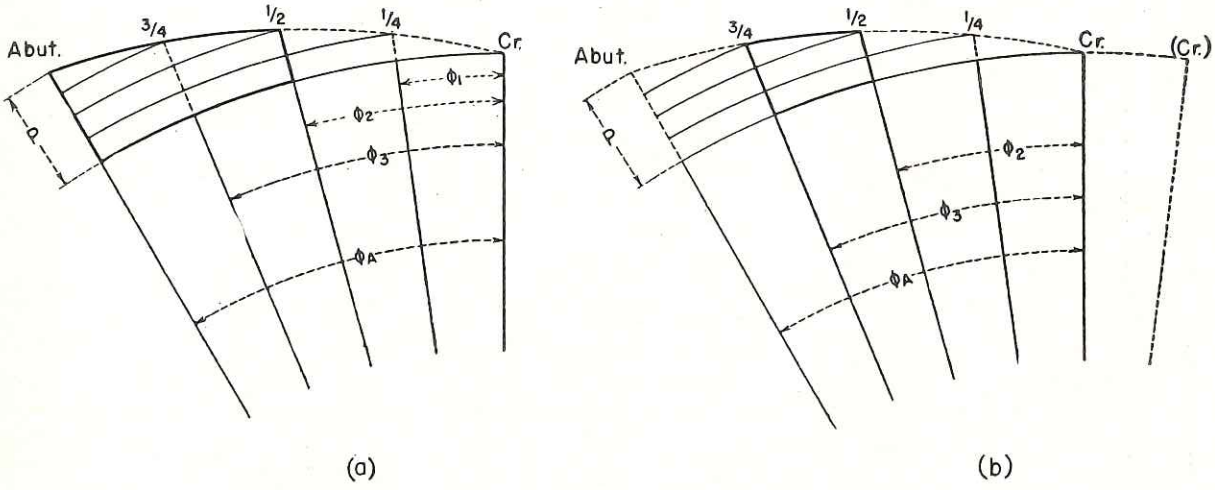


FIGURE 46—DETERMINATION OF FORMULAS FOR LOAD 5, TABLE 9

for load 5 at the 1/2 point. Since D-terms are deflections due to load at the left of the point considered, it can be seen that the tabular D-term at the 1/2 point, for the load between the 1/2 point and the abutment, is the tabular value at the 1/2 point for load 5 for angle ϕ_A, see figure 46-a. Likewise, the tabular D-term at the 1/2 point for the load between the 1/2 and 3/4 points, ϕ_2 to ϕ_3, is equal to 3/4 of the D-term at the 3/4 point for load 4 for angle ϕ_A. This statement is evident if it is assumed that the half-arch is moved to the right, with the load stationary; so that it extends from the 3/4 to Cr, as shown in figure 46-b. This makes the load ordinate at the 3/4 point equal to 3/4 of P, and the true 1/2 point corresponds to the 3/4 point of the assumed position of the arch. Then the tabular D-term for voussoir ϕ_A to ϕ_3 is

$$\Delta D', \phi_A \text{ to } \phi_3, = D \text{ for load 5 at 1/2 Pt.} - 3/4\ D \text{ for load 4 at 3/4 Pt.},$$

and for voussoir ϕ_3 *to* ϕ_2

$$\Delta D', \phi_3 \text{ to } \phi_2, = 3/4\ D \text{ for load 4 at 3/4 Pt.}$$

These may now be inserted directly in equations 183 to 191, chapter V. By following similar procedures, formulas can be determined for D-term values, or prime values, at other arch points for other unit loads.

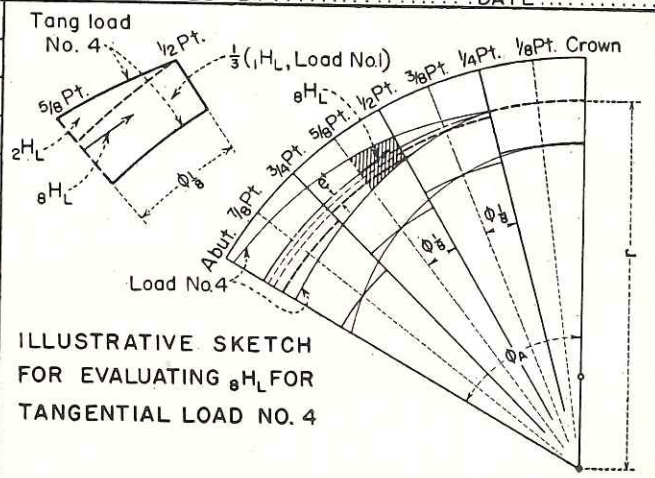
.....DAM STUDY NO.....

ARCH AT EL..... TANG. LOAD NO.....

COMPUTED BY..... DATE.....

FORMULAS FOR COMPUTING ECCENTRIC THRUSTS

	Tang. Load No. 1	Tang. Load No. 2	Tang. Load No. 3	Tang. Load No. 4	Tang. Load No. 5
${}_1H_L$	$-Pr \sin \phi_{1/8}$	0	0	0	$-\frac{Pr}{\phi_A}$ vers $\phi_{1/8}$
${}_2H_L$	$-Pr \sin \phi_{3/8}$	0	0	$-\frac{Pr}{\phi_3}$ vers $\phi_{1/8}$	$-\frac{Pr}{\phi_A}$ vers $\phi_{3/8}$
${}_3H_L$	$-Pr \sin \phi_{5/8}$	0	$-\frac{Pr}{\phi_2}$ vers $\phi_{1/8}$	$-\frac{Pr}{\phi_3}$ vers $\phi_{3/8}$	$-\frac{Pr}{\phi_A}$ vers. $\phi_{5/8}$
${}_4H_L$	$-Pr \sin \phi_{7/8}$	$-\frac{Pr}{\phi_1}$ vers $\phi_{1/8}$	$-\frac{Pr}{\phi_2}$ vers $\phi_{3/8}$	$-\frac{Pr}{\phi_3}$ vers $\phi_{5/8}$	$-\frac{Pr}{\phi_A}$ vers $\phi_{7/8}$
${}_5H_L$	${}_1H_L$	0	0	${}_2H_L$	${}_1H_L+\frac{1}{4}({}_1H_L$, Load No.1)
${}_6H_L$	${}_2H_L$	0	${}_3H_L$	${}_3H_L$	${}_2H_L+\frac{1}{4}({}_2H_L$, Load No.1)
${}_7H_L$	${}_3H_L$	${}_4H_L$	${}_4H_L$	${}_4H_L$	${}_3H_L+\frac{1}{4}({}_3H_L$, Load No.1)
${}_8H_L$	${}_1H_L$	0	${}_3H_L$	${}_2H_L+\frac{1}{3}({}_1H_L$, Load No.1)	${}_1H_L+\frac{1}{2}({}_1H_L$, Load No.1)
${}_9H_L$	${}_2H_L$	${}_4H_L$	${}_4H_L$	${}_3H_L+\frac{1}{3}({}_2H_L$, Load No.1)	${}_2H_L+\frac{1}{2}({}_2H_L$, Load No.1)
${}_{10}H_L$	${}_1H_L$	${}_4H_L$	${}_3H_L+\frac{1}{2}({}_1H_L$, Load No.1)	${}_2H_L+\frac{2}{3}({}_1H_L$, Load No.1)	${}_1H_L+\frac{3}{4}({}_1H_L$, Load No.1)



ILLUSTRATIVE SKETCH FOR EVALUATING ${}_8H_L$ FOR TANGENTIAL LOAD NO. 4

——·——= Arc through crown ℄ - radius (r)
– – – – – –= Arc through mid-point of voussoir.
– – – – – –= Voussoir mid-point arc extended.
Tangential load applied from left to right along arc through crown ℄
e_o=e'=eccentricity of thrust with reference to arc through mid-point voussoir.
Φ = angle from crown to point.
$\Phi_{\frac{1}{8}}=\frac{1}{8}$ of Φ_A, $\Phi_{\frac{3}{8}}=\frac{3}{8}$ of Φ_A, etc.
${}_1H_L$, ${}_2H_L$, etc. are thrusts accruing along arc of crown ℄ between $\frac{1}{4}$ points of arc and centers of voussoirs.

TRIGONOMETRIC FUNCTIONS

POINT	Φ	SIN Φ	VERS Φ
$\frac{1}{8}$			
$\frac{3}{8}$			
$\frac{5}{8}$			
$\frac{7}{8}$			

MOMENTS ($H_L e'$) DUE TO ECCENTRIC THRUSTS

POINT	CROWN		$\frac{1}{4}$ POINT		$\frac{1}{2}$ POINT		$\frac{3}{4}$ POINT	
	H_L	$H_L e'$	H_L	$H_L e'$	H_L	$H_L e'$	H_L	$H_L e'$
$\frac{1}{8}$	${}_1H_L$							
$\frac{3}{8}$	${}_2H_L$		${}_5H_L$					
$\frac{5}{8}$	${}_3H_L$		${}_6H_L$		${}_8H_L$			
$\frac{7}{8}$	${}_4H_L$		${}_7H_L$		${}_9H_L$		${}_{10}H_L$	

FIGURE 47--COMPUTATION FORM FOR MOMENTS DUE TO ECCENTRICITY OF TANGENTIAL LOADS ($H_L e'$)

The remaining computations for load constants require no further explanation except those for moments due to eccentricity of tangential loads. Calculations for $H_L e'$ quantities may be made on the computation form shown in figure 47. This form may require a little clarification. The H_L terms are tangential thrusts at the centers of the voussoirs. Referring to the table at the bottom of the form, each H_L is the thrust due to the tangential load beween the 1/4 point given at the top of the table and the center of the voussoir indicated at the side. Thus ${}_8H_L$ is the tangential thrust at the 5/8 point, center of the third voussoir, due to the load between that point and the 1/2 point. This load is illustrated in the sketch for tangential load 4, shown in the upper right corner of figure 47. In the upper part of the figure is a tabulation of formulas for the H_L terms for various loads. These are based on equations 151 and 159 of chapter V.

135. Other Calculations.—After arch and load constants have been determined on the supplementary computation forms, general forms can be used for the remaining calculations. Formulas for x and y, however, must be changed to $x = r_P \sin \phi$ and $y = r_O - r_P \cos \phi$ as explained in chapter V, section 109. These are the arch point coordinates with the crown as the origin. They are used on sheet 2 in solving the crown forces, and on sheet 3 in computing total moments. In addition, values of x_A and y_A for coordinates of the abutment with arch points as origins are required. These may be computed on sheet 1, on the two vacant lines under point $\phi_A - \phi$. Their formulas are $x_A = r_A \sin (\phi_A - \phi)$ and $y_A = r_P - r_A \cos (\phi_A - \phi)$ in which ϕ_A is the angle from the crown to the abutment and ϕ is the angle from the crown to the point considered. These coordinates are used in calculating deflections and temperature D-terms. Another change to be made in the use of the general forms is in the values of M_L, H_L, and V_L on sheet 3. These must be calculated as described in section 109, chapter V. Inasmuch as calculations of arch and load constants are made on supplementary forms, values of tabular parts and multipliers are not placed on the general forms.

ARCH WITH FILLETS AT DOWNSTREAM FACE

136. General Statement.—An arch with abutment fillets at the downstream face is a special case of a variable thickness arch. Hence, computations for an analysis of this type of arch are similar to those for

a variable-thickness arch. The computations may be made on the same forms, using the method and formulas described in sections 110 to 114, chapter V.

Arch forces and deflections are calculated for the arch crown, the point midway between the arch crown and the fillet crown, the fillet crown, the 1/2 point of the fillet section, and the abutment. This procedure applies to the usual case where the fillet starts near the 1/2 point. If it begins near the 3/4 point of the half-arch, a similar arrangement of points can be used.

CURVES AND TABULATIONS

137. General.—After analyses have been made for all unit loads on an arch, curves are plotted showing resulting radial, tangential, and angular movements. Tabulations are also prepared showing resulting forces, moments, and deflections.

As stated in section 127, curves of deflections are convenient for checking purposes. Errors in calculations of deflections are indicated by pronounced variations from smooth curves. Studies of curves for various unit loads on an arch, together with considerations of corresponding curves for other arches in the same dam, often establish the general trend of deflection curves and may indicate other errors.

Another check is possible with deflection curves, due to the interesting relation between radial deflection and angular movement curves for the same load. The angular movement at any point approximately represents the slope of the radial deflection curve at that point. Thus the maximum angular movement and the maximum slope of the radial deflection curve are at the same point. Similarly the maximum radial deflection occurs at the point of zero angular movement. These relations are particularly evident if the arch is long and thin, so that the greater parts of the movements are due to bending. In short and thick arches these relations do not hold, because large parts of the radial deflections are produced by shear detrusions.

138. Symmetrical Unit Loads.—Typical deflection curves for symmetrical unit loads on an arch element are shown in figures 48, 49, and 50. The dimensions of the element are shown below the curves. Although the curves are for a variable-thickness arch, they are typical for uniform-thickness arches and fillet arches.

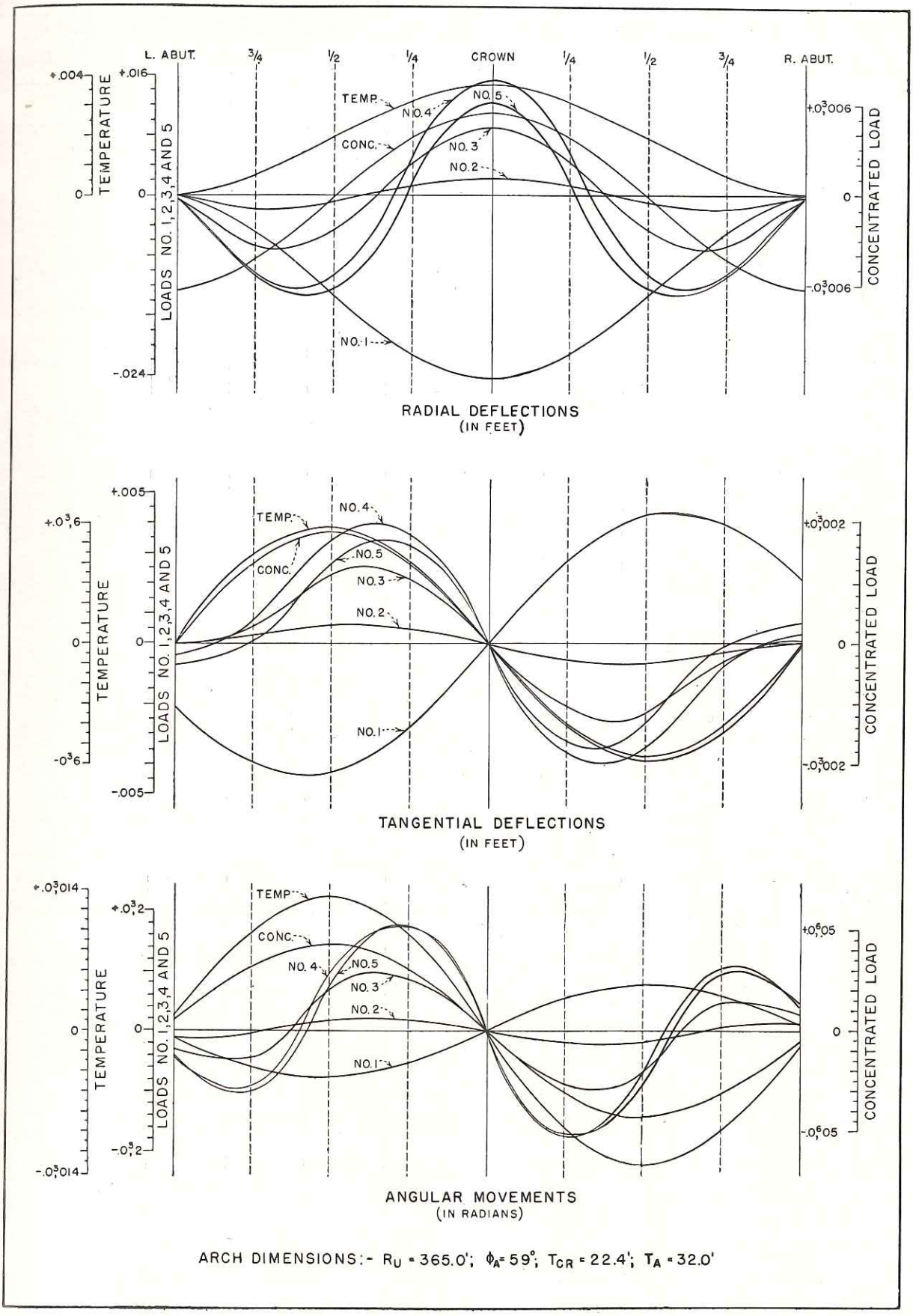


FIGURE 48—TYPICAL ARCH DEFLECTION CURVES FOR SYMMETRICAL UNIT RADIAL LOADS AND +1° F. TEMPERATURE CHANGE

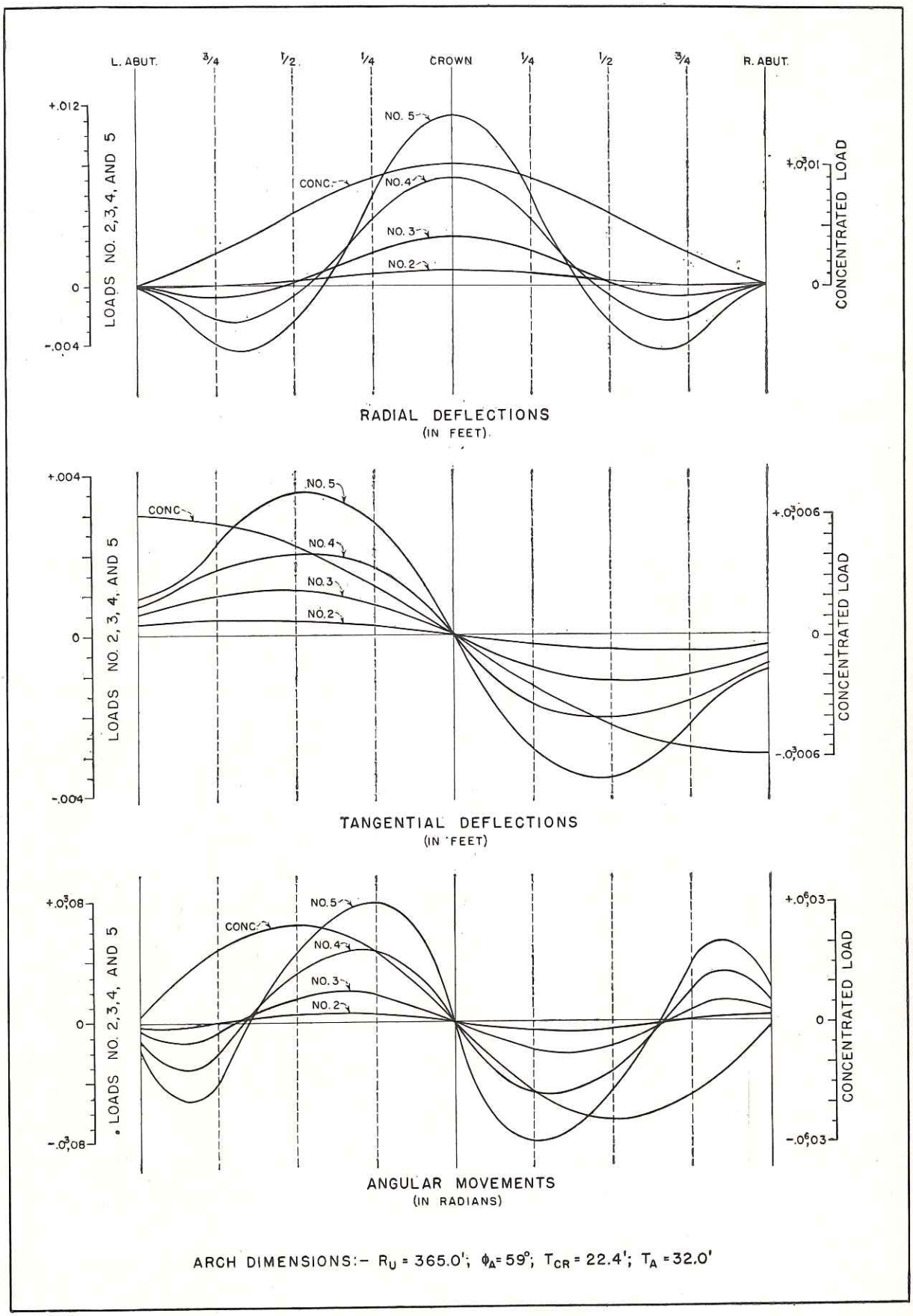


FIGURE 49—TYPICAL ARCH DEFLECTION CURVES FOR SYMMETRICAL UNIT TANGENTIAL LOADS

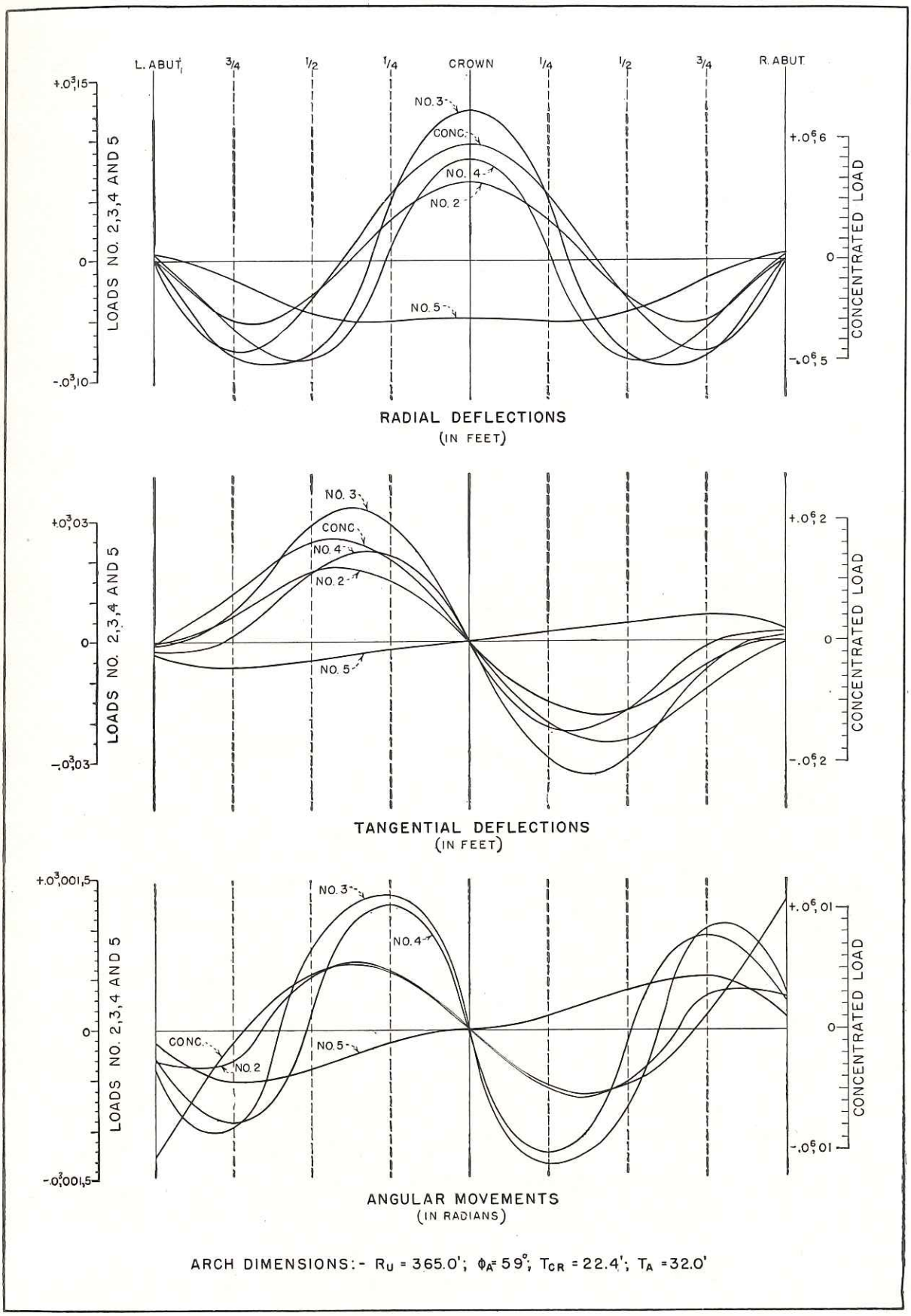


FIGURE 50—TYPICAL ARCH DEFLECTION CURVES FOR SYMMETRICAL UNIT TWIST LOADS

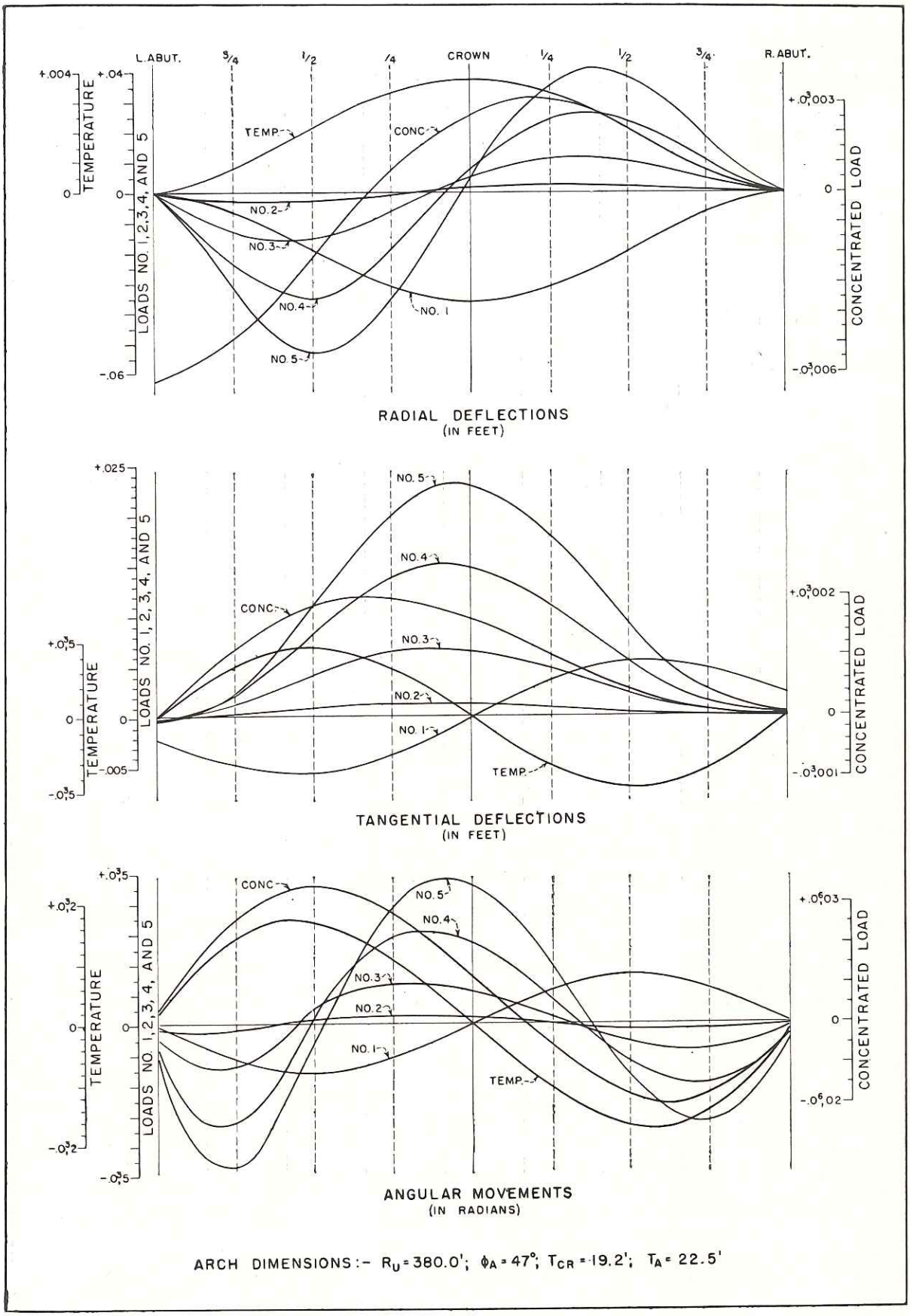


FIGURE 51—TYPICAL ARCH DEFLECTION CURVES FOR NONSYMMETRICAL UNIT RADIAL LOADS AND +1° F. TEMPERATURE CHANGE

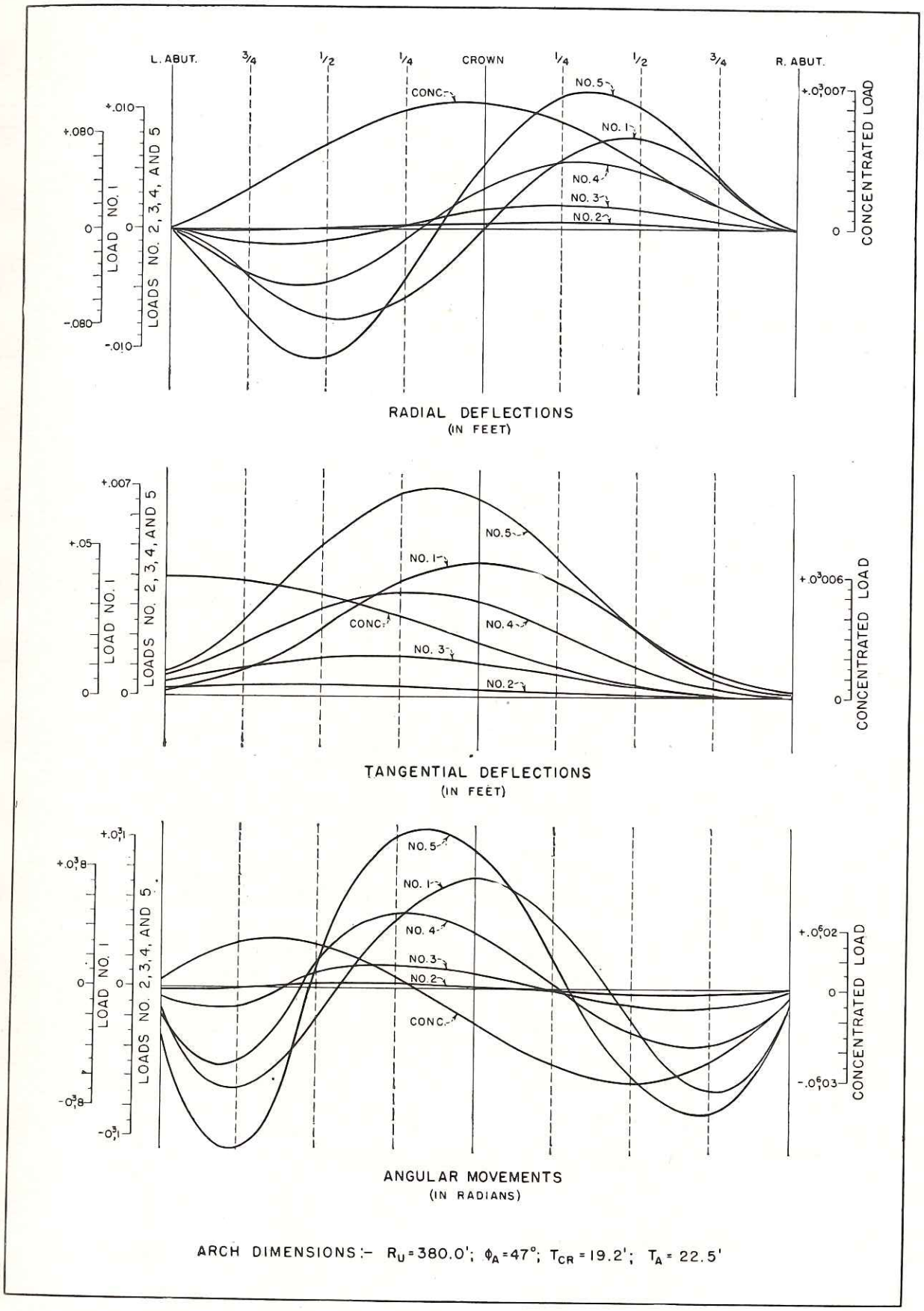


FIGURE 52—TYPICAL ARCH DEFLECTION CURVES FOR NONSYMMETRICAL UNIT TANGENTIAL LOADS

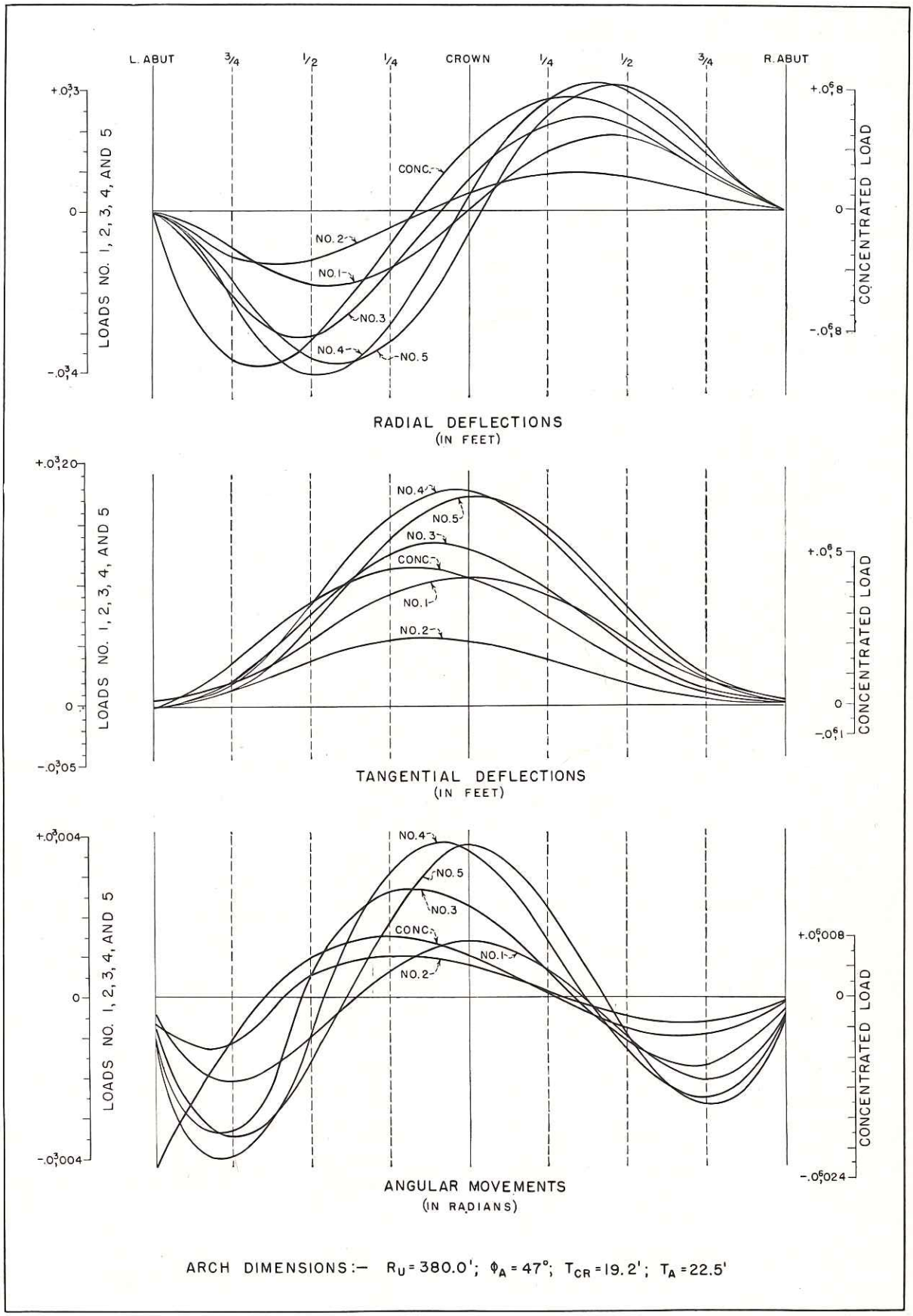


FIGURE 53—TYPICAL ARCH DEFLECTION CURVES FOR NONSYMMETRICAL UNIT TWIST LOADS

139. Nonsymmetrical Unit Loads.—Figures 51, 52, and 53 show typical deflection curves for nonsymmetrical unit loads on a symmetrical arch. These curves are for a variable thickness arch with dimensions similar to those for the arch represented in figures 48 to 50. The curves for unit triangular and concentrated loads are for loads on the left part of the arch only. If the curves are reversed, so that the left abutment becomes the right abutment and vice versa, and if the changes of sign indicated on table 8, section 130, are applied, the curves will be those for unit triangular and concentrated loads on the right side of the arch. The deflections for uniform loads remain the same.

140. Arch Forces and Deflections.—Moments, thrusts, and shears caused by unit loads, at the crown and abutment points are tabulated for use in calculating stresses. Values at other arch points are included if stresses are desired at such points. Tabulations of arch point deflections are also made for all unit loads. These are used in deflection adjustments as described in chapter VII.

CHAPTER VII—LOADS AND ADJUSTMENTS

INTRODUCTION

141. General Statement.—Trial load adjustments consist of dividing external loads between the arch and cantilever systems and applying internal self-balancing tangential and twist loads to the two systems, until geometrical continuity is established throughout the structure. To accomplish this, required loads are estimated and gradually determined by trial until deflections and rotations of arches and cantilevers agree at conjugate points. Radial, tangential, and angular movements due to forces acting on the dam are considered respectively in radial, tangential, and twist adjustments.

142. Arches and Cantilevers.—In beginning a trial load analysis the number and location of sample arches and cantilevers to be used must be determined. If the dam is reasonably symmetrical about the maximum cantilever section, only half the structure is considered in the analysis. In this case about five to seven arches and four to seven cantilevers are used. If the parts of the dam on the left and right sides of the maximum section are considerably different, a nonsymmetrical analysis, in which the entire dam is considered, is required. More cantilevers, usually nine to eleven, are used for this type of analysis.

One cantilever is located at the maximum section and the others on the arch abutments, as indicated in chapter II. With this selection of elements, it may not always be feasible to bring arch and cantilever deflections into reasonable agreement in some parts of the dam. In such cases additional elements may be necessary. A sharp break in abutment slope often requires additional cantilevers. After sample arches and cantilevers are selected, properties and dimensions needed in analyzing elements are determined.

143. Constants and Structural Data.—Constants and structural data required for a trial load analysis are given in the following list:

1. Dimensions of arches and cantilevers.
2. Modulus of elasticity of concrete in tension and compression.

3. Modulus of elasticity of concrete in shear.
4. Poisson's ratio of concrete.
5. Modulus of elasticity of abutment rock in tension and compression.
6. Poisson's ratio of abutment rock.
7. Dimensions of developed base of dam.
8. Slopes of arch abutments and cantilever foundations.
9. Unit weight of concrete.
10. Unit weight of water.
11. Elevation of reservoir water surface.
12. Elevation of tailwater surface.
13. Acceleration, direction, and period of vibration of assumed earthquake shock.
14. Temperature changes at elevations of arches.
15. Coefficient of thermal expansion for concrete.
16. Unit weight and elevation of silt accumulations.
17. Elevation and pressure of ice loads.

FORCES ACTING ON A DAM

144. General.—Forces acting on a dam include weight, temperature changes, and some combination of external loads produced by reservoir water, tailwater, uplift pressure, silt, ice, and earthquake shocks.

145. Concrete Weight.—Evaluations of weights, moments, and deflections due to concrete loads are described in chapter IV. If the dam is built in vertical sections and construction joints grouted after completion, effects of concrete weight are taken by the cantilevers alone. In this case concrete weight does not affect trial load adjustments and need not be considered until stresses are computed. If grouting is started before completion, deflections due to weights of concrete added subsequently must be included in the analysis. Likewise, when cracking of the cantilevers is assumed to take place, concrete weight becomes a factor in computing deflections of vertical elements. This subject is treated in chapter IV.

146. Temperature.—Seasonal variations of water and air temperatures at the faces of the dam cause changes in concrete temperature. These changes cause stresses and movements in the arches. Increases in concrete

temperature cause upstream movements which work against the water load, while decreases cause downstream movements which work with the water load. The critical condition, from the viewpoint of trial load analyses, is usually produced by the maximum drop from grouting temperature occurring at time of maximum reservoir water surface. A temperature curve showing maximum ranges in concrete temperatures for arches of different thickness is given in figure 54. The curve is based on an average of theoretical values and experimental data collected from a large number of existing dams.[5] If construction joints are assumed to be grouted at the

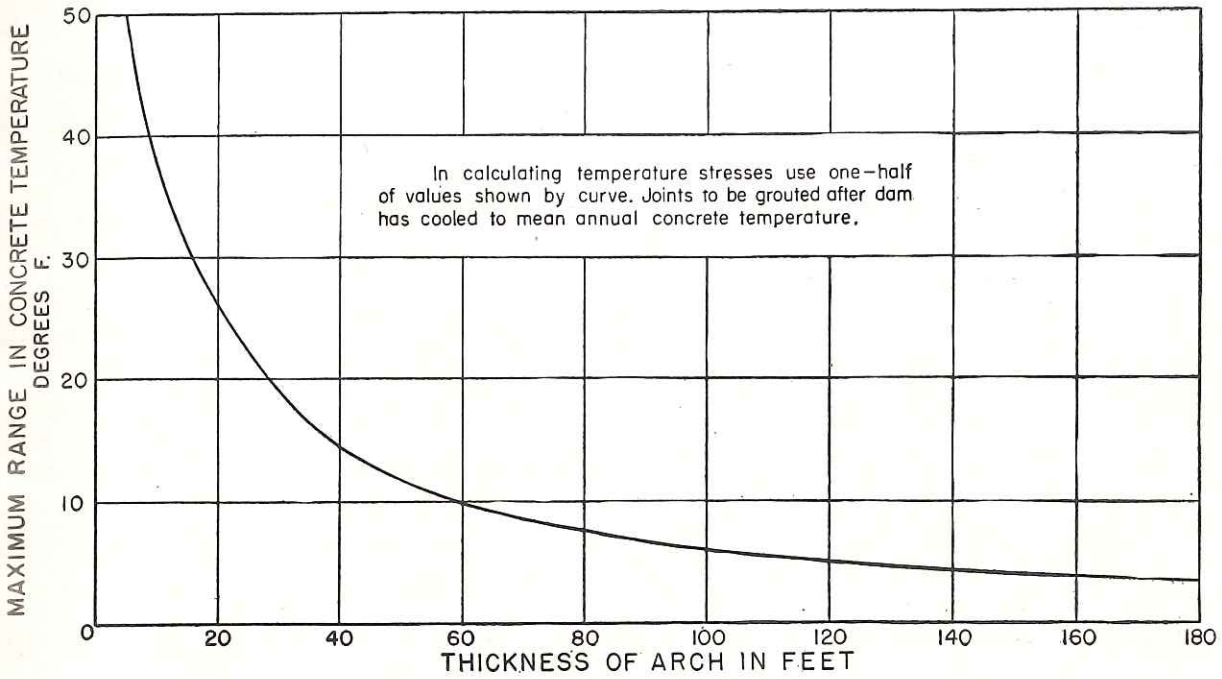


FIGURE 54—MAXIMUM RANGE IN CONCRETE TEMPERATURE

time the concrete has cooled to mean annual temperature, one-half the values shown by the curve should be applied to the arches. Chapter VI describes computations for a unit temperature load of + 1° F.

In determining temperature loads proper allowances must be made for any setting heat which may not have been generated or dissipated at the time of grouting the construction joints. Recently, dissipation of setting heat has been greatly facilitated by artificial cooling processes. This development has made possible the cooling of concrete to temperatures below mean annual, before grouting, thereby improving the distribution of stresses and permitting a more economical design.

---

[5]Houk, Ivan E., "Temperature Variations in Concrete Dams", West. Const. News, Dec. 10, 1930, pp. 601 to 608.

147. Water Pressure.—Reservoir water at the upstream face and tailwater at the downstream face of a dam produce hydrostatic pressures that are calculated by the formula,

$$p = W_W h \qquad (225)$$

in which p is the unit hydrostatic pressure, W_W, the unit weight of water, and h the vertical distance below the water surface.

148. Uplift Pressure.—Uplift pressure usually occurs at the base of a concrete dam, even though elaborate foundation grouting and draining provisions may have been made. However, measurements at existing dams have thus far failed to show the development of serious uplift pressure in the interior of massive concrete dams, free from cracks at the upstream face. If tension stresses, great enough to cause cracking, do not occur at the upstream face, the development of uplift forces simply means a slight transfer of horizontal water load from the cantilever elements of the dam to the arch elements, without important changes in stress magnitude and distribution. If cracking occurs at the upstream face, as may be the case in relatively thin arch dams, the cantilever elements become much more flexible, appreciable portions of water load are transferred to the arch elements, and stress conditions may be materially altered. Consequently, uplift forces should be included in trial load analyses whenever preliminary calculations indicate appreciable cracking at the upstream face.

The intensity of uplift pressure is usually assumed to be distributed in one of two ways; first, as a straight line varying from full reservoir pressure at the upstream face to zero, or tailwater pressure, at the downstream face; or, second, as a combination of two straight lines varying from full reservoir pressure at the upstream face to some proportion of reservoir pressure at the location of the drains, then to zero, or tailwater pressure, at the downstream face. Total uplift forces at the base of the dam, or at horizontal sections chosen for analysis, are obtained by assuming uplift pressures to be effective on certain percentages of the total horizontal areas, the percentage usually being taken higher for the second method of distribution. Comprehensive studies of all available measurements of uplift pressure intensities at existing dams showed that, using the first method of distribution, it seldom is advisable to apply the assumed pressures to less than one-third the area, or seldom necessary to apply them to more than two-thirds the area.[6] However, in the case of comparatively

---

[6]Houk, Ivan E., "Uplift Pressure in Gravity Dams," West. Const. News, July 25, 1930, pp. 344 to 349.

porous foundations, such as sandstone, it may be desirable to apply the assumed uplift pressures to the entire areas.

149. Silt.—Silt accumulations at the dam are usually assumed to act in the same way as vertical water loads. The unit weight of silt is taken as the weight of saturated earth. Increases in horizontal pressure due to silt flows, such as may occur at diversion dams or relatively low structures, may be allowed for by using fluid pressures greater than that of water, the amount of the increase depending on the proportion of silt expected.

150. Ice.—Forces exerted on a dam by ice at the surface of the reservoir are rather indeterminate because the surface layer usually buckles before crushing strength is reached. Pressures from 10 to 25 tons per lineal foot are often assumed as probable ice loads. Since ice layers usually occur at levels somewhat lower than the maximum reservoir surface assumed for design purposes, stresses for ice conditions are often lower than those caused by full reservoir loads.

151. Earthquake Shocks.—A solution for the effects of earthquake shocks on concrete dams has been published and is briefly discussed herein.[7] Two causes account for the added forces on a dam during an earthquake; first, accelerations of the mass of the dam; and, second, changes in water pressure. The dynamic action of the mass of the dam is transformed into a problem in statics by introducing horizontal inertia forces, expressed as mass times acceleration reversed, D'Alembert's principle. The horizontal inertia force of the concrete may be expressed by the formula

$$V_E = W_C \lambda \qquad (226)$$

in which V_E is the horizontal inertia force in the direction of the assumed earthquake shock; W_C, the weight of the concrete or dead load; and λ, the ratio of the maximum horizontal acceleration of the foundation to gravity. λ is usually assumed to be *0.1*. Since maximum horizontal accelerations due to an earthquake are assumed to act in a radial direction at the maximum cantilever section, the acceleration of the mass at any point in the dam is resolved into radial and tangential components.

The dynamic action of bodies of water adjacent to the faces of the dam cause changes in water pressures, which are assumed to have a para-

---

[7]Westergaard, H. M., "Water Pressures on Dams During Earthquakes", Trans. Am. Soc. C. E., Vol. 98, 1933, pp. 418 to 433.

bolic variation from the water surface to the base. This change in water pressure is determined by the formulas,

$$p_E = C\lambda\sqrt{zh} \tag{227}$$

$$C = \frac{51\ lb./cu.\ ft.}{\sqrt{1 - 0.72\left(\frac{z\ sec.}{1000\ N\ ft.}\right)^2}} \tag{228}$$

in which p_E is the change in water pressure due to earthquake shock; C, a coefficient depending on the ratio of z to N; z, the distance from the water surface to the base of the maximum section; h, the vertical distance below the water surface; and N, the period of the horizontal foundation vibrations, usually assumed to be one second. Effects of vertical earthquake accelerations are of minor importance in analyses of arch dams and are generally ignored.

Earthquake forces usually produce most severe effects when acting downstream. However, if conditions are such that decreases in downstream forces cause a severe effect, the direction of foundation acceleration can be reversed.

In trial load adjustments earthquake forces may be handled as additional radial and tangential loads. These loads may be calculated by the formulas,

$$p_R = p_E + W\lambda \cos\phi \tag{229}$$

$$p_T = W\lambda \sin\phi \tag{230}$$

in which p_R is the additional radial load; p_T, the additional tangential load; ϕ, the horizontal angle between the maximum cantilever section and the point considered; and W, the weight of an element extending through the dam, bounded by two horizontal planes, one foot apart, and two vertical radial planes, one foot apart at the upstream face. Water pressures caused by earthquake motion are usually combined with static water pressures and the total applied as radial loads.

APPLICATION OF EXTERNAL LOADS

152. General.—In the analysis, external loads are supported by the systems of arches and cantilevers which have been assumed to replace the dam. Since each system occupies the full volume of the dam, external loads may be applied to either system or divided between the two. In any case ensuing adjustments balance the loads between the two systems. However, the system of load application given in the following sections is found expedient.

153. Horizontal Water Loads.—Horizontal components of combined static and earthquake water loads are divided into two parts, one acting on the arches and the other on the cantilevers. The division is made by trial in the radial adjustment, so that radial deflections of arches and cantilevers are reasonably close at conjugate points. As different trial loads are applied to the two systems it is necessary to have the sum of the arch and cantilever loads at each point equal to the total horizontal radial load at that point. This may be done conveniently by plotting arch loads and total loads on each developed arch center line. Then the difference between the arch load and the total load represents the cantilever load.

154. Vertical Water Loads.—Vertical components of water loads are placed on the cantilevers as initial loads. Deflections due to these vertical loads may be considered as an initial condition of the cantilever structure before the horizontal trial loads are applied. Then by combining deflections due to the trial load with initial deflections, the total trial load deflections are obtained. These are the deflections to be compared with arch deflections.

155. Temperature Loads.—Temperature changes are assigned to the arches. Temperature deflections are combined with deflections for trial arch loads to give the total arch movements used in the adjustments. Other special loads resisted by the dam may be considered in the same way; that is, the loads may be assigned to either the arches or the cantilevers, and the deflections due to the loads combined with the deflections of the trial loads.

156. Concentrated Loads.—In order to compute abutment deformations, it is necessary to introduce concentrated loads at the arch abutments. These represent loads coming down from the cantilever ele-

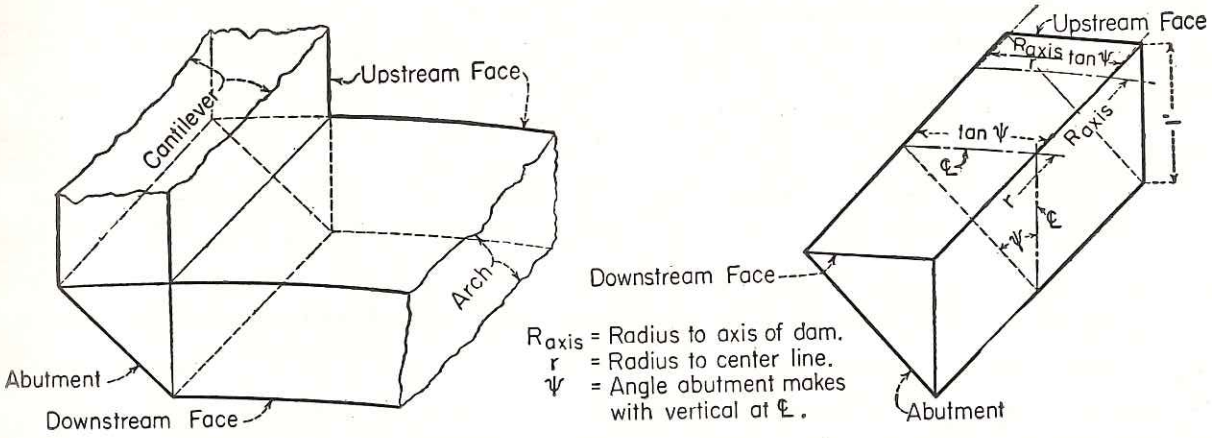


FIGURE 55—CONTACT OF ARCH AND CANTILEVER WITH ABUTMENT

ments resting thereon. From figure 55 it may be seen that at the center line the width of a cantilever, resting on an arch of unit height, is equal to $tan\ \psi$. Since the assumed unit cantilever used in the analysis is one foot wide at the axis, the radial shear at the base of a cantilever is multiplied by $\frac{R_{axis}}{r} tan\ \psi$ to give the correct amount of concentrated radial load to be applied to an arch one foot high. For the same reason, tangential shears and twisting moments at cantilever foundations are corrected by the factor $\frac{R_{axis}}{r} tan\ \psi$ to determine, respectively, concentrated tangential and twist loads for the arches. In the adjustments, abutment movements of the arches are added algebraically to deflections of corresponding cantilevers, since cantilever foundations move with arch abutments.

PREPARATORY STEPS

157. General.—Considerable delay in the analysis may be avoided by completing all preliminary work before starting adjustments. After selecting sample arches and cantilevers, each arch is drawn to a suitable scale. Since arch formulas are based on radial abutments, actual surfaces of contact between arches and abutment rock must be modified somewhat. Radial abutment planes are drawn, approximating actual planes of contact. A radial plane through the point where the centerline of the arch intersects the abutment ordinarily is sufficient. The abutment angle, ϕ_A, is taken to the nearest whole degree, in order to use the tables of arch and load constants without interpolating. At the same time, the angles of abutment planes, ψ, are determined and tabulated on the foundation form, figure 14, chapter III.

158. Foundation.—The foundation form, figure 14, chapter III, is filled out for each arch abutment and each separate cantilever foundadation. Reference should be made to the text of chapter III for cases where b is replaced by $b/2$ in the evaluation of foundation constants. The foundation constants are then transferred to figure 26, chapter IV, for the cantilever analysis, and to figure 40, chapter VI, for the arch analysis.

159. Cantilevers.—The following tabulations are made for uncracked cantilever elements.

1. For each sample cantilever, requisite data are entered in figure 24, chapter IV. This form is completed with the aid of the

curves in figure 16. Data are tabulated for the elevation of each sample arch.

2. Radial-side cantilevers are transformed to parallel-side cantilevers by the graphical method shown in figure 17. From the equivalent parallel-side cantilevers, weights and moments of concrete and vertical water loads are obtained. A mechanical integrator may be used for this purpose. Weights and moments are tabulated on figure 25, using separate sheets for concrete and vertical water loads.

3. Shears and moments are computed for earthquake inertia forces from data on figure 24. These shears and moments are tabulated for each cantilever, in the same manner as weights and moments of concrete and vertical water loads.

4. Loads so far considered are those initially applied to the cantilever structure. From them, initial deflections of the cantilevers are computed using equations 59 to 63, chapter IV. Computations of deflections are conveniently made on the forms shown in figure 26, using the headings at *a* for radial loads and at *c* for tangential loads. Care should be taken to include foundation movements as directed in the text accompanying equations 59 to 63.

5. Again using the forms of figure 26, cantilever deflections are computed for each unit radial, tangential, and twist load. Moments and shears for unit loads are calculated as indicated in figure 18, chapter IV.

6. A table of deflections is prepared, giving the deflection of each cantilever at each arch elevation for each unit load.

160. Arches.—The procedure outlined below constitutes the preparatory steps necessary for analyses of arch elements.

1. Plans of arches are examined to determine whether they are to be classed as uniform thickness, variable thickness, fillet arches, or other types.

2. If of uniform thickness, arches are ordinarily analyzed as symmetrical. In this case the necessary data are tabulated on figure 40, chapter VI, and carried through subsequent computations on the general arch forms as directed for symmetrical arches in chapter VI. Arch constants require computation but once. Load constants require a separate set of computations for each load.

For the lower arches of a dam, all unit loads are not usually required. Often a uniform load, a number 5, and possibly a number 3

load are sufficient. After a little experience, the loads required may be estimated from the number of cantilevers intersecting the arch. Other unit loads are not computed unless difficulties in adjustment show the need for additional loads.

3. Arch deflection curves, similar to figures 48 to 53, chapter VI, are plotted as a check on the computations. The other checks listed in section 127 are also made.

4. A table of moments, thrusts, and shears at the crown and abutments is prepared for the unit loads considered. Unit load deflections are also tabulated at all arch points.

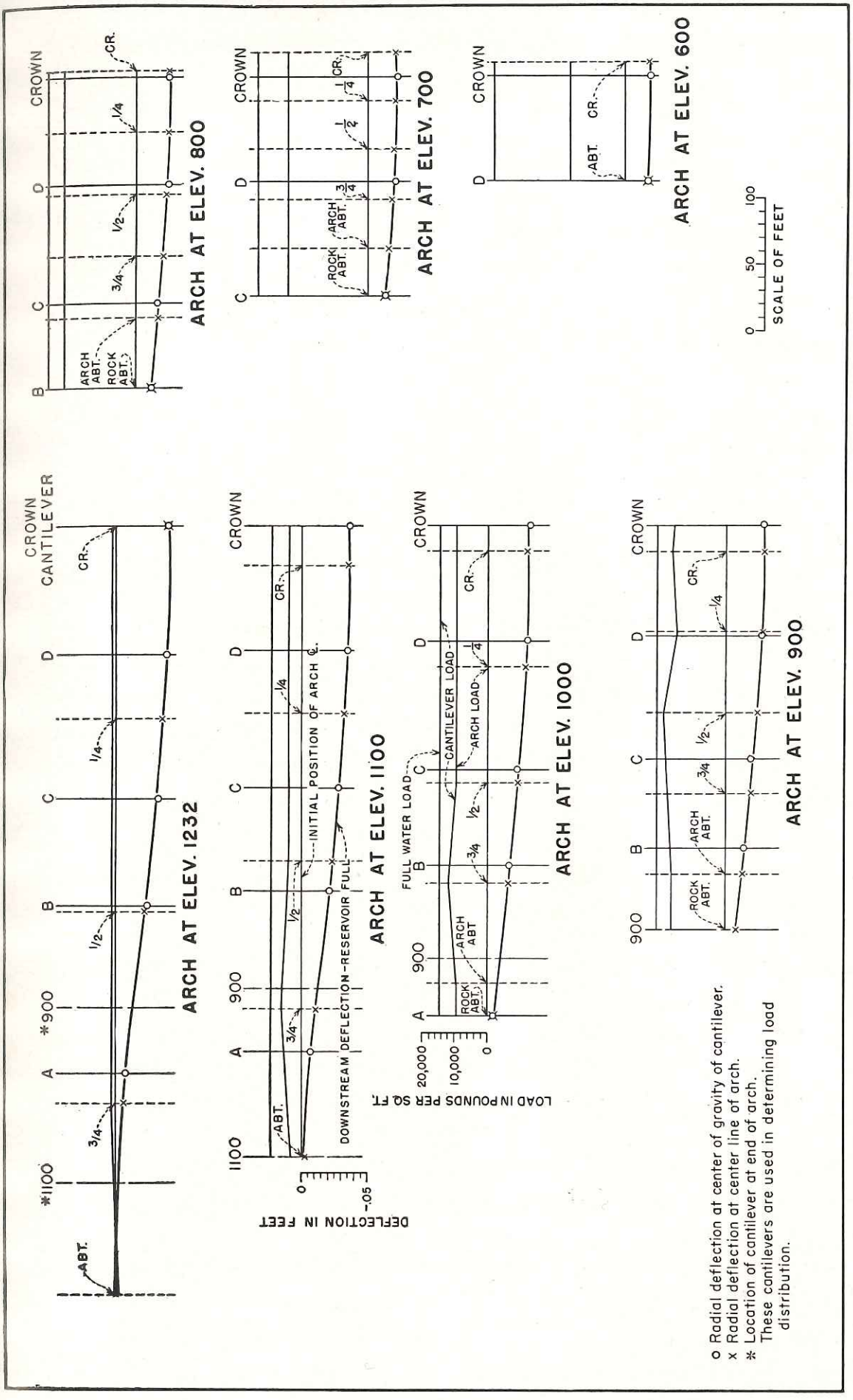
5. Temperature deflections are computed for the temperature changes assumed. These deflections are initial deflections of the arch elements.

6. When the arches are of variable thickness or other non-uniform types, the method to be followed is outlined under the proper heading in chapters V and VI. Except for the computation of arch and load constants, the procedure for variable thickness arches is the same as that given for uniform thickness arches. Before entering general arch forms, voussoir dimensions are tabulated on figure 44, chapter VI. Computations of arch and load constants may then be made on figures 44, 45, and 47, with the aid of table 9 and the accompanying text. Evaluated constants are inserted in the general arch forms and subsequent operations carried on as before.

RADIAL ADJUSTMENT

161. Procedure.—The radial adjustment may now be started. At the beginning, arches and cantilevers have initial deflections due to initial loads. The horizontal radial load must be divided by trial between the arch and cantilever systems, so that agreement of radial deflections is secured. Since cantilevers are not usually located at arch quarter-points, the division may be made graphically. Horizontal radial loads are plotted on the developed arch center lines as shown in figure 56. Cantilevers and arch quarter-points are indicated by full and dotted lines respectively. A trial arch loading is chosen and plotted on the loading diagram. The cantilever loading, which is the difference between total load and arch load, is measured directly on the diagram at each cantilever. Both cantilever and arch loads are made up by summing unit loads.

Below the developed arch center line, initial deflections of the elements are plotted, using different symbols to distinguish between the arch and



o Radial deflection at center of gravity of cantilever.
x Radial deflection at center line of arch.
* Location of cantilever at end of arch.
These cantilevers are used in determining load distribution.

FIGURE 56—RADIAL LOADS AND DEFLECTIONS—BOULDER DAM

cantilever as in figure 56. As the loading is changed, deflections are replotted from the initial positions. For the first estimate of load distribution, crown cantilever deflections are adjusted to crown arch deflections. After the crown cantilever is adjusted, deflections of other arch and cantilever points are computed and plotted. By comparing these deflections, additional radial loads can be estimated and applied to the structure, until resulting arch and cantilever deflections are approximately the same at conjugate points. After a few trials, deflections are usually in close agreement, which completes the first stage of the radial adjustment.

Upon completion of the radial adjustment, stresses are computed. If excessive tension stresses are found in the cantilevers, the adjustment is repeated, assuming cracking in the cantilever elements. Figures 27 and 28, chapter IV, are used for computing deflections of cracked cantilevers. Elevations at which cracking occurs may be determined from values of the ratio F tabulated in figure 27. The method, which is simply an extension of the middle third theorem to radial-side cantilevers, is given in sections 76 and 77, chapter IV.

162. Readjustments.—Subsequent readjustments of radial deflections are necessary because loads introduced in tangential and twist adjustments produce additional radial arch and cantilever movements. Final radial loads and deflections for Boulder Dam, as determined by adjustments and readjustments, are shown in figure 56.

The radial adjustment constitutes the most important step in the trial load analysis. Although tangential and twist adjustments are usually advisable, they are of secondary importance when compared with the radial adjustment. The reason for this is evident from the fact that radial movements are much larger than tangential or angular movements. An analysis based on an adjustment of radial deflections only is often made as a preliminary study. Such an analysis has been found to give a fair indication of maximum stresses. This type of analysis is used to compare preliminary designs for a dam, the complete analysis being used to determine stresses in the adopted design.

TANGENTIAL ADJUSTMENT

163. General.—The radial adjustment is followed by the tangential adjustment, by which movements of arches and cantilevers are adjusted in the tangential direction. At the beginning of the tangential adjustment, tangential movements consist of initial cantilever deflections

due to earthquake; and, in addition, relative deflections of arches and cantilevers caused by the radial adjustment.

164. Movements due to Radial Adjustment.—An analysis of the first stage of the radial adjustment shows that radial arch loads deflect points on the arch center lines in both tangential and radial directions, while radial cantilever loads usually produce negligible tangential movements in the cantilevers. Therefore, conjugate points of the two systems of elements are out of agreement in tangential directions. Initial tangential arch movements are obtained by combining tangential deflections for the unit loads used in the first stage of the radial adjustment. Since each cantilever moves with the arch abutment on which it rests, the radial adjustment causes tangential cantilever movements corresponding to the tangential movements of the arch abutments.

165. Application of Loads.—Equal and opposite tangential loads, one on the arch and the other on the cantilever, are introduced to remove relative tangential deflections. Required amounts of tangential loads are estimated and applied to the two systems by successive trials. As in the radial adjustment, unit loads have been found useful in determining tangential loads and deflections.

One of the requirements of the tangential adjustment is that self-balancing tangential loads must be equal and opposite at all points in the two systems of elements. A method that satisfies this condition is to plot arch loads on developed arch center lines, and then measure cantilever loadings at the intersections of the arches and cantilevers. These measurements give the loads to be applied to the cantilevers.

An estimate of the tangential load distribution to apply to the structure is simplified if only one or two cantilevers near the half-points of the arches are first investigated. After these elements are brought into agreement with the arches by estimated or trial tangential loads, deflections of other arch and cantilever points are computed. Relative tangential deflections of all points can then be compared and additional loads estimated that will result in approximately the same deflections at conjugate points. A close agreement of deflections is generally secured after a few trials. Subsequent readjustments of the tangential deflections are required because twist adjustment and radial readjustment loads produce additional tangential movements.

166. Results of Tangential Adjustment.—Results of the tangential adjustment for Boulder Dam are shown in figure 57. The drawing gives

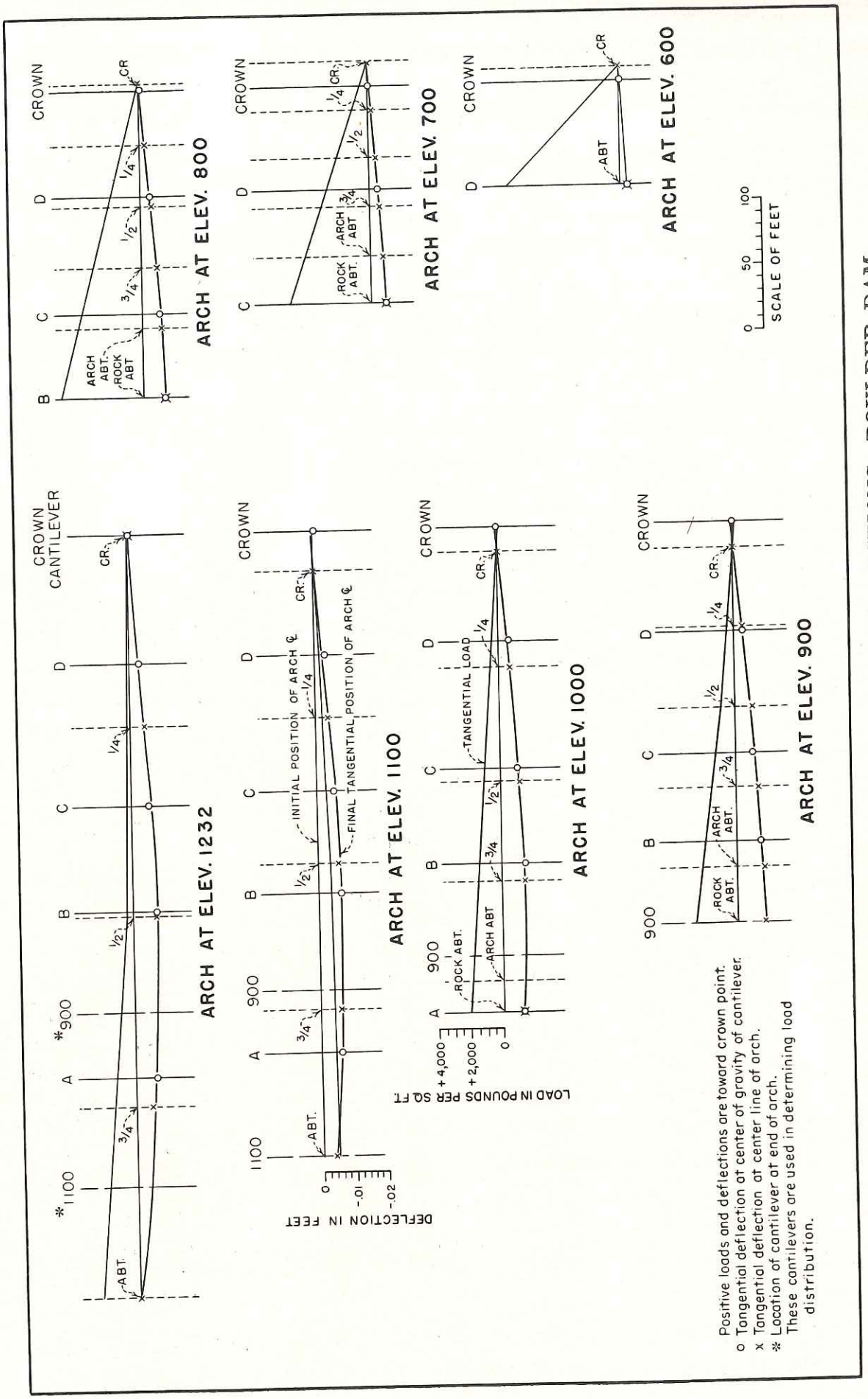


FIGURE 57—TANGENTIAL LOADS AND DEFLECTIONS—BOULDER DAM

initial positions and final tangential displacements of the arch center lines; also amounts of tangential loads required for the adjustment. Positive tangential loads and movements shown in figure 57 act toward the arch crown points. Negative loads and movements act toward the abutment points. Following this convention of signs, it is seen that tangential loads on the arches act toward the crown, while actual tangential displacements due to all loads on the dam are toward the abutments. In most dams tangential loads and deflections are small compared with radial loads and deflections. This indicates that tangential loads have a comparatively small effect on the total movements of a dam. However, tangential loads have appreciable effects on arch stresses, because they change the arch thrusts without causing appreciable movements in the structure.

TWIST ADJUSTMENT

167. General.—Rotations of arch and cantilever elements about vertical axes are considered in the twist adjustment. Loads used in the first stages of the radial and tangential adjustments produce angular movements in both arches and cantilevers. However, angular movements in the cantilevers, due to radial loads, are usually found small enough to be neglected. This being the case, initial angular movements of the cantilevers are determined by summating movements due to applied unit tangential loads, and then combining these movements with abutment movements of arches that abut on the same foundations.

168. Application of Loads.—Radial and tangential adjustments cause relative rotations which are removed by introducing equal and opposite twist loads on the arches and the cantilevers. Required amounts of twist loads are estimated and gradually adjusted by trial.

For the first set of trial twist loads it has been found convenient to consider only one or two cantilevers near the half-points of the arches. After an approximate agreement has been secured at these points, other arch and cantilever points are considered. Each succeeding trial should produce a better agreement between arch and cantilever angular movements, until they are close enough to complete the first stage of twist adjustment. Since radial and tangential readjustments both produce additional angular movements in the arches and cantilevers, subsequent twist readjustments are required to restore agreement between angular movements in the two systems of elements.

Final twist loads and angular movements for Boulder Dam, after completing necessary readjustments, are shown in figure 58. Final rota-

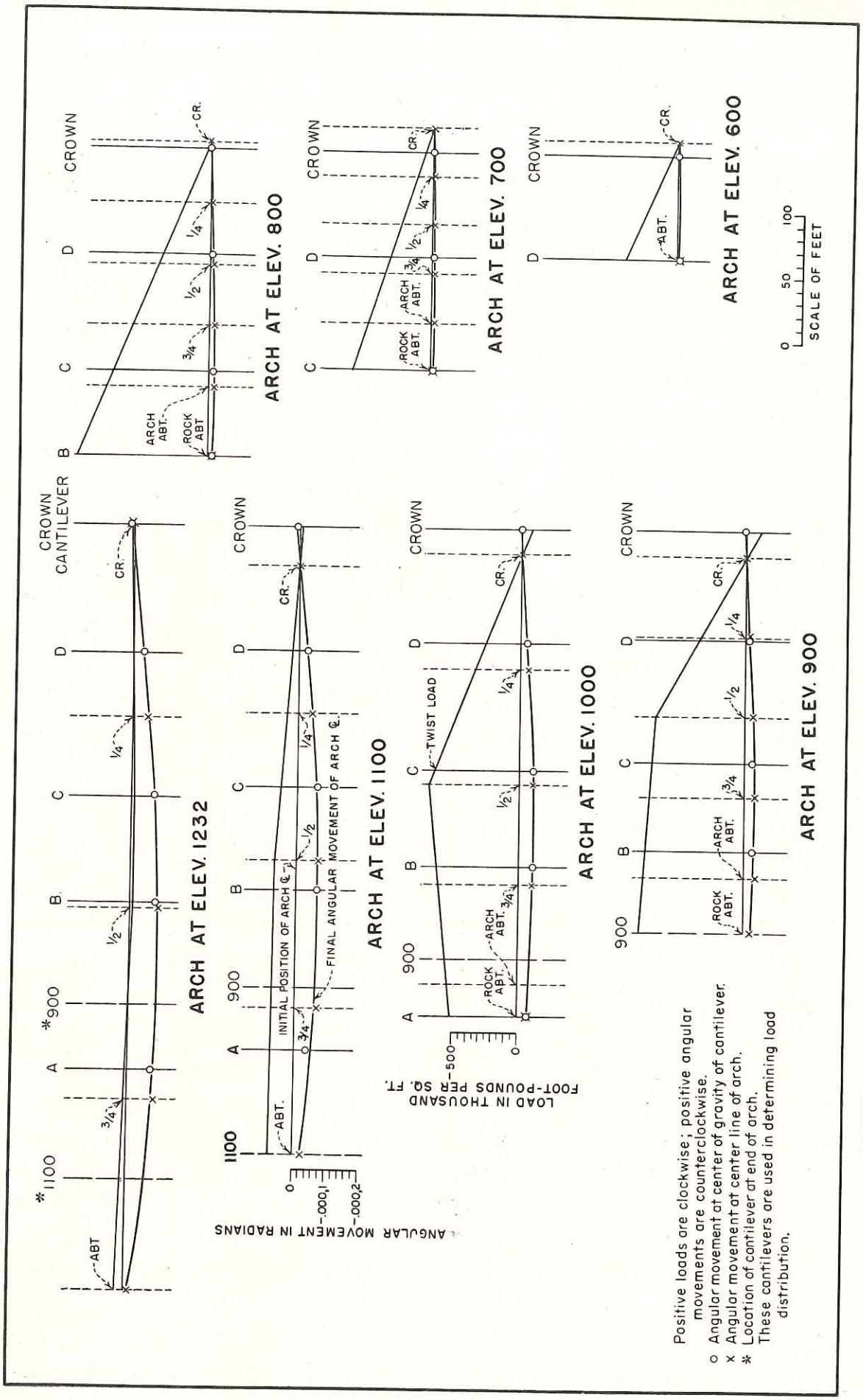

ARCH AT ELEV. 800
ARCH AT ELEV. 700
ARCH AT ELEV. 600
ARCH AT ELEV. 1232
ARCH AT ELEV. 1100
ARCH AT ELEV. 1000
ARCH AT ELEV. 900
CROWN
CROWN CANTILEVER
CR.
ARCH ABT.
ROCK ABT
ABT.
TWIST LOAD
INITIAL POSITION OF ARCH ℄
FINAL ANGULAR MOVEMENT OF ARCH ℄
ANGULAR MOVEMENT IN RADIANS
LOAD IN THOUSAND FOOT-POUNDS PER SQ. FT.
SCALE OF FEET
Positive loads are clockwise; positive angular movements are counterclockwise.
o Angular movement at center of gravity of cantilever.
x Angular movement at center line of arch.
* Location of cantilever at end of arch.
These cantilevers are used in determining load distribution.

FIGURE 58—TWIST LOADS AND ANGULAR MOVEMENTS—BOULDER DAM

tions are clockwise if viewed from above. Twisting moments applied to the arches are in the opposite direction, counterclockwise, except for a portion of the top arch. Twisting moments applied to the cantilevers are equal and opposite to arch twisting moments.

169. Radial Cantilever Deflections Due to Twist Loads.—In addition to rotating the elements of a dam, twist loads produce radial and tangential deflections in the arches and radial deflections in the cantilevers. Twist loads, being horizontal couples, act on the arches as bending moments, thereby producing bending deflections. Magnitudes of bending deflections may easily be computed from twist loads. Computation of radial and tangential deflections of the arches completes the twist adjustment insofar as the horizontal couples are concerned.

From the theory of twisted structures, couples also exist in vertical radial planes. These twist the arches and bend the cantilevers. As stated in chapter II, an additional adjustment is not required since rotations in vertical radial planes are in adjustment when rotations in horizontal planes are in adjustment, providing radial agreement is maintained. However, cantilever deflections due to vertical couples must be computed in order that radial agreement may be restored in the radial readjustment.

Vertical couples are computed from values of horizontal twist loads, utilizing the equality, from slab theory, between twisting moments on mutually perpendicular planes, see reference 2, chapter II. Twist loads, being distributed loads, require that this relationship be modified. For distributed loads, horizontal rates of change of horizontal twisting moments are equal to vertical rates of change of vertical twisting moments. Since cantilever bending moments of the twisted structure are equal to vertical twisting moments, the proposition may be restated as follows. Rates of change of applied twisting moments along arch center lines are equal to rates of change of cantilever bending moments along cantilever center lines. It should be noted that bending moments here referred to are those due to twist alone.

Moments due to twist effects have been found relatively large in most dams. In Boulder Dam they amounted to approximately 25 percent of the moments caused by radial loads. The steps to be followed in calculating the above deflections are given below.

1. Twisting moments are computed for all cantilevers at all arch elevations.

2. Twisting moments are differentiated along arch center lines. This gives rates of change of twisting moments along arch center lines,

which are equal to rates of change of radial bending moments in the cantilevers.

3. Rates of change are integrated from the top of each cantilever to the lower elevations, thus giving bending moments in the cantilevers.

4. A double integration of the M/EI curve from the base of each cantilever gives the desired cantilever deflections.

170. Effects of Adjustment.—Twist resistance in the ordinary type of dam, as indicated by the twist adjustment, stiffens the structure, thereby reducing movements and stresses. Effects of twist are usually greater than effects of tangential shear in dams located at relatively narrow canyon sites. However, they are relatively small when compared with the results of the radial adjustment.

SEQUENCE OF ADJUSTMENTS

171. Readjustments Required.—When the first twist adjustment is completed, movements not considered in previous adjustments are radial deflections due to tangential and twist loads, and tangential deflections due to twist loads. The radial movements are considered in the first radial readjustment, which introduces additional radial loads, causing new tangential and twist movements. Next, the first tangential readjustment is made to correct discrepancies caused by twist loads and the first radial readjustment loads. Likewise, a twist readjustment and subsequent radial, tangential, and twist readjustments may be required because each readjustment unbalances previous adjustments. However, the effects converge rapidly. After a very few radial, tangential, and twist readjustments are made further changes in deflection do not appreciably affect earlier adjustments.

172. Complete Analysis.—A complete analysis, including all adjustments and readjustments, determines the total movements of the dam and the amount and distribution of the different loads. With this information, moments, thrusts, and shears can be determined for the arch and cantilever elements. With forces and moments known, arch, cantilever, and principal stresses can be calculated by formulas given in chapter VIII.

ADJUSTMENTS FOR POISSON'S RATIO

173. General.—The trial load analysis, as described in the preceding sections on adjustments, gives data required to calculate stresses

in arch and cantilever elements. The Poisson's ratio effect of the stresses causes additional movements and stresses in the arches and cantilevers. Since the movements are not the same for the arches and cantilevers, a new set of adjustments is required to consider effects of Poisson's ratio.

174. Cantilever Movements.—Cantilever movements due to Poisson's ratio are caused by arch stresses and radial stresses. Arch stresses can be easily computed, but radial stresses require laborious solutions of long equations. However, radial stresses can be readily calculated at the upstream and downstream faces. If a linear distribution of stress is assumed between the faces, approximate stresses can be calculated. These are reasonably accurate for this step of the analysis.

Strains due to arch and radial stresses are determined and the products of the strains and Poisson's ratio are computed. The products are strains in the cantilevers due to effects of Poisson's ratio. Horizontal radial deflections due to strains can then be determined. These are the initial cantilever movements used in adjustments for Poisson's ratio.

175. Arch Movements.—Movements in the arches due to Poisson's ratio are caused by cantilever stresses and radial stresses. Here again, it is expedient to use approximate radial stresses. Strains due to such stresses are determined and products of strains and Poisson's ratio calculated. The values cannot be used directly as arch strains because the arches are statically indeterminate elements. However, the arch can be cut at the crown and movements due to strains computed for each half of the arch. Then arch moments, thrusts, shears, and deflections can be calculated as described in chapter V. Resulting deflections are initial arch movements to be used in adjustments for Poisson's ratio.

176. Adjustments.—Adjustments are similar to those described in preceding sections, except that initial arch and cantilever movements due to Poisson's ratio are plotted on adjustment sheets as initial movements. Equal and opposite loads are applied by successive trials until agreement has been restored between the arch and cantilever elements.

If the cantilevers are assumed to crack, a different procedure is followed, since actual trial loads, instead of unit loads, must be applied to the cracked cantilever if the amount of cracking is to be determined. For this case, initial movements due to Poisson's ratio are combined with final movements as determined by the first set of adjustments. The loads needed to restore agreement between the arches and cantilevers are introduced in additional readjustments until the desired agreement is secured.

177. Effects of Grouting Joints.—Contraction joints in a dam are usually grouted after the structure has been built to its final height. In this case the weight of the structure has no Poisson's ratio effect on arch stresses, because the movements simply tend to close the open contraction joints. However, if sections of the dam are grouted before the final height is reached, weights of concrete placed after grouting must be considered in the Poisson's ratio adjustments. This must be kept in mind at the time stresses are calculated for determining initial Poisson's ratio movements.

178. Effects on Stresses and Movements.—Poisson's ratio adjustments have shown relatively small effects in dams investigated by trial load methods. In general, compressive stresses are increased, or tensile stresses decreased, at the upstream face; and compressive stresses decreased at the downstream face. Movements in the dam are not appreciably changed by Poisson's ratio adjustments.

CHAPTER VIII—EVALUATION OF STRESSES

INTRODUCTION

179. General Statement.—The purpose of this chapter is to derive equations which can be used in evaluating stresses at any point in an arch dam. Equations are derived in terms of forces and moments acting on cantilever and arch elements. Obviously, forces and moments are known after deflection adjustments are completed.

Ordinarily, normal and principal stresses at the faces and shearing stresses in the interior of the dam are the only values determined in a stress analysis. Although maximum tensile and compressive stresses occur at the faces, assuming a linear variation between the upstream and downstream slopes, often the stress condition should be investigated at some interior point in the structure, particularly if high stresses might cause failure at some plane or zone of weakness. Considering these possibilities, it is evident that the value of any stress at any location might be desired. Hence, equations are developed for principal tensile and compressive stresses at the surface or in the interior of the dam. Three principal stresses occur at a point. These determine the ellipsoid of stress, which gives a complete picture of stress conditions at the point.

In developing stress equations, ordinary stress formulas are taken from mechanics of materials text books whenever applicable. These are used in the derivations, with necessary changes in notation and form. Special formulas for stress conditions, not covered in engineering text books, are developed in this chapter. No computation forms are given in this chapter since methods of computation are evident from the stress equations, and forms may easily be prepared as needed.

180. Notation and Definitions.—The notation and definitions used in derivations of stress equations are listed below.

$'$ = symbols referring to coordinate axes X', Y', and Z'.

$''$ = symbols referring to Y'' axis in figures 61 and 62.

x, y, z = coordinates; origin at downstream face; x positive toward abutment; y positive toward upstream face; z positive downward.

U, D = subscripts referring to upstream and downstream faces of horizontal and vertical sections, respectively.

R_U, R_{axis}, r = radius to upstream face, axis of dam, and center line of arch, respectively. ft.

T = thickness of dam in radial direction. ft.

lg = distance from upstream face to center of gravity of horizontal cantilever section. ft.

A_c, A_a = area of horizontal cantilever section and vertical arch section, respectively. ft.^{2}

I_c, I_a = moment of inertia of horizontal cantilever section and vertical arch section, respectively. ft.^{4}

ϕ_U = angle upstream edge of cantilever makes with vertical.

ϕ_D = angle downstream edge of cantilever makes with vertical.

β_U = angle a line tangent to the extrados makes with normal to radial arch section.

β_D = angle a line tangent to the intrados makes with normal to radial arch section.

p = normal pressure at face of dam. lb./ft.^{2}

W = total weight of cantilever one foot wide at the axis, including vertical water load. lb.

M = total moment of cantilever, one foot wide at the axis, about center of gravity of horizontal section. Positive moment causes compression at the upstream face. ft.-lb.

V_{ca} = total horizontal radial shear on a horizontal cantilever section one foot wide at the axis. Positive shear acts upstream. lb.

V_c = total horizontal radial shear on a horizontal cantilever section one foot wide at the center line. lb.

V_a = total horizontal radial shear on vertical arch section one foot high. Positive shear acts upstream. lb.

V_{Ta} = total horizontal tangential shear force on horizontal cantilever section one foot wide at the axis. Positive shear acts toward abutment. lb.

M_{Tw} = total twisting moment on horizontal cantilever section one foot wide at the axis. Positive moment acts counterclockwise on left side of dam. ft.-lb.

M_a = total horizontal moment for an arch one foot high. Positive moment causes compression at the upstream face. ft.-lb.

H_a = total horizontal thrust for an arch one foot high, compression positive. lb.

σ_X = horizontal arch stress normal to vertical radial plane, compression positive. lb./ft.^{2}

σ_Y = horizontal radial stress normal to vertical tangential plane, compression positive. lb./ft.^{2}

σ_Z = vertical cantilever stress normal to horizontal plane, compression positive. lb./ft.^{2}

τ_{XY} = horizontal arch shear stress acting in radial direction on a vertical radial plane. Direction of positive shear shown on drawings. lb./ft.^{2}

τ_{ZX} = horizontal cantilever shear stress acting in tangential direction on a horizontal plane. lb./ft.^{2}

τ_{ZY} = horizontal cantilever shear stress acting in radial direction on a horizontal plane. lb./ft.^{2}

σ_P = principal stress at face of dam. lb./ft.^{2}

α = angle principal stress makes with Z' axis at face of dam.

θ = angle between a circumferential inclined plane and the corresponding vertical plane.

τ_1 = horizontal shear stress on inclined plane. lb./ft.^{2}

τ_2 = shear stress perpendicular to τ_1 on inclined plane. lb./ft.^{2}

$\tau_{MAX.}$ = maximum shear stress on inclined plane. lb./ft.^{2}

δ = angle maximum shear stress makes with τ_1.

σ_N = normal stress on inclined plane. lb./ft.^{2}

j, m, n = direction cosines of any inclined plane referred to X, Y, and Z axes, respectively.

j', m', n' = direction cosines of maximum shear stress, $\tau_{MAX.}$, in an inclined plane referred to X, Y, and Z axes, respectively.

S = total stress on an inclined plane. lb./ft.^{2}

S_1, S_2, S_3 = components of total stress on inclined plane in X, Y, and Z directions, respectively. lb./ft.^{2}

STRESSES AT FACES OF DAM

181. Procedure.—Since maximum stresses in a dam usually occur at the faces of the structure, an analysis ordinarily requires only the evaluation of stresses at the upstream and downstream faces. In considering stresses at the faces, the problem is simplified because shear stress is zero in the plane of the face, and one principal stress is normal to the face.

In the following derivations it is assumed that cantilever forces and moments, W, M, V_{ca}, V_{Ta}, M_{Tw}, and arch forces and moments, H_a, M_a, V_a, are known. Equations are derived for the left part of the dam, looking upstream. These can also be used for the right part of the dam if signs of forces and moments follow the convention used in the trial load analysis.

Stresses are first determined for three mutually perpendicular planes: a vertical radial plane, YZ; a vertical tangential plane, XZ; and a horizontal plane, XY. There are six stress components on these three planes. Three stresses, horizontal arch stress σ_X, vertical cantilever stress σ_Z, and tangential cantilever shear stress τ_{ZX}, are assumed to have a linear variation from upstream face to downstream face. Radial cantilever shear stresses τ_{ZY}, radial arch shear stresses τ_{XY}, and radial stresses normal to vertical tangential planes σ_Y, are derived by equating forces acting along X, Y, and Z axes, see figures 59 and 60. Equations are derived first for the downstream face, and later for the upstream face.

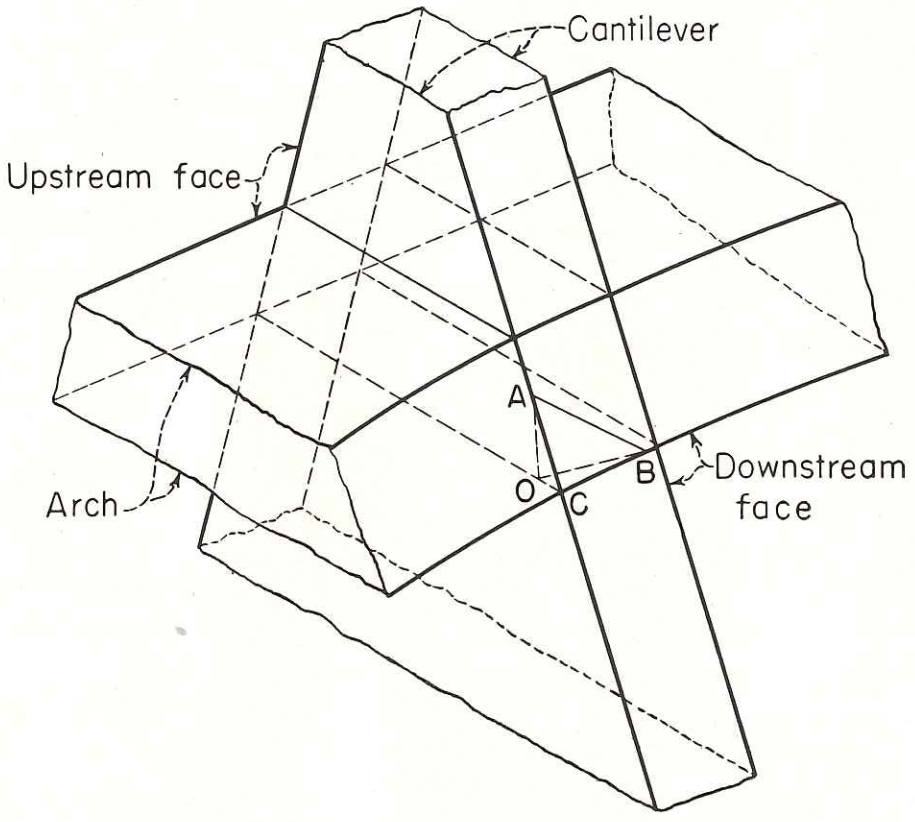


FIGURE 59—INTERSECTION OF ARCH AND CANTILEVER ELEMENTS

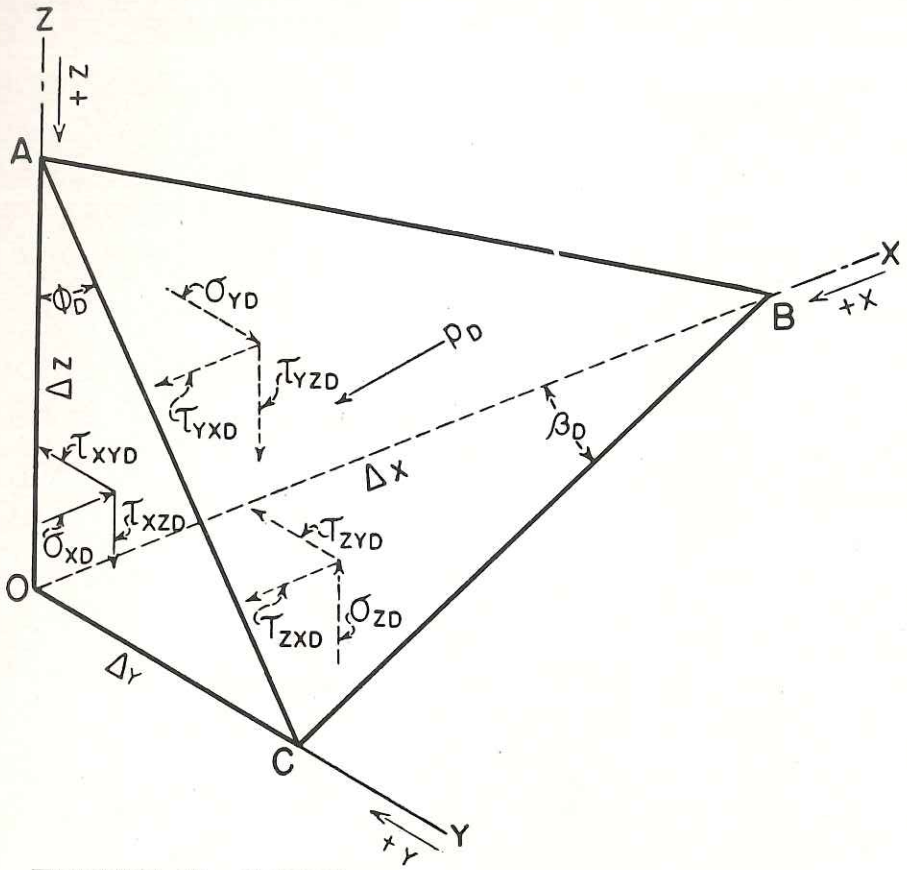


FIGURE 60—STRESSES AT TETRAHEDRON ABCO

182. Stresses at Downstream Face.—Equations for stresses at the downstream face are given below.

Vertical cantilever stress on a horizontal plane,

$$\sigma_{ZD} = \frac{W}{A_c} - \frac{M}{I_c}(T - lg) \tag{231}$$

Horizontal arch stress on a vertical radial plane,

$$\sigma_{XD} = \frac{H_a}{A_a} - \frac{M_a}{I_a} \cdot \frac{T}{2} \tag{232}$$

Horizontal cantilever shear stress acting in a tangential direction on a horizontal plane,

$$\tau_{ZXD} = \tau_{XZD} = \frac{-V_{Ta}}{A_c} - \frac{-M_{Tw}}{I_c}(T - lg) \tag{233}$$

Consider an element at the downstream face, see figure 60, with six stresses acting on three mutually perpendicular planes, and normal pressure p_D acting on the face ABC.

Equating forces in the X direction,

$$p_D \frac{\Delta Z \Delta Y}{2} = \sigma_{XD} \frac{\Delta Z \Delta Y}{2} - \tau_{YXD} \frac{\Delta Z \Delta X}{2} - \tau_{ZXD} \frac{\Delta Y \Delta X}{2}$$

Equating forces in the Y direction,

$$p_D \frac{\Delta Z \Delta X}{2} = \sigma_{YD} \frac{\Delta Z \Delta X}{2} - \tau_{XYD} \frac{\Delta Z \Delta Y}{2} - \tau_{ZYD} \frac{\Delta Y \Delta X}{2}$$

Equating forces in the Z direction,

$$p_D \frac{\Delta x \Delta y}{2} = \sigma_{zD} \frac{\Delta x \Delta y}{2} - \tau_{xzD} \frac{\Delta z \Delta y}{2} - \tau_{yzD} \frac{\Delta z \Delta x}{2}$$

Substituting $\Delta z = \Delta y \cot \phi_D$, $\Delta x = \Delta y \cot \beta_D$ and dividing by $\frac{(\Delta y)^2}{2}$

$$p_D \cot \phi_D = \sigma_{xD} \cot \phi_D - \tau_{yxD} \cot \phi_D \cot \beta_D - \tau_{zxD} \cot \beta_D$$

$$p_D \cot \phi_D \cot \beta_D = \sigma_{yD} \cot \phi_D \cot \beta_D - \tau_{xyD} \cot \phi_D - \tau_{zyD} \cot \beta_D$$

$$p_D \cot \beta_D = \sigma_{zD} \cot \beta_D - \tau_{xzD} \cot \phi_D - \tau_{yzD} \cot \phi_D \cot \beta_D$$

Since $\tau_{xyD} = \tau_{yxD}$, $\tau_{xzD} = \tau_{zxD}$, and $\tau_{yzD} = \tau_{zyD}$, the unknowns can be solved, giving

$$\tau_{xyD} = \tau_{yxD} = (\sigma_{xD} - p_D) \tan \beta_D - \tau_{xzD} \tan \phi_D \qquad (234)$$

$$\tau_{yzD} = \tau_{zyD} = (\sigma_{zD} - p_D) \tan \phi_D - \tau_{xzD} \tan \beta_D \qquad (235)$$

$$\sigma_{yD} = p_D + \tau_{xyD} \tan \beta_D + \tau_{yzD} \tan \phi_D \qquad (236)$$

The next step is to find the stresses referred to three new coordinate axes, X', Y', Z', parallel to BC, OE, and AE, respectively, see figure 61. In considering the tetrahedron $ABCO$ in figure 61, the plane ABC is the downstream face of the dam, the YZ plane is rotated about the Z axis until the line AD is perpendicular to BC, and the angle the line AD makes with the vertical Z axis is designated by ϕ'_D.

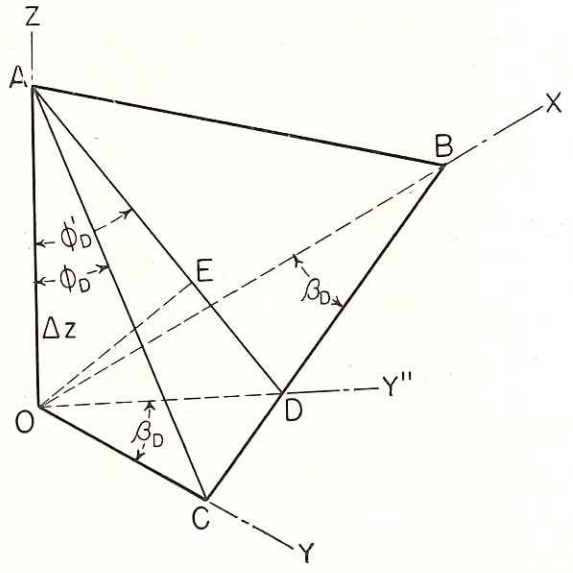


FIGURE 61—TETRAHEDRON ABCO, SHOWING AXES AND ANGLES

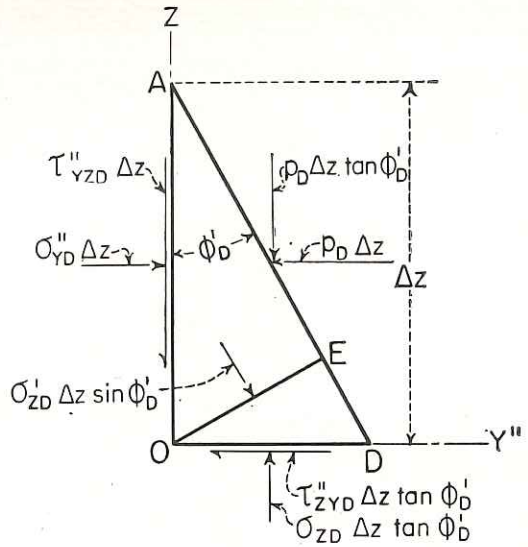


FIGURE 62—STRESSES AT SECTION ADO

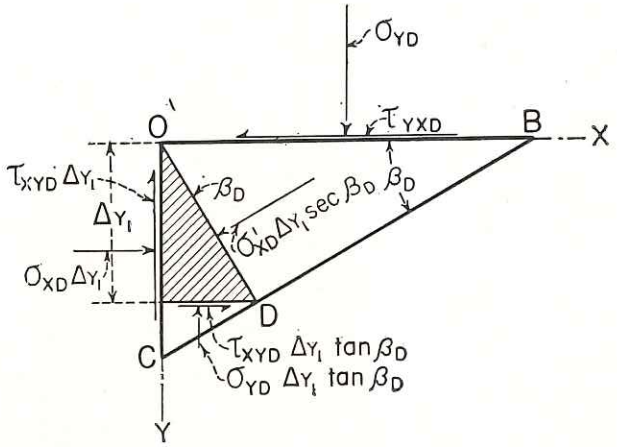


FIGURE 63—STRESSES IN PLANE BCO

$$OC = \Delta z \tan\phi_D = OD \sec\beta_D$$

$$OD = \Delta z \frac{\tan\phi_D}{\sec\beta_D} = \Delta z \tan\phi'_D$$

$$\phi'_D = \tan^{-1}(\tan\phi_D \cos\beta_D) \qquad (237)$$

Considering section ADO in figure 62,

$$\tau''_{YZD} = (\sigma_{ZD} - p_D)\tan\phi'_D = \tau''_{ZYD}$$

$$\sigma'_{ZD}\,\Delta z \sin\phi'_D = \sigma_{ZD}\,\Delta z \tan\phi'_D \cos\phi'_D + \tau''_{ZYD}\,\Delta z \tan\phi'_D \sin\phi'_D$$

$$\sigma'_{ZD} = \sigma_{ZD} + (\sigma_{ZD} - p_D)\tan^2\phi'_D$$

$$\sigma'_{ZD} = \sigma_{ZD}\sec^2\phi'_D - p_D \tan^2\phi'_D \qquad (238)$$

Considering plane BCO in figure 63,

$$\sigma'_{XD}\,\Delta Y_1 \sec\beta_D = \sigma_{XD}\,\Delta Y_1 \cos\beta_D + \tau_{XYD}\,\Delta Y_1 \tan\beta_D \cos\beta_D + \tau_{XYD}\,\Delta Y_1 \sin\beta_D + \sigma_{YD}\,\Delta Y_1 \tan\beta_D \sin\beta_D$$

$$\sigma'_{XD} = \sigma_{XD}\cos^2\beta_D + \sigma_{YD}\sin^2\beta_D + 2\,\tau_{XYD}\sin\beta_D\cos\beta_D \qquad (239)$$

Considering figure 64, the shear stress parallel to CB, on plane BCO, is

$$\tau_{CBD} = \tau_{ZXD}\cos\beta_D - \tau_{ZYD}\sin\beta_D$$

Considering figure 65, the shear stress parallel to AD is the resultant shear stress on ADO because the shear stress perpendicular to it is zero.

$$\tau'_{XZD} = \tau_{CBD}\sec\phi'_D$$

$$\tau'_{XZD} = \tau'_{ZXD} = (\tau_{XZD}\cos\beta_D - \tau_{ZYD}\sin\beta_D)\sec\phi'_D \qquad (240)$$

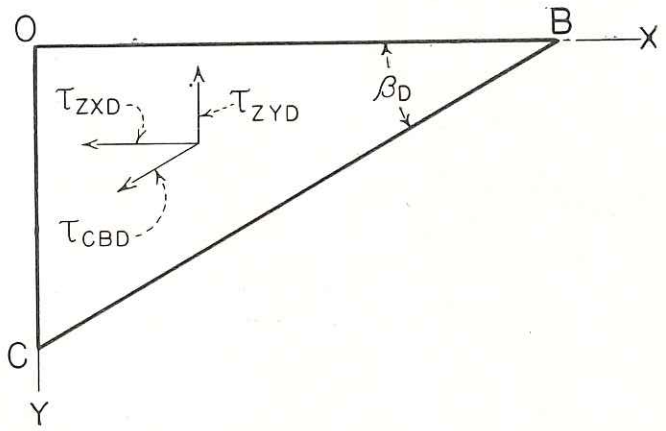


FIGURE 64—SHEAR STRESS ON PLANE BCO

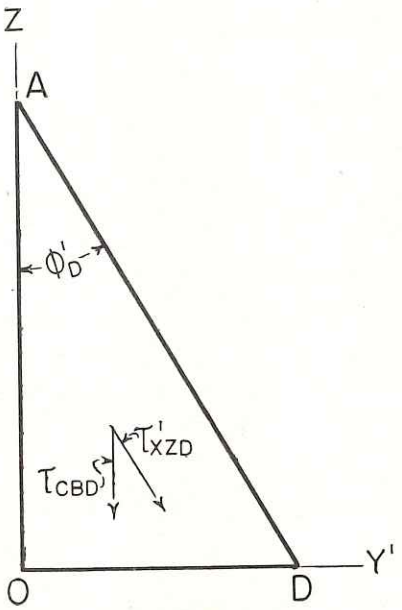


FIGURE 65—SHEAR STRESS ON SECTION ADO

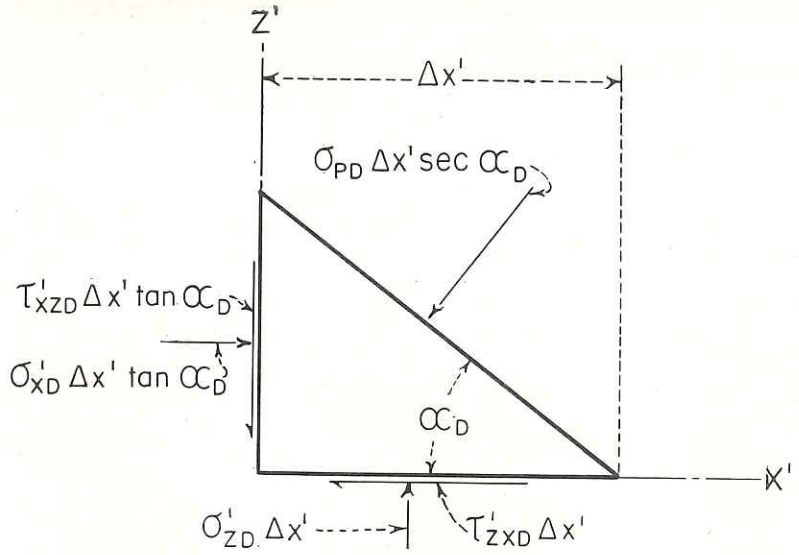


FIGURE 66—STRESSES IN X'Z' PLANE

Considering figure 66, two of the principal stresses are given by the formula,

$$\sigma_{PD} = \frac{\sigma'_{ZD} + \sigma'_{XD}}{2} \pm \sqrt{\left(\frac{\sigma'_{ZD} - \sigma'_{XD}}{2}\right)^2 + (\tau'_{XZD})^2} \qquad (241)$$

Where $(\sigma'_{ZD} - \sigma'_{XD}) > O$, use + for radical sign.

Where $(\sigma'_{ZD} - \sigma'_{XD}) < O$, use — for radical sign.

$$\tan 2\alpha_D = - \frac{2\tau'_{XZD}}{\sigma'_{ZD} - \sigma'_{XD}} \qquad (242)$$

If $\tan 2\alpha_D$ is +, $0 < \alpha_D < 45°$

If $\tan 2\alpha_D$ is −, $-45° < \alpha_D < 0$

Positive angles are measured clockwise from the Z' axis on the left part of the dam and counterclockwise on the right part of the dam, looking upstream.

The two values of σ_{PD} given by equation 241 and the normal pressure p_D are the three principal stresses at the face of the dam. The maximum shear stress acts on a plane that bisects the angle between the largest and the smallest of the principal stresses. It is equal to half the difference between the principal stresses.

A summary of stresses calculated at the downstream face of the dam, with symbols and equation numbers, is tabulated below.

	Symbol	Equation
Vertical cantilever stress	σ_{ZD}	231
Horizontal arch stress	σ_{XD}	232
Tangential shear on horizontal plane	τ_{ZXD}	233
Radial shear on vertical plane	τ_{XYD}	234
Radial shear on horizontal plane	τ_{ZYD}	235

	Symbol	Equation
Horizontal radial stress	σ_{YD}	236
Cantilever stress parallel to Z' axis	σ'_{ZD}	238
Arch stress parallel to intrados	σ'_{XD}	239
Tangential shear parallel to intrados	τ'_{ZXD}	240
Principal stress	σ_{PD}	241

If the extrados and intrados curves are concentric circular arches, the angle ϕ'_D becomes ϕ_D and equation 238 gives the cantilever stress parallel to the downstream face.

183. Stresses at Upstream Face.—Stresses at the upstream face are derived in the same way as for the downstream face. The resulting equations are

$$\sigma_{ZU} = \frac{W}{A_c} + \frac{M}{I_c} \cdot lg \tag{243}$$

$$\sigma_{XU} = \frac{H_a}{A_a} + \frac{M_a}{I_a} \cdot \frac{T}{2} \tag{244}$$

$$\tau_{XZU} = \tau_{ZXU} = \frac{-V_{Ta}}{A_c} + \frac{-M_{Tw}}{I_c} \cdot lg \tag{245}$$

$$\tau_{XYU} = \tau_{YXU} = -(\sigma_{XU} - p_U)\tan\beta_U + \tau_{XZU}\tan\phi_U \tag{246}$$

$$\tau_{YZU} = \tau_{ZYU} = -(\sigma_{ZU} - p_U)\tan\phi_U + \tau_{XZU}\tan\beta_U \tag{247}$$

$$\sigma_{YU} = p_U - \tau_{XYU}\tan\beta_U - \tau_{YZU}\tan\phi_U \tag{248}$$

$$\phi'_U = \tan^{-1}(\tan\phi_U \cos\beta_U) \tag{249}$$

$$\sigma'_{ZU} = \sigma_{ZU}\sec^2\phi'_U - p_U\tan^2\phi'_U \tag{250}$$

$$\sigma'_{XU} = \sigma_{XU}\cos^2\beta_U + \sigma_{YU}\sin^2\beta_U - 2\tau_{XYU}\sin\beta_U\cos\beta_U \tag{251}$$

$$\tau'_{XZU} = \tau'_{ZXU} = (\tau_{XZU}\cos\beta_U + \tau_{YZU}\sin\beta_U)\sec\phi'_U \tag{252}$$

$$\sigma_{PU} = \frac{\sigma'_{ZU} + \sigma'_{XU}}{2} \pm \sqrt{\left(\frac{\sigma'_{ZU} - \sigma'_{XU}}{2}\right)^2 + (\tau'_{XZU})^2} \tag{253}$$

$$\tan 2\alpha_U = -\frac{2\tau'_{XZU}}{\sigma'_{ZU} - \sigma'_{XU}} \tag{254}$$

STRESSES ON HORIZONTAL AND VERTICAL PLANES

184. Procedure.—After stresses have been evaluated for the faces of a dam, equations may be derived for stresses on horizontal planes and vertical radial and tangential planes at any interior point in the dam. Since horizontal arch stresses σ_X, vertical cantilever stresses σ_Z, and tangential cantilever shear stresses τ_{ZX}, are assumed to have a linear variation from the upstream face to the downstream face, equations can be derived for radial cantilever shear stresses τ_{ZY}, radial arch shear stresses τ_{XY}, and radial stresses normal to the vertical tangential planes σ_Y. Since τ_{XY} equals τ_{YX}, τ_{XZ} equals τ_{ZX}, and τ_{YZ} equals τ_{ZY}; it is evident that the six stresses, σ_X, σ_Y, σ_Z, τ_{XY}, τ_{XZ}, and τ_{YZ}, determine all stresses acting on three mutually perpendicular planes. These planes are a vertical radial plane YZ, a vertical tangential plane XZ, and a horizontal plane XY.

185. Normal Cantilever and Arch Stresses.—Vertical cantilever stresses on a horizontal plane, see figure 67, may be calculated by the equation,

$$\sigma_z = \frac{W}{A_c} + \frac{M}{I_c}\left[Y-(T-lg)\right] = \sigma_{zD} + \frac{M}{I_c}\cdot Y \qquad (255)$$

Horizontal arch stresses on a vertical radial plane, see figure 68, may be calculated by the equation,

$$\sigma_x = \frac{H_a}{A_a} + \frac{M_a}{I_a}\left(Y-\frac{T}{2}\right) = \sigma_{xD} + \frac{M_a}{I_a}\cdot Y \qquad (256)$$

186. Cantilever Shear Stresses.—Horizontal cantilever shear stresses, acting in tangential directions on horizontal planes, may be computed by the equation,

$$\tau_{zx} = \tau_{xz} = \frac{-V_{Ta}}{A_c} + \frac{-M_{Tw}}{I_c}\left[Y-(T-lg)\right] = \tau_{xzD} + \frac{-M_{Tw}}{I_c}\cdot Y \qquad (257)$$

Formulas for horizontal cantilever shear stresses, acting in radial directions on horizontal planes, see figure 67, may be derived in the following manner: assume a parabolic distribution of shear and let

$$\tau_{zY} = \tau_{YZ} = a_1 + b_1 Y + c_1 Y^2$$

When $Y = 0$, $\tau_{YZD} = a_1$ (258)

When $Y = T$, $\tau_{YZU} = \tau_{YZD} + b_1 T + c_1 T^2$ (259)

The total shear on a parallel-side horizontal plane, one foot wide, between the origin and any point Y is,

$$\int_0^Y \tau_{YZ}\, dY = a_1 Y + b_1 \frac{Y^2}{2} + c_1 \frac{Y^3}{3}$$

In considering horizontal planes of cantilevers with radial sides, it is assumed that the total shear on a plane, one foot wide at the arch

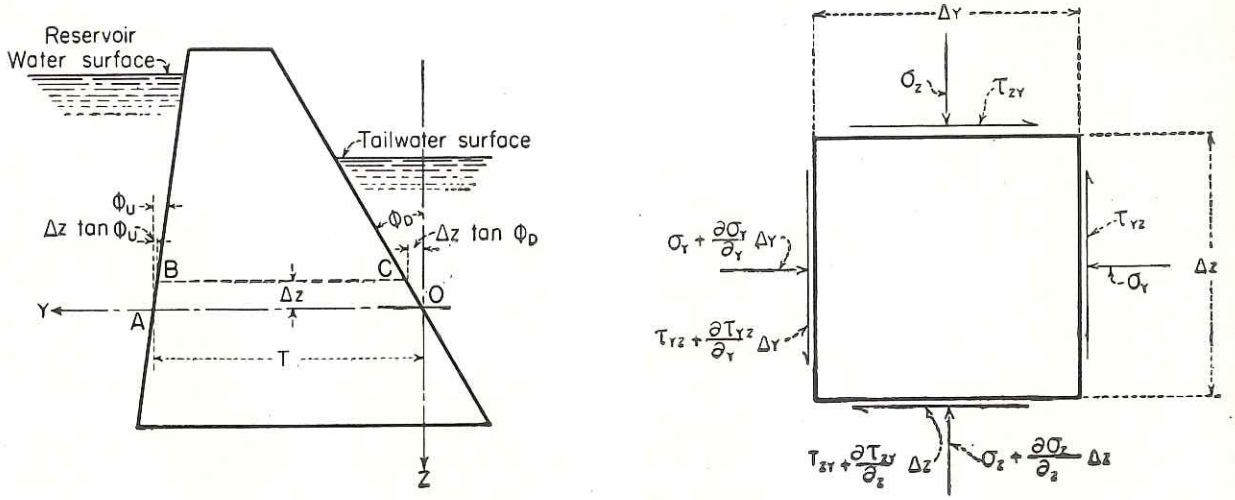


FIGURE 67—VERTICAL CANTILEVER ELEMENT

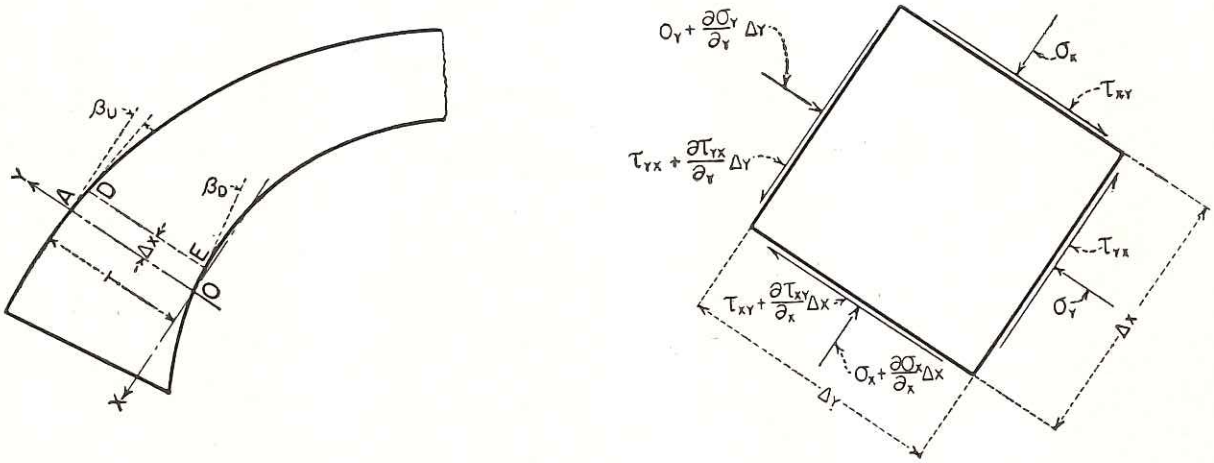


FIGURE 68—HORIZONTAL ARCH ELEMENT

center line, is equal to the total shear on a horizontal plane with parallel sides one foot apart.

$$V_c = + \frac{R_{axis}}{r} V_{ca} \tag{260}$$

$$\int_0^T \tau_{YZ}\, dY = -V_c = \tau_{YZD}\, T + b_1 \frac{T^2}{2} + c_1 \frac{T^3}{3}$$

$$b_1 = -\frac{2V_c}{T^2} - \frac{2\tau_{YZD}}{T} - \frac{2c_1 T}{3} \tag{261}$$

Substituting the above value of b_1 in equation 259,

$$c_1 = \frac{3\tau_{YZU}}{T^2} + \frac{3\tau_{YZD}}{T^2} + \frac{6V_c}{T^3} \tag{262}$$

Substituting the above value of c_1 in equation 261,

$$b_1 = \frac{-2V_c}{T^2} - \frac{2\tau_{YZD}}{T} - \frac{2\tau_{YZU}}{T} - \frac{2\tau_{YZD}}{T} - \frac{4V_c}{T^2}$$

$$b_1 = \frac{-6V_c}{T^2} - \frac{2\tau_{YZU}}{T} - \frac{4\tau_{YZD}}{T} \qquad (263)$$

Therefore

$$\tau_{YZ} = \tau_{YZD} - Y\left(\frac{6V_c}{T^2} + \frac{2\tau_{YZU}}{T} + \frac{4\tau_{YZD}}{T}\right) + Y^2\left(\frac{3\tau_{YZU}}{T^2} + \frac{3\tau_{YZD}}{T^2} + \frac{6V_c}{T^3}\right)$$

$$\tau_{ZY} = \tau_{YZ} = \tau_{YZD} - Y\left[\frac{1}{T}\left(\frac{6V_c}{T} + 2\tau_{YZU} + 4\tau_{YZD}\right)\right] + Y^2\left[\frac{1}{T^2}\left(3\tau_{YZU} + 3\tau_{YZD} + \frac{6V_c}{T}\right)\right] \qquad (264)$$

Values of τ_{YZU} and τ_{YZD} are given by equations 247 and 235, respectively.

187. Radial Shear in Arch Elements.—An equation for horizontal arch shear stress acting in a radial direction on a vertical radial plane, see figure 68, may be derived by the procedure followed in deriving equation 264. The formula obtained is

$$\tau_{XY} = \tau_{YX} = \tau_{XYD} - Y\left[\frac{1}{T}\left(\frac{6V_a}{T} + 2\tau_{XYU} + 4\tau_{XYD}\right)\right] + Y^2\left[\frac{1}{T^2}\left(3\tau_{XYU} + 3\tau_{XYD} + \frac{6V_a}{T}\right)\right] \qquad (265)$$

Values of τ_{XYU} and τ_{XYD} are given by equations 246 and 234, respectively.

188. Horizontal Radial Stresses.—Horizontal radial stresses, acting on vertical tangential planes, see figures 67 and 68, may be computed by the formula,

$$\sigma_Y = \int_o^Y \frac{\partial \tau_{YZ}}{\partial Z}\, dY + \int_o^Y \frac{\partial \tau_{XY}}{\partial X}\, dY + \sigma_{YD} \qquad (266)$$

Integral terms in equation 266 may be computed from values of τ_{YZ} and τ_{XY} given by equations 264 and 265, respectively. Since Y is measured from the downstream face, it is a function of Z and must be so considered in differentiating τ_{YZ} and τ_{XY}. For example, the bracket term multiplied by Y, equation 264, is the product of two functions of Z and is

differentiated by the usual rule as given below.

$$\frac{\partial}{\partial z}\, Y\left[\frac{1}{T}\left(\frac{6V_c}{T} + 2\tau_{YZU} + 4\tau_{YZD}\right)\right]$$

$$= Y\left[6\,\frac{T^2\frac{\partial V_c}{\partial z} - 2V_c T\frac{\partial T}{\partial z}}{T^4} + 2\,\frac{T\frac{\partial \tau_{YZU}}{\partial z} - \tau_{YZU}\frac{\partial T}{\partial z}}{T^2} + 4\,\frac{T\frac{\partial \tau_{YZD}}{\partial z} - \tau_{YZD}\frac{\partial T}{\partial z}}{T^2}\right]$$

$$+ \frac{\partial Y}{\partial z}\left[\frac{1}{T}\left(\frac{6V_c}{T} + 2\tau_{YZU} + 4\tau_{YZD}\right)\right] \quad (267)$$

Following this method, τ_{YZ} is first differentiated with respect to z.

$$\frac{\partial \tau_{YZ}}{\partial z} = \frac{\partial Y}{\partial z}\left[\frac{6Y\tau_{YZU}}{T^2} + \frac{6Y\tau_{YZD}}{T^2} + \frac{12YV_c}{T^3} - \frac{2\tau_{YZU}}{T} - \frac{4\tau_{YZD}}{T} - \frac{6V_c}{T^2}\right]$$

$$+ \frac{\partial T}{\partial z}\left[\frac{2Y\tau_{YZU}}{T^2} + \frac{4Y\tau_{YZD}}{T^2} + \frac{12YV_c}{T^3} - \frac{6Y^2\tau_{YZU}}{T^3} - \frac{6Y^2\tau_{YZD}}{T^3} - \frac{18Y^2V_c}{T^4}\right]$$

$$+ \frac{\partial \tau_{YZU}}{\partial z}\left[\frac{3Y^2}{T^2} - \frac{2Y}{T}\right] + \frac{\partial \tau_{YZD}}{\partial z}\left[1 + \frac{3Y^2}{T^2} - \frac{4Y}{T}\right] - \frac{\partial V_c}{\partial z}\left[\frac{6Y}{T^2} - \frac{6Y^2}{T^3}\right] \quad (268)$$

This expression is then integrated with respect to Y.

$$\int_o^Y \frac{\partial \tau_{YZ}}{\partial z}\, dY = \frac{\partial Y}{\partial z}\left[\frac{3Y^2\tau_{YZU}}{T^2} + \frac{3Y^2\tau_{YZD}}{T^2} + \frac{6Y^2V_c}{T^3} - \frac{2Y\tau_{YZU}}{T} - \frac{4Y\tau_{YZD}}{T} - \frac{6YV_c}{T^2}\right]$$

$$+ \frac{\partial T}{\partial z}\left[\frac{Y^2\tau_{YZU}}{T^2} + \frac{2Y^2\tau_{YZD}}{T^2} + \frac{6Y^2V_c}{T^3} - \frac{2Y^3\tau_{YZU}}{T^3} - \frac{2Y^3\tau_{YZD}}{T^3} - \frac{6Y^3V_c}{T^4}\right]$$

$$+ \frac{\partial \tau_{YZU}}{\partial z}\left[\frac{Y^3}{T^2} - \frac{Y^2}{T}\right] + \frac{\partial \tau_{YZD}}{\partial z}\left[Y + \frac{Y^3}{T^2} - \frac{2Y^2}{T}\right] - \frac{\partial V_c}{\partial z}\left[\frac{3Y^2}{T^2} - \frac{2Y^3}{T^3}\right] \quad (269)$$

$$\int_o^Y \frac{\partial \tau_{YZ}}{\partial z}\, dY = \frac{\partial Y}{\partial z}\cdot\frac{Y}{T}\left[\frac{3Y}{T}(\tau_{YZU} + \tau_{YZD}) - \frac{6V_c}{T}\left(1 - \frac{Y}{T}\right) - 2\tau_{YZU} - 4\tau_{YZD}\right]$$

$$+ \frac{\partial T}{\partial z}\cdot\frac{Y^2}{T^2}\left[\tau_{YZU} + 2\tau_{YZD} + \frac{6V_c}{T}\left(1 - \frac{Y}{T}\right) - \frac{Y}{T}(2\tau_{YZU} + 2\tau_{YZD})\right]$$

$$+ \frac{\partial \tau_{YZU}}{\partial z}\cdot Y\left[\frac{Y^2}{T^2} - \frac{Y}{T}\right] + \frac{\partial \tau_{YZD}}{\partial z}\cdot Y\left[1 + \frac{Y^2}{T^2} - \frac{2Y}{T}\right] - \frac{\partial V_c}{\partial z}\cdot\frac{Y^2}{T^2}\left[3 - \frac{2Y}{T}\right] \quad (270)$$

$$\frac{\partial Y}{\partial z} = \tan\phi_D \quad (271)$$

$$\frac{\partial T}{\partial z} = \tan\phi_U + \tan\phi_D \quad (272)$$

$\frac{\partial \tau_{YZU}}{\partial z}$ and $\frac{\partial \tau_{YZD}}{\partial z}$ are evaluated by assuming a linear variation of stress between arch elevations.

$$-\frac{\partial V_C}{\partial Z} = \text{Horizontal cantilever trial load at section, times } \frac{R_U}{r} \qquad (273)$$

The same procedure is followed in deriving a formula for the second integral term of equation 266, obtaining

$$\int_0^Y \frac{\partial \tau_{XY}}{\partial X} dY = \frac{\partial Y}{\partial X} \cdot \frac{Y}{T}\left[\frac{3Y}{T}(\tau_{XYU} + \tau_{XYD}) - \frac{6V_a}{T}\left(1 - \frac{Y}{T}\right) - 2\tau_{XYU} - 4\tau_{XYD}\right]$$

$$+ \frac{\partial T}{\partial X} \cdot \frac{Y^2}{T^2}\left[\tau_{XYU} + 2\tau_{XYD} + \frac{6V_a}{T}\left(1 - \frac{Y}{T}\right) - \frac{Y}{T}(2\tau_{XYU} + 2\tau_{XYD})\right]$$

$$+ \frac{\partial \tau_{XYU}}{\partial X} \cdot Y\left[\frac{Y^2}{T^2} - \frac{Y}{T}\right] + \frac{\partial \tau_{XYD}}{\partial X} \cdot Y\left[1 + \frac{Y^2}{T^2} - \frac{2Y}{T}\right] - \frac{\partial V_a}{\partial X} \cdot \frac{Y^2}{T^2}\left[3 - \frac{2Y}{T}\right] \qquad (274)$$

$$\frac{\partial Y}{\partial X} = \tan \beta_D \qquad (275)$$

$$\frac{\partial T}{\partial X} = \tan \beta_U + \tan \beta_D \qquad (276)$$

Values of $\frac{\partial \tau_{XYU}}{\partial X}$ and $\frac{\partial \tau_{XYD}}{\partial X}$ are determined by assuming a linear variation of stress between trial cantilevers.

$$-\frac{\partial V_A}{\partial X} = \text{Horizontal arch trial load at section.} \qquad (277)$$

If temperature changes are being considered in the analysis, equation 277 must be amplified to include temperature effects.

The derivation of σ_{YD} is explained in section 182, and its value is given by equation 236. When Y equals T, σ_Y becomes σ_{YU}, which is determined by equation 248.

The sum of equations 270, 274, and 236 gives equation 266.

189. Summary of Stresses.—A summary of stresses calculated on horizontal and vertical planes, with symbols and equation numbers, is tabulated below.

	Symbol	Equation
Vertical cantilever stress	σ_Z	255
Horizontal arch stress	σ_X	256
Tangential shear on horizontal plane	τ_{ZX}	257
Radial shear on horizontal plane, upstream face	τ_{ZYU}	247
Radial shear on horizontal plane, downstream face	τ_{ZYD}	235
Radial shear on horizontal plane	τ_{ZY}	264

	Symbol	Equation
Radial shear on vertical plane, upstream face	τ_{XYU}	246
Radial shear on vertical plane, downstream face	τ_{XYD}	234
Radial shear on vertical plane	τ_{XY}	265
Radial horizontal stress on vertical plane	σ_Y	266
Radial horizontal stress, upstream face	σ_{YU}	248
Radial horizontal stress, downstream face	σ_{YD}	236

STRESSES ON INCLINED CIRCUMFERENTIAL AREA

190. General.—If a dam is to be increased in height, it may be necessary to add new concrete on the downstream face. As this introduces a possible plane of weakness, investigations of normal and shear stresses along the face are required.

Equations for normal and shear stresses on an inclined circumferential area, one parallel to the X axis and inclined to the Y and Z axes, are determined by using the six stresses, σ_X, σ_Y, σ_Z, τ_{XY}, τ_{XZ}, and τ_{YZ}, considering the elemental area as a plane. Forces acting on an element, see figure 69, are equated to derive stress equations. It is assumed that forces on planes BFC and AED are self balancing. It must be remembered that τ_{XY} equals τ_{YX}, τ_{XZ} equals τ_{ZX}, and τ_{YZ} equals τ_{ZY}.

191. Stress Equations.—Considering the forces in figure 69b, the following stress equations are found:

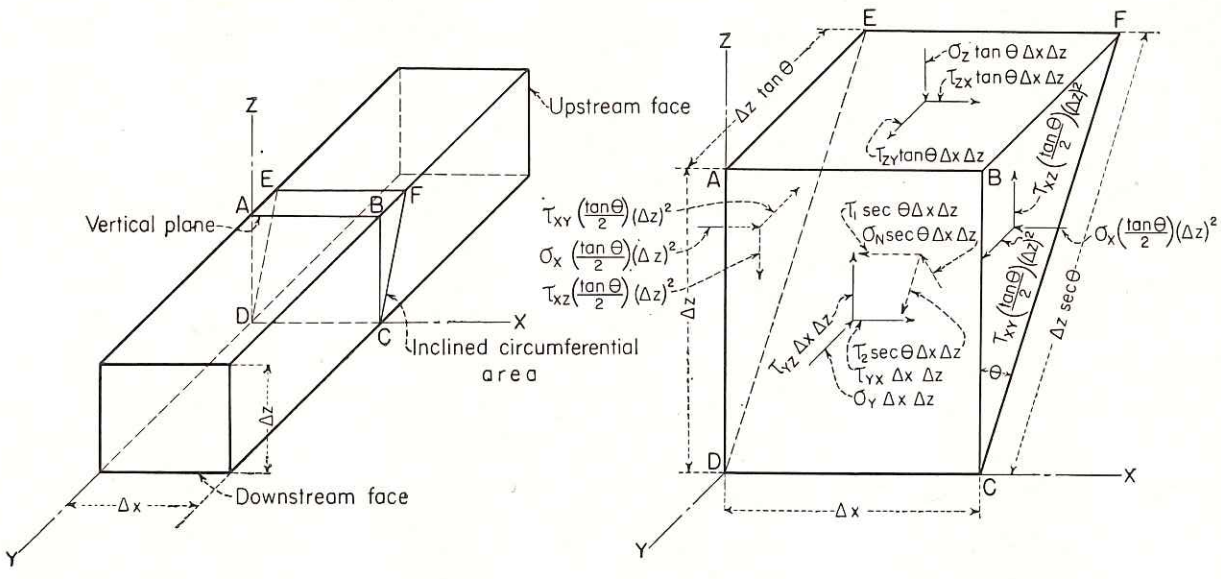


FIGURE 69—STRESSES ON INCLINED CIRCUMFERENTIAL AREA

Horizontal shear stress on inclined plane,

$$\tau_1 \sec\theta\,\Delta x\,\Delta z = \tau_{XY}\,\Delta x\,\Delta z + \tau_{XZ}\tan\theta\,\Delta x\,\Delta z$$

$$\tau_1 = \tau_{XY}\cos\theta + \tau_{XZ}\sin\theta \tag{278}$$

Shear stress on inclined plane, acting at right angles to horizontal shear stress,

$$\tau_2 \sec\theta\Delta x\Delta z = \tau_{ZY}\cos\theta\Delta x\,\Delta z - \tau_{ZY}\tan\theta\sin\theta\Delta x\Delta z$$
$$-\sigma_Z\tan\theta\cos\theta\,\Delta x\Delta z + \sigma_Y\sin\theta\,\Delta x\Delta z$$

$$\tau_2 = \tau_{ZY}(\cos^2\theta - \sin^2\theta) - \sin\theta\cos\theta\,(\sigma_Z - \sigma_Y) \tag{279}$$

Maximum shear stress in inclined plane,

$$\tau_{MAX.} = \sqrt{\tau_1^2 + \tau_2^2} \tag{280}$$

Angle maximum shear stress makes with τ_1,

$$\delta = \tan^{-1}\left(\frac{\tau_2}{\tau_1}\right) \tag{281}$$

Normal stress on inclined plane,

$$\sigma_N \sec\theta\,\Delta x\Delta z = \sigma_Z\tan\theta\sin\theta\,\Delta x\Delta z + \sigma_Y\cos\theta\,\Delta x\Delta z$$
$$-\tau_{ZY}\,\Delta x\Delta z\,(\tan\theta\cos\theta + \sin\theta)$$

$$\sigma_N = \sigma_Z\sin^2\theta + \sigma_Y\cos^2\theta - 2\tau_{ZY}\sin\theta\cos\theta \tag{282}$$

The preceding formulas for stresses on an inclined circumferential area may be checked by substituting direction cosines, given in section 192, in equations 282 and 280.

STRESSES ON OBLIQUE PLANE

192. General.—Equations for normal and shear stresses on an oblique plane are derived so that stresses in any direction can be determined at any point in the dam.

Consider the tetrahedron *OABC* cut from the dam as shown in figure 70. The three mutually perpendicular planes are taken as coordinate planes, and the unit stresses, not the total forces, are shown on the faces. If *O* is the point where stresses on the *XY*, *XZ*, and *YZ* planes are known, stresses on the plane *ABC* approach the stresses on a plane through *O* parallel to plane *ABC* if the tetrahedron *OABC* is made infinitesimal. Since the element is made very small, body forces can be neglected, and stresses over the sides can be assumed as uniformly distributed.

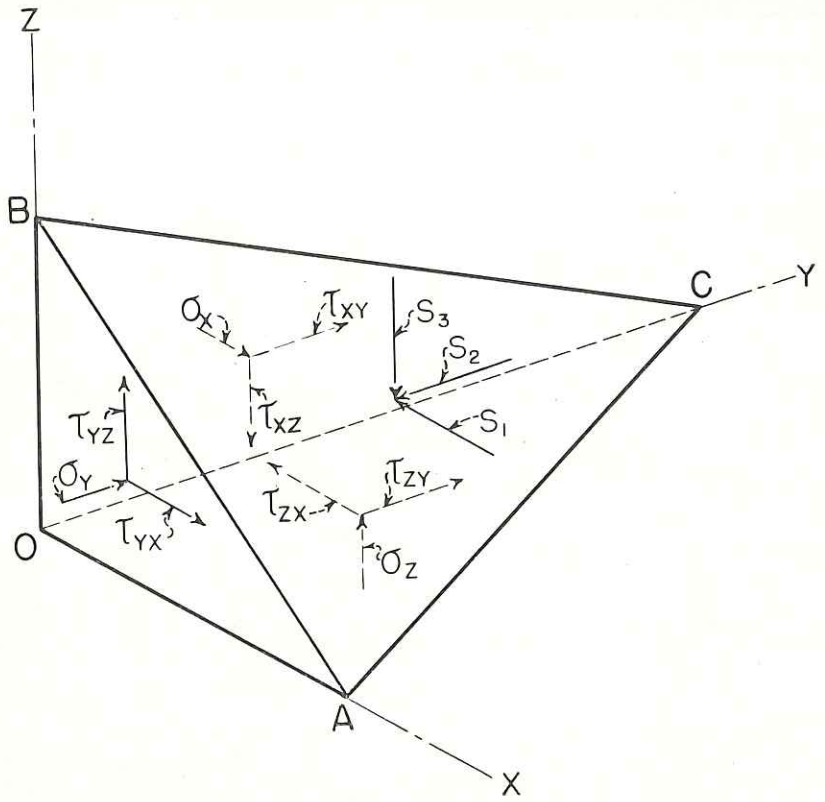


FIGURE 70—ELEMENTAL TETRAHEDRON IN INTERIOR OF DAM

Let j, m, and n denote the direction cosines of the oblique plane ABC. Then,

j = cosine of the angle between the normal to ABC and the X axis.
m = cosine of the angle between the normal to ABC and the Y axis.
n = cosine of the angle between the normal to ABC and the Z axis.

Since the angle between two planes is equal to the angle between normals to the planes,

$$\text{Area } OBC = j(\text{Area } ABC) \qquad \text{Area } OAB = m(\text{Area } ABC)$$
$$\text{Area } OAC = n(\text{Area } ABC)$$

By equating forces in X, Y, and Z directions, the following equations can be written:

$$j\sigma_X + m\tau_{YX} - n\tau_{ZX} = S_1 \tag{283}$$

$$m\sigma_Y + n\tau_{ZY} + j\tau_{XY} = S_2 \tag{284}$$

$$n\sigma_Z - j\tau_{XZ} + m\tau_{YZ} = S_3 \tag{285}$$

S_1, S_2, and S_3 are components of the resultant stress on ABC in the X, Y, and Z directions, respectively. By using these values, normal and maximum shear stresses on plane ABC can be derived.

193. Stress Equations.—Normal stress on the oblique plane, see figure 71, is determined by the equation,

$$\sigma_N = j S_1 + m S_2 + n S_3$$

$$\sigma_N = j^2\sigma_X + jm\tau_{YX} - jn\tau_{ZX} + m^2\sigma_Y + mn\tau_{ZY} + jm\tau_{XY} + n^2\sigma_Z - jn\tau_{XZ} + mn\tau_{YZ}$$

$$\sigma_N = j^2\sigma_X + m^2\sigma_Y + n^2\sigma_Z + 2jm\tau_{XY} + 2mn\tau_{YZ} - 2jn\tau_{XZ} \qquad (286)$$

The maximum shear stress on the oblique plane can be determined as follows: if S = the total stress on plane ABC

$$S^2 = S_1^2 + S_2^2 + S_3^2$$

Then the square of the maximum shear stress can be written,

$$\tau^2_{MAX.} = S^2 - \sigma_N^2, \text{ or}$$

$$\tau^2_{MAX.} = S_1^2 + S_2^2 + S_3^2 - (j^2S_1^2 + m^2S_2^2 + n^2S_3^2 + 2jmS_1S_2 + 2jnS_1S_3 + 2mnS_2S_3)$$

or

$$\tau^2_{MAX} = (1-j^2)S_1^2 + (1-m^2)S_2^2 + (1-n^2)S_3^2 - 2jmS_1S_2 - 2jnS_1S_3 - 2mnS_2S_3 \qquad (287)$$

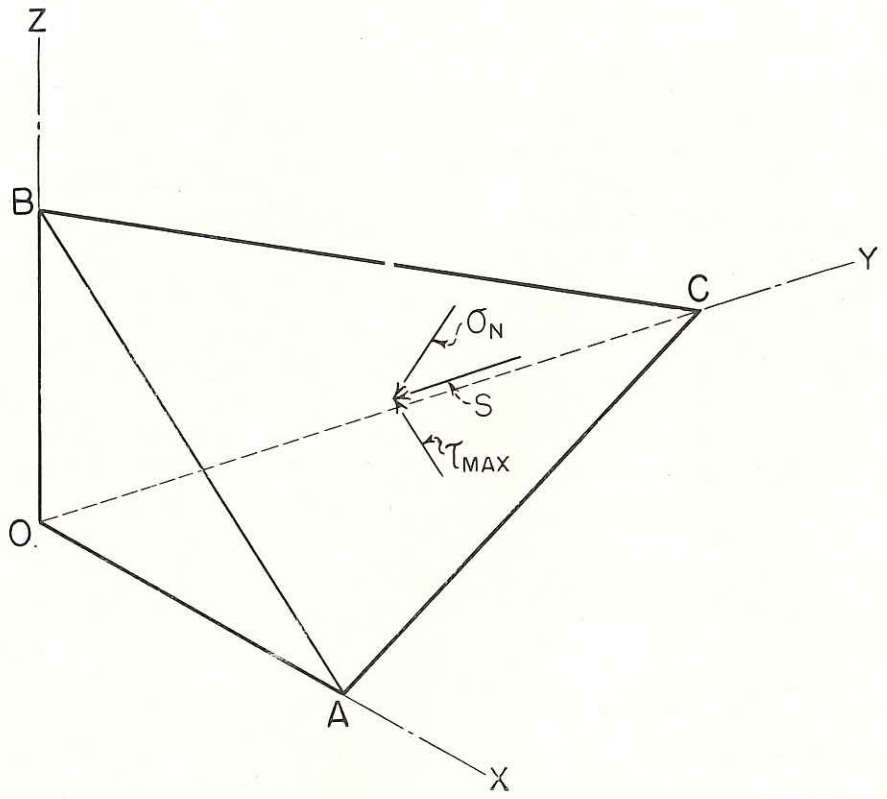


FIGURE 71—STRESSES ON OBLIQUE PLANE

Therefore, $\tau_{MAX.}$ can be determined by using equations 283, 284, 285, and 287.

Since S_1, S_2, and S_3 are components of the total stress S, and $\sigma_N j$, $\sigma_N m$, and $\sigma_N n$ are components of the stress σ_N in the X, Y, and Z directions, respectively, direction cosines of the maximum shear stress are,

$$j' = \frac{S_1 - \sigma_N j}{\tau_{MAX.}} \tag{288}$$

$$m' = \frac{S_2 - \sigma_N m}{\tau_{MAX.}} \tag{289}$$

$$n' = \frac{S_3 - \sigma_N n}{\tau_{MAX.}} \tag{290}$$

194. Principal Stresses in Interior of Dam.—If a complete stress picture is required at any point in a dam, principal stresses and maximum shear stresses should be determined. Again consider the tetrahedron $OABC$ in figure 70 and let ABC be a principal plane with the principal stress, S, acting normal to the plane. By substituting jS for S_1, mS for S_2, and nS for S_3 in equations 283, 284, and 285 the following equations can be written:

$$j(\sigma_X - S) + m\tau_{YX} - n\tau_{ZX} = 0 \tag{291}$$

$$m(\sigma_Y - S) + n\tau_{ZY} + j\tau_{XY} = 0 \tag{292}$$

$$n(\sigma_Z - S) - j\tau_{XZ} + m\tau_{YZ} = 0 \tag{293}$$

If j, m, and n are eliminated and the equations are solved for S, the following cubic equation is derived:

$$(\sigma_X - S)(\sigma_Y - S)(\sigma_Z - S) - (\sigma_X - S)\tau^2_{YZ} - (\sigma_Y - S)\tau^2_{XZ} - (\sigma_Z - S)\tau^2_{XY} - 2\tau_{XY}\tau_{YZ}\tau_{XZ} = 0$$

This equation may be written,

$$S^3 - (\sigma_X + \sigma_Y + \sigma_Z)S^2 + (\sigma_X\sigma_Y + \sigma_Y\sigma_Z + \sigma_X\sigma_Z - \tau^2_{XY} - \tau^2_{YZ} - \tau^2_{XZ})S$$
$$-(\sigma_X\sigma_Y\sigma_Z - 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau^2_{YZ} - \sigma_Y\tau^2_{XZ} - \sigma_Z\tau^2_{XY}) = 0 \tag{294}$$

The three roots of equation 294 give the principal stresses. By substituting the stresses in equations 291, 292, and 293 and using the relation $(j^2 + m^2 + n^2 = 1)$, the three sets of direction cosines for the three principal planes can be found. The maximum shear stress acts on a plane that bisects the angle between the largest and the smallest principal stresses, and is equal to half the difference between these two principal stresses.

CHAPTER IX—TABLES USED IN ARCH ANALYSES

INTRODUCTION

195. General Statement.—The tables presented in this chapter were compiled to simplify and expedite calculations for circular arches. Formulas for quantities in the tables are derived in chapter V. The use of tabular values in arch computations is described in chapter VI. Methods used in calculating tabular quantities, together with discussions of the tables, are included in this chapter. Although a great amount of time and labor was involved in compiling the tables, the work was more than justified by resultant savings in time required for conducting the complicated trial load analyses required in arch dam designs.

196. Arrangement.—All tabulations are given to seven significant figures except some values in tables 11 to 15, inclusive, which give trigonometric functions and integrals needed for calculating load constants. These are given to twelve places because several significant figures are lost when the values are used in computing subsequent functions and integrals.

Some tabular values have several zeros following the decimal point. For purposes of condensation, such values are designated in the tables by superior numbers. For example, *.000,012,984,18* is shown as *.0^{4}12,984,18*. The superior number after the zero gives the number of zeros between the decimal point and the first significant figure.

CONSTANTS FOR CIRCULAR ARCHES

197. Use of Table.—Quantities in table 10, Constants for Circular Arches, are used in determining arch movements due to moment, thrust, and radial shear at a section. Detailed discussions of arch constants and outlines of their derivations are given in chapter V. An arch constant, as defined in chapter V, is the product, or the sum of products, of a multiplier and a trigonometric function. Trigonometric parts used in determining arch constants have been evaluated and are tabulated in table 10.

TABLE 10—CONSTANTS FOR CIRCULAR ARCHES

ϕ	SIN ϕ	COS ϕ	A_1	B_1	B_3		C_1	B_2		C_2	
° ′			(Radians)	(ϕ−SIN ϕ)	1ST TERM	2ND TERM	(VERS ϕ)	1ST TERM	2ND TERM	1ST TERM	2ND TERM
0 15	.004,363,309	.999,990,5	.004,363,323	$.0^{7}13,845,24$	$.0^{13}79,078,03$	.004,363,379	$.0^{5}9,519,279$	$.0^{10}45,308,34$	$.0^{4}19,038,47$	$.0^{7}27,690,40$	.013,089,91
0 30	.008,726,535	.999,961,9	.008,726,646	$.0^{6}110,761,6$	$.0^{11}2,530,480$	.008,727,089	$.0^{4}38,076,94$	$.0^{9}724,926,5$	$.0^{4}76,152,42$	$.0^{6}221,520,7$	.026,179,50
0 45	.013,089,60	.999,914,3	.013,089,97	$.0^{6}373,818,6$	$.0^{10}19,215,61$	.013,091,46	$.0^{4}85,672,43$	$.0^{8}3,669,882$	$.0^{3}171,337,5$	$.0^{6}747,618,0$	.039,268,41
1 00	.017,452,41	.999,847,7	.017,453,29	$.0^{6}886,082,6$	$.0^{10}80,973,15$	.017,456,84	$.0^{3}152,304,8$	$.0^{7}11,598,38$	$.0^{3}304,586,5$	$.0^{5}1,772,084$	.052,356,33
1 15	.021,814,89	.999,762,0	.021,816,62	$.0^{5}1,730,615$	$.0^{9}247,105,4$	.021,823,54	$.0^{3}237,972,9$	$.0^{7}28,315,55$	$.0^{3}475,889,2$	$.0^{5}3,460,984$	.065,442,93
1 30	.026,176,95	.999,657,3	.026,179,94	$.0^{5}2,990,472$	$.0^{9}614,862,1$	.026,191,90	$.0^{3}342,675,0$	$.0^{7}58,713,09$	$.0^{3}685,232,6$	$.0^{5}5,980,329$	.078,527,85
1 45	.030,538,51	.999,533,6	.030,543,26	$.0^{5}4,748,700$	$.0^{8}1,328,920$	.030,562,25	$.0^{3}466,409,2$	$.0^{6}108,768,8$	$.0^{3}932,600,8$	$.0^{5}9,496,071$	.091,610,79
2 00	.034,899,50	.999,390,8	.034,906,59	$.0^{5}7,088,337$	$.0^{8}2,590,859$	.034,934,93	$.0^{3}609,173,0$	$.0^{6}185,545,9$	.001,217,975	$.0^{4}14,174,08$	.104,691,4
2 15	.039,259,82	.999,229,0	.039,269,91	$.0^{4}10,092,41$	$.0^{8}4,668,633$	.039,310,27	$.0^{3}770,963,8$	$.0^{6}297,192,6$	.001,541,333	$.0^{4}20,180,15$	.117,769,4
2 30	.043,619,39	.999,048,2	.043,633,23	$.0^{4}13,843,93$	$.0^{8}7,906,030$	.043,688,59	$.0^{3}951,7784$	$.0^{6}452,941,1$	.001,902,651	$.0^{4}27,679,96$	.130,844,3
2 45	.047,978,13	.998,848,4	.047,996,55	$.0^{4}18,425,91$	$.0^{7}12,732,13$	.048,070,23	.001,151,614	$.0^{6}663,106,9$	.002,301,901	$.0^{4}36,839,08$	.143,916,0
3 00	.052,335,96	.998,629,5	.052,359,88	$.0^{4}23,921,32$	$.0^{7}19,670,77$	.052,455,52	.001,370,465	$.0^{6}939,087,5$	.002,739,052	$.0^{4}47,822,96$	.156,984,0
3 15	.056,692,79	.998,391,7	.056,723,20	$.0^{4}30,413,13$	$.0^{7}29,349,95$	.056,844,79	.001,608,329	$.0^{5}1,293,362$	.003,214,072	$.0^{4}60,796,90$	.170,048,0
3 30	.061,048,54	.998,134,8	.061,086,52	$.0^{4}37,984,28$	$.0^{7}42,511,28$	.061,238,38	.001,865,202	$.0^{5}1,739,489$	.003,726,924	$.0^{4}75,926,06$	.183,107,7
3 45	.065,403,13	.997,858,9	.065,449,85	$.0^{4}46,717,72$	$.0^{7}60,01941$	.065,636,60	.002,141,077	$.0^{5}2,292,105$	.004,277,569	$.0^{4}93,375,42$	.196,162,8
4 00	.069,756,47	.997,564,1	.069,813,17	$.0^{4}56,696,34$	$.0^{7}82,871,43$	.070,039,79	.002,435,950	$.0^{5}2,966,925$	.004,865,966	$.0^{3}113,309,8$	.209,212,9
4 15	.074,108,49	.997,250,2	.074,176,49	$.0^{4}68,003,01$	$.0^{6}112,206,2$	.074,448,28	.002,749,815	$.0^{5}3,780,741$	.005,492,068	$.0^{3}135,893,8$	.222,257,7
4 30	.078,459,10	.996,917,3	.078,539,82	$.0^{4}80,720,61$	$.0^{6}149,314,0$	.078,862,40	.003,082,666	$.0^{5}4,751,416$	.006,155,830	$.0^{3}161,291,9$	.235,296,9
4 45	.082,808,21	.996,565,5	.082,903,14	$.0^{4}94,931,96$	$.0^{6}195,645,4$	.083,282,48	.003,434,498	$.0^{5}5,897,887$	.006,857,199	$.0^{3}189,668,3$	.248,330,1
5 00	.087,155,74	.996,194,7	.087,266,46	$.0^{3}110,719,9$	$.0^{6}252,821,0$	.087,708,84	.003,805,302	$.0^{5}7,240,160$	.007,596,124	$.0^{3}221,186,9$	.261,357,0
5 15	.091,501,62	.995,804,9	.091,629,79	$.0^{3}128,167,1$	$.0^{6}322,640,8$	.092,141,81	.004,195,072	$.0^{5}8,799,316$	.008,372,546	$.0^{3}256,011,5$	.274,377,3
5 30	.095,845,75	.995,396,2	.095,993,11	$.0^{3}147,356,3$	$.0^{6}407,093,6$	.096,581,72	.004,603,802	$.0^{4}10,597,50$	.009,186,408	$.0^{3}294,305,6$	.287,390,7
5 45	.100,188,1	.994,968,5	.100,356,4	$.0^{3}168,370,4$	$.0^{6}508,364,6$	.101,028,9	.005,031,482	$.0^{4}12,657,90$	.010,037,65	$.0^{3}336,232,4$	.300,396,8
6 00	.104,528,5	.994,521,9	.104,719,8	$.0^{3}191,291,9$	$.0^{6}628,848,6$	.105,483,7	.005,478,105	$.0^{4}15,004,82$	.010,926,20	$.0^{3}381,954,9$	.313,395,4
6 15	.108,866,9	.994,056,3	.109,083,1	$.0^{3}216,203,4$	$.0^{6}771,155,3$	.109,946,4	.005,943,662	$.0^{4}17,663,56$	.011,852,00	$.0^{3}431,635,6$	.326,386,0
6 30	.113,203,2	.993,571,9	.113,446,4	$.0^{3}243,187,6$	$.0^{6}938,119,6$	.114,417,3	.006,428,144	$.0^{4}20,660,52$	.012,814,97	$.0^{3}485,437,1$	.339,368,3
6 45	.117,537,4	.993,068,5	.117,809,7	$.0^{3}272,327,1$	$.0^{5}1,132,813$	.118,896,8	.006,931,543	$.0^{4}24,023,14$	.013,815,04	$.0^{3}543,521,3$	.352,342,2
7 00	.121,869,3	.992,546,2	.122,173,0	$.0^{3}303,704,2$	$.0^{5}1,358,549$	.123,385,1	.007,453,848	$.0^{4}27,779,93$	.014,852,14	$.0^{3}606,049,9$	.365,307,0
7 15	.126,199,0	.992,004,9	.126,536,4	$.0^{3}337,401,6$	$.0^{5}1,618,896$	.127,882,7	.007,995,050	$.0^{4}31,960,42$	.015,926,18	$.0^{3}673,184,4$	.378,262,8
7 30	.130,526,2	.991,444,9	.130,899,7	$.0^{3}373,501,7$	$.0^{5}1,917,685$	.132,389,9	.008,555,139	$.0^{4}36,595,20$	.017,037,09	$.0^{3}745,085,7$	.391,208,9
7 45	.134,850,9	.990,865,9	.135,263,0	$.0^{3}412,086,8$	$.0^{5}2,259,017$	.136,906,8	.009,134,103	$.0^{4}41,715,92$	.018,184,77	$.0^{3}821,914,5$	.404,145,2
8 00	.139,173,1	.990,268,1	.139,626,3	$.0^{3}453,239,2$	$.0^{5}2,647,273$	.141,434,0	.009,731,931	$.0^{4}47,355,24$	.019,369,15	$.0^{3}903,831,1$	.417,071,4
8 15	.143,492,6	.989,651,4	.143,989,7	$.0^{3}497,041,3$	$.0^{5}3,087,128$	.145,971,7	.010,348,61	$.0^{4}53,546,90$	.020,590,13	$.0^{3}990,995,5$	.429,987,0
8 30	.147,809,4	.989,015,9	.148,353,0	$.0^{3}543,575,3$	$.0^{5}3,583,551$	.150,520,1	.010,984,14	$.0^{4}60,325,63$	.021,847,62	.001,083,567	.442,891,8
8 45	.152,123,4	.988,361,5	.152,716,3	$.0^{3}592,923,4$	$.0^{5}4,141,821$	.155,079,7	.011,638,49	$.0^{4}67,727,22$	.023,141,52	.001,181,705	.455,785,5
9 00	.156,434,5	.987,688,3	.157,079,6	$.0^{3}645,167,6$	$.0^{5}4,767,532$	.159,650,8	.012,311,66	$.0^{4}75,788,48$	.024,471,74	.001,285,568	.468,667,8
9 15	.160,742,6	.986,996,4	.161,443,0	$.0^{3}700,390,2$	$.0^{5}5,466,608$	.164,233,6	.013,003,63	$.0^{4}84,547,24$	.025,838,17	.001,395,314	.481,538,2
9 30	.165,047,6	.986,285,6	.165,806,3	$.0^{3}758,673,1$	$.0^{5}6,245,302$	.168,828,5	.013,714,40	$.0^{4}94,042,36$	.027,240,71	.001,511,101	.494,396,6
9 45	.169,349,5	.985,556,1	.170,169,6	$.0^{3}820,098,2$	$.0^{5}7,110,215$	.173,435,8	.014,443,94	$.0^{3}104,313,7$	.028,679,25	.001,633,086	.507,242,6
10 00	.173,648,2	.984,807,8	.174,532,9	$.0^{3}884,747,5$	$.0^{5}8,068,297$	.178,055,8	.015,192,25	$.0^{3}115,402,2$	.030,153,69	.001,761,427	.520,075,9
10 15	.177,943,5	.984,040,7	.178,896,2	$.0^{3}952,702,9$	$.0^{5}9,126,867$	.182,688,8	.015,959,30	$.0^{3}127,349,7$	.031,663,91	.001,896,279	.532,896,2
10 30	.182,235,5	.983,254,9	.183,259,6	.001,024,046	$.0^{4}10,293,59$	.187,335,2	.016,745,09	$.0^{3}140,199,1$	.033,209,79	.002,037,798	.545,703,1
10 45	.186,524,0	.982,450,4	.187,622,9	.001,098,859	$.0^{4}11,576,55$	.191,995,2	.017,549,60	$.0^{3}153,994,3$	.034,791,22	.002,186,141	.558,496,4
11 00	.190,809,0	.981,627,2	.191,986,2	.001,177,222	$.0^{4}12,984,18$	.196,669,1	.018,372,82	$.0^{3}168,780,2$	.036,408,07	.002,341,461	.571,275,7
11 15	.195,090,3	.980,785,3	.196,349,5	.001,259,219	$.0^{4}14,525,33$	.201,357,4	.019,214,72	$.0^{3}184,602,7$	.038,060,23	.002,503,912	.584,040,8
11 30	.199,367,9	.979,924,7	.200,712,9	.001,344,930	$.0^{4}16,209,26$	.206,060,2	.020,075,30	$.0^{3}201,508,7$	.039,747,57	.002,673,650	.596,791,3
11 45	.203,641,8	.979,045,5	.205,076,2	.001,434,436	$.0^{4}18,045,57$	.210,777,8	.020,954,53	$.0^{3}219,546,1$	.041,469,96	.002,850,826	.609,526,9
12 00	.207,911,7	.978,147,6	.209,439,5	.001,527,819	$.0^{4}20,044,49$	.215,510,7	.021,852,40	$.0^{3}238,763,7$	.043,227,27	.003,035,594	.622,247,3
12 15	.212,177,7	.977,231,1	.213,802,8	.001,625,161	$.0^{4}22,216,41$	.220,259,1	.022,768,89	$.0^{3}259,211,3$	.045,019,36	.003,228,106	.634,952,3
12 30	.216,439,6	.976,296,0	.218,166,2	.001,726,543	$.0^{4}24,572,31$	.225,023,2	.023,703,99	$.0^{3}280,939,6$	.046,846,11	.003,428,513	.647,641,4
12 45	.220,697,4	.975,342,3	.222,529,5	.001,832,045	$.0^{4}27,123,60$	.229,803,4	.024,657,68	$.0^{3}304,000,6$	.048,707,36	.003,636,966	.660,314,5
13 00	.224,951,1	.974,370,1	.226,892,8	.001,941,748	$.0^{4}29,882,15$	.234,600,0	.025,629,94	$.0^{3}328,446,8$	.050,602,98	.003,853,615	.672,971,2
13 15	.229,200,4	.973,379,3	.231,256,1	.002,055,735	$.0^{4}32,860,27$	.239,413,4	.026,620,74	$.0^{3}354,331,9$	.052,532,82	.004,078,610	.685,611,2
13 30	.233,445,4	.972,369,9	.235,619,4	.002,174,085	$.0^{4}36,070,75$	.244,243,7	.027,630,08	$.0^{3}381,710,7$	.054,496,74	.004,312,100	.698,234,2
13 45	.237,685,9	.971,342,1	.239,982,8	.002,296,880	$.0^{4}39,526,88$	.249,091,2	.028,657,93	$.0^{3}410,638,4$	.056,494,58	.004,554,233	.710,839,9
14 00	.241,921,9	.970,295,7	.244,346,1	.002,424,200	$.0^{4}43,242,42$	.253,956,4	.029,704,27	$.0^{3}441,171,9$	.058,526,20	.004,805,157	.723,428,0
14 15	.246,153,3	.969,230,9	.248,709,4	.002,556,125	$.0^{4}47,231,62$	.258,839,5	.030,769,09	$.0^{3}473,368,5$	.060,591,44	.005,065,019	.735,998,2
14 30	.250,380,0	.968,147,6	.253,072,7	.002,692,737	$.0^{4}51,509,26$	.263,740,7	.031,852,36	$.0^{3}507,286,4$	.062,690,15	.005,333,966	.748,550,3
14 45	.254,601,9	.967,045,9	.257,436,1	.002,834,117	$.0^{4}56,090,62$	.268,660,4	.032,954,06	$.0^{3}542,985,1$	.064,822,15	.005,612,142	.761,083,9
15 00	.258,819,0	.965,925,8	.261,799,4	.002,980,343	$.0^{4}60,991,49$	.273,598,8	.034,074,17	$.0^{3}580,524,7$	.066,987,30	.005,899,694	.773,598,8
15 15	.263,031,2	.964,787,3	.266,162,7	.003,131,496	$.0^{4}66,228,22$	.278,556,2	.035,212,68	$.0^{3}619,966,3$	.069,185,42	.006,196,765	.786,094,6
15 30	.267,238,4	.963,630,5	.270,526,0	.003,287,658	$.0^{4}71,817,66$	.283,533,0	.036,369,55	$.0^{3}661,372,0$	.071,416,35	.006,503,498	.798,571,1
15 45	.271,440,4	.962,455,2	.274,889,4	.003,448,907	$.0^{4}77,777,23$	.288,529,4	.037,544,76	$.0^{3}704,804,6$	.073,679,92	.006,820,037	.811,028,0
16 00	.275,637,4	.961,261,7	.279,252,7	.003,615,325	$.0^{4}84,124,90$	.293,545,7	.038,738,30	$.0^{3}750,328,1$	.075,975,95	.007,146,524	.823,465,0
16 15	.279,829,0	.960,049,9	.283,616,0	.003,786,989	$.0^{4}90,879,20$	.298,582,2	.039,950,15	$.0^{3}798,007,1$	.078,304,28	.007,483,100	.835,881,8
16 30	.284,015,3	.958,819,7	.287,979,3	.003,963,982	$.0^{4}98,059,22$	.303,639,1	.041,180,27	$.0^{3}847,907,1$	.080,664,72	.007,829,905	.848,278,2
16 45	.288,196,3	.957,571,4	.292,342,6	.004,146,382	$.0^{3}105,684,6$	.308,716,8	.042,428,64	$.0^{3}900,094,7$	.083,057,09	.008,187,079	.860,653,8
17 00	.292,371,7	.956,304,8	.296,706,0	.004,334,268	$.0^{3}113,775,7$	.313,815,5	.043,695,24	$.0^{3}954,637,2$	.085,481,21	.008,554,761	.873,008,4
17 15	.296,541,6	.955,019,9	.301,069,3	.004,527,721	$.0^{3}122,353,2$	.318,935,5	.044,980,06	.001,011,603	.087,936,91	.008,933,089	.885,341,7
17 30	.300,705,8	.953,717,0	.305,432,6	.004,726,820	$.0^{3}131,438,7$	.324,077,0	.046,283,05	.001,071,060	.090,423,98	.009,322,200	.897,653,5
17 45	.304,864,3	.952,395,8	.309,795,9	.004,931,643	$.0^{3}141,054,2$	.329,240,4	.047,604,20	.001,133,080	.092,942,24	.009,722,232	.909,943,4
18 00	.309,017,0	.951,056,5	.314,159,3	.005,142,271	$.0^{3}151,222,4$	.334,425,9	.048,943,48	.001,197,732	.095,491,50	.010,133,32	.922,211,2
18 15	.313,163,8	.949,699,1	.318,522,6	.005,358,782	$.0^{3}161,966,5$	.339,633,8	.050,300,87	.001,265,089	.098,071,57	.010,555,60	.934,456,6
18 30	.317,304,7	.948,323,7	.322,885,9	.005,581,255	$.0^{3}173,310,4$	.344,864,3	.051,676,34	.001,335,222	.100,682,2	.010,989,20	.946,679,3
18 45	.321,439,5	.946,930,1	.327,249,2	.005,809,769	$.0^{3}185,278,8$	.350,117,8	.053,069,87	.001,408,206	.103,323,3	.011,434,26	.958,879,2
19 00	.325,568,2	.945,518,6	.331,612,6	.006,044,403	$.0^{3}197,896,7$	.355,394,4	.054,481,42	.001,484,113	.105,994,6	.011,890,91	.971,055,9
19 15	.329,690,6	.944,089,0	.335,975,9	.006,285,236	$.0^{3}211,190,2$	.360,694,4	.055,910,98	.001,563,019	.108,695,9	.012,359,28	.983,209,1
19 30	.333,806,9	.942,641,5	.340,339,2	.006,532,345	$.0^{3}225,185,5$	.366,018,2	.057,358,51	.001,644,999	.111,427,0	.012,839,50	.995,338,6
19 45	.337,916,7	.941,176,0	.344,702,5	.006,785,809	$.0^{3}239,910,0$	.371,365,9	.058,823,98	.001,730,131	.114,187,7	.013,331,71	1.007,444,2
20 00	.342,020,1	.939,692,6	.349,065,9	.007,045,707	$.0^{3}255,391,4$	.376,737,9	.060,307,38	.001,818,490	.116,977,8	.013,836,02	1.019,525,5
20 15	.346,117,1	.938,191,3	.353,429,2	.007,312,116	$.0^{3}271,658,2$	.382,134,3	.061,808,66	.001,910,156	.119,797,0	.014,352,57	1.031,582,4
20 30	.350,207,4	.936,672,2	.357,792,5	.007,585,115	$.0^{3}288,739,7$	.387,555,5	.063,327,81	.002,005,206	.122,645,2	.014,881,49	1.043,614,5
20 45	.354,291,0	.935,135,2	.362,155,8	.007,864,782	$.0^{3}306,665,7$	.393,001,6	.064,864,79	.002,103,721	.125,522,1	.015,422,90	1.055,621,7
21 00	.358,367,9	.933,580,4	.366,519,1	.008,151,193	$.0^{3}325,466,9$	.398,473,0	.066,419,57	.002,205,780	.128,427,6	.015,976,92	1.067,603,6
21 15	.362,438,0	.932,007,9	.370,882,5	.008,444,428	$.0^{3}345,174,4$	.403,969,8	.067,992,13	.002,311,465	.131,361,3	.016,543,68	1.079,560,0
21 30	.366,501,2	.930,417,6	.375,245,8	.008,744,562	$.0^{3}365,820,3$	.409,492,4	.069,582,43	.002,420,857	.134,323,1	.017,123,30	1.091,490,8
21 45	.370,557,4	.928,809,6	.379,609,1	.009,051,675	$.0^{3}387,437,4$	.415,040,9	.071,190,45	.002,534,040	.137,312,8	.017,715,91	1.103,395,5
22 00	.374,606,6	.927,183,9	.383,972,4	.009,365,842	$.0^{3}410,058,9$	.420,615,7	.072,816,15	.002,651,096	.140,330,1	.018,321,63	1.115,274,1
22 15	.378,648,6	.925,540,5	.388,335,8	.009,687,141	$.0^{3}433,719,2$	.426,216,9	.074,459,50	.002,772,108	.143,374,8	.018,940,56	1.127,126,1
22 30	.382,683,4	.923,879,5	.392,699,1	.010,015,65	$.0^{3}458,453,1$	.431,844,8	.076,120,47	.002,897,163	.146,446,6	.019,572,85	1.138,951,6

TABLE 10—Continued

φ	SIN φ	COS φ	A_1	B_1	B_3		C_1	B_2		C_2	
° ′			(Radians)	(φ−SIN φ)	1ST TERM	2ND TERM	(VERS φ)	1ST TERM	2ND TERM	1ST TERM	2ND TERM
22 45	.386,711,0	.922,201,0	.397,062,4	.010,351,44	$.0^3$484,296,3	.437,499,6	.077,799,03	.003,026,344	.149,545,4	.020,218,59	1.150,750,0
23 00	.390,731,1	.920,504,9	.401,425,7	.010,694,60	$.0^3$511,285,1	.443,181,6	.079,495,15	.003,159,739	.152,670,8	.020,877,91	1.162,521,4
23 15	.394,743,9	.918,791,2	.405,789,1	.011,045,19	$.0^3$539,456,6	.448,890,9	.081,208,79	.003,297,434	.155,822,7	.021,550,93	1.174,265,3
23 30	.398,749,1	.917,060,1	.410,152,4	.011,403,31	$.0^3$568,848,9	.454,627,9	.082,939,93	.003,439,516	.159,000,8	.022,237,76	1.185,981,6
23 45	.402,746,7	.915,311,5	.414,515,7	.011,769,01	$.0^3$599,500,5	.460,392,7	.084,688,52	.003,586,073	.162,204,9	.022,938,51	1.197,670,1
24 00	.406,736,6	.913,545,5	.418,879,0	.012,142,38	$.0^3$631,450,9	.466,185,6	.086,454,54	.003,737,194	.165,434,7	.023,653,30	1.209,330,5
24 15	.410,718,9	.911,762,0	.423,242,3	.012,523,49	$.0^3$664,740,4	.472,006,8	.088,237,96	.003,892,968	.168,690,0	.024,382,24	1.220,962,5
24 30	.414,693,2	.909,961,3	.427,605,7	.012,912,42	$.0^3$699,409,9	.477,856,5	.090,038,73	.004,053,486	.171,970,5	.025,125,44	1.232,566,1
24 45	.418,659,7	.908,143,2	.431,969,0	.013,309,25	$.0^3$735,501,1	.483,735,0	.091,856,83	.004,218,838	.175,276,0	.025,883,00	1.244,141,0
25 00	.422,618,3	.906,307,8	.436,332,3	.013,714,05	$.0^3$773,056,8	.489,642,4	.093,692,21	.004,389,115	.178,606,2	.026,655,05	1.255,686,8
25 15	.426,568,7	.904,455,1	.440,695,6	.014,126,90	$.0^3$812,120,2	.495,579,0	.095,544,85	.004,564,410	.181,960,9	.027,441,67	1.267,203,6
25 30	.430,511,1	.902,585,3	.445,059,0	.014,547,86	$.0^3$852,735,6	.501,544,9	.097,414,72	.004,744,813	.185,339,8	.028,242,99	1.278,690,9
25 45	.434,445,3	.900,698,2	.449,422,3	.014,977,02	$.0^3$894,948,0	.507,540,5	.099,301,76	.004,930,420	.188,742,7	.029,059,10	1.290,148,6
26 00	.438,371,1	.898,794,0	.453,785,6	.015,414,46	$.0^3$938,803,1	.513,565,8	.101,206,0	.005,121,323	.192,169,3	.029,890,11	1.301,576,6
26 15	.442,288,7	.896,872,7	.458,148,9	.015,860,24	$.0^3$984,347,6	.519,621,2	.103,127,3	.005,317,616	.195,619,3	.030,736,13	1.312,974,5
26 30	.446,197,8	.894,934,4	.462,512,3	.016,314,44	.001,031,629	.525,706,8	.105,065,6	.005,519,394	.199,092,5	.031,597,25	1.324,342,3
26 45	.450,098,4	.892,978,9	.466,875,6	.016,777,13	.001,080,695	.531,822,7	.107,021,1	.005,726,753	.202,588,6	.032,473,57	1.335,679,6
27 00	.453,990,5	.891,006,5	.471,238,9	.017,248,40	.001,131,596	.537,969,3	.108,993,5	.005,939,789	.206,107,4	.033,365,20	1.346,986,3
27 15	.457,873,9	.889,017,1	.475,602,2	.017,728,31	.001,184,381	.544,146,7	.110,982,9	.006,158,597	.209,648,5	.034,272,23	1.358,262,2
27 30	.461,748,6	.887,010,8	.479,965,5	.018,216,93	.001,239,101	.550,355,1	.112,989,2	.006,383,276	.213,211,8	.035,194,76	1.369,507,1
27 45	.465,614,5	.884,987,6	.484,328,9	.018,714,35	.001,295,808	.556,594,6	.115,012,4	.006,613,922	.216,796,9	.036,132,89	1.380,720,8
28 00	.469,471,6	.882,947,6	.488,692,2	.019,220,63	.001,354,553	.562,865,6	.117,052,4	.006,850,633	.220,403,5	.037,086,70	1.391,903,2
28 15	.473,319,7	.880,890,7	.493,055,5	.019,735,85	.001,415,392	.569,168,1	.119,109,3	.007,093,508	.224,031,5	.038,056,30	1.403,053,9
28 30	.477,158,8	.878,817,1	.497,418,8	.020,260,08	.001,478,377	.575,502,4	.121,182,9	.007,342,646	.227,680,5	.039,041,78	1.414,173,0
28 45	.480,988,8	.876,726,8	.501,782,2	.020,793,39	.001,543,564	.581,868,6	.123,273,2	.007,598,146	.231,350,2	.040,043,22	1.425,260,0
29 00	.484,809,6	.874,619,7	.506,145,5	.021,335,86	.001,611,008	.588,266,9	.125,380,3	.007,860,109	.235,040,4	.041,060,72	1.436,315,0
29 15	.488,621,2	.872,496,0	.510,508,8	.021,887,56	.001,680,767	.594,697,5	.127,504,0	.008,128,634	.238,750,7	.042,094,36	1.447,337,7
29 30	.492,423,6	.870,355,7	.514,872,1	.022,448,57	.001,752,899	.601,160,6	.129,644,3	.008,403,823	.242,481,0	.043,144,24	1.458,327,9
29 45	.496,216,5	.868,198,8	.519,235,5	.023,018,95	.001,827,461	.607,656,3	.131,801,2	.008,685,776	.246,230,8	.044,210,44	1.469,285,5
30 00	.500,000,0	.866,025,4	.523,598,8	.023,598,78	.001,904,514	.614,184,9	.133,974,6	.008,974,596	.250,000,0	.045,293,04	1.480,210,3
30 15	.503,774,0	.863,835,5	.527,962,1	.024,188,12	.001,984,118	.620,746,3	.136,164,5	.009,270,385	.253,788,2	.046,392,13	1.491,102,0
30 30	.507,538,4	.861,629,2	.532,325,4	.024,787,06	.002,066,334	.627,341,0	.138,370,8	.009,573,245	.257,595,2	.047,507,78	1.501,960,7
30 45	.511,293,1	.859,406,4	.536,688,7	.025,395,66	.002,151,224	.633,968,9	.140,593,6	.009,883,279	.261,420,6	.048,640,09	1.512,786,0
31 00	.515,038,1	.857,167,3	.541,052,1	.026,013,99	.002,238,851	.640,630,3	.142,832,7	.010,200,59	.265,264,2	.049,789,14	1.523,577,9
31 15	.518,773,3	.854,911,9	.545,415,4	.026,642,13	.002,329,279	.647,325,4	.145,088,1	.010,525,28	.269,125,7	.050,954,99	1.534,336,2
31 30	.522,498,6	.852,640,2	.549,778,7	.027,280,15	.002,422,573	.654,054,2	.147,359,8	.010,857,46	.273,004,8	.052,137,73	1.545,060,7
31 45	.526,213,9	.850,352,2	.554,142,0	.027,928,11	.002,518,799	.660,816,9	.149,647,8	.011,197,23	.276,901,1	.053,337,43	1.555,751,3
32 00	.529,919,3	.848,048,1	.558,505,4	.028,586,10	.002,618,024	.667,613,7	.151,951,9	.011,544,69	.280,814,4	.054,554,17	1.566,407,7
32 15	.533,614,5	.845,727,8	.562,868,7	.029,254,17	.002,720,315	.674,444,7	.154,272,2	.011,899,95	.284,744,5	.055,788,02	1.577,030,0
32 30	.537,299,6	.843,391,4	.567,232,0	.029,932,40	.002,825,740	.681,310,1	.156,608,6	.012,263,12	.288,690,9	.057,039,06	1.587,617,9
32 45	.540,974,5	.841,039,0	.571,595,3	.030,620,86	.002,934,370	.688,210,0	.158,961,0	.012,634,30	.292,653,4	.058,307,35	1.598,171,3
33 00	.544,639,0	.838,670,6	.575,958,7	.031,319,62	.003,046,274	.695,144,6	.161,329,4	.013,013,59	.296,631,7	.059,592,96	1.608,690,0
33 15	.548,293,2	.836,286,2	.580,322,0	.032,028,75	.003,161,524	.702,113,9	.163,713,8	.013,401,11	.300,625,5	.060,895,97	1.619,174,0
33 30	.551,937,0	.833,885,8	.584,685,3	.032,748,31	.003,280,192	.709,118,2	.166,114,2	.013,796,96	.304,634,4	.062,216,44	1.629,623,0
33 45	.555,570,2	.831,469,6	.589,048,6	.033,478,39	.003,402,351	.716,157,5	.168,530,4	.014,201,25	.308,658,3	.063,554,43	1.640,037,0
34 00	.559,192,9	.829,037,6	.593,411,9	.034,219,04	.003,528,075	.723,232,0	.170,962,4	.014,614,08	.312,696,7	.064,910,01	1.650,415,8
34 15	.562,804,9	.826,589,7	.597,775,3	.034,970,34	.003,657,440	.730,341,8	.173,410,3	.015,035,56	.316,749,4	.066,283,24	1.660,759,3
34 30	.566,406,2	.824,126,2	.602,138,6	.035,732,36	.003,790,521	.737,487,0	.175,873,8	.015,465,80	.320,816,0	.067,674,19	1.671,067,4
34 45	.569,996,8	.821,646,9	.606,501,9	.036,505,15	.003,927,395	.744,667,7	.178,353,1	.015,904,91	.324,896,3	.069,082,91	1.681,339,9
35 00	.573,576,4	.819,152,0	.610,865,2	.037,288,80	.004,068,140	.751,884,2	.180,848,0	.016,352,99	.328,989,9	.070,509,46	1.691,576,8
35 15	.577,145,2	.816,641,6	.615,228,6	.038,083,37	.004,212,835	.759,136,4	.183,358,4	.016,810,16	.333,096,6	.071,953,91	1.701,777,9
35 30	.580,703,0	.814,115,5	.619,591,9	.038,888,93	.004,361,559	.766,424,5	.185,884,5	.017,276,52	.337,215,9	.073,416,30	1.711,943,1
35 45	.584,249,7	.811,574,0	.623,955,2	.039,705,54	.004,514,394	.773,748,6	.188,426,0	.017,752,18	.341,347,7	.074,896,69	1.722,072,2
36 00	.587,785,3	.809,017,0	.628,318,5	.040,533,28	.004,671,421	.781,108,8	.190,983,0	.018,237,25	.345,491,5	.076,395,14	1.732,165,3
36 15	.591,309,6	.806,444,6	.632,681,9	.041,372,21	.004,832,722	.788,505,2	.193,555,4	.018,731,85	.349,647,1	.077,911,69	1.742,222,2
36 30	.594,822,8	.803,856,9	.637,045,2	.042,222,39	.004,998,381	.795,938,0	.196,143,1	.019,236,07	.353,814,1	.079,446,40	1.752,242,7
36 45	.598,324,6	.801,253,8	.641,408,5	.043,083,90	.005,168,483	.803,407,1	.198,746,2	.019,750,02	.357,992,3	.080,999,32	1.762,226,9
37 00	.601,815,0	.798,635,5	.645,771,8	.043,956,80	.005,343,113	.810,912,8	.201,364,5	.020,273,83	.362,181,3	.082,570,49	1.772,174,5
37 15	.605,294,0	.796,002,0	.650,135,1	.044,841,16	.005,522,357	.818,455,1	.203,998,0	.020,807,59	.366,380,8	.084,159,96	1.782,085,5
37 30	.608,761,4	.793,353,3	.654,498,5	.045,737,04	.005,706,303	.826,034,0	.206,646,7	.021,351,42	.370,590,5	.085,767,78	1.791,959,9
37 45	.612,217,3	.790,689,6	.658,861,8	.046,644,51	.005,895,039	.833,649,8	.209,310,4	.021,905,43	.374,810,0	.087,393,99	1.801,797,4
38 00	.615,661,5	.788,010,8	.663,225,1	.047,563,64	.006,088,655	.841,302,4	.211,989,2	.022,469,72	.379,039,1	.089,038,63	1.811,598,1
38 15	.619,093,9	.785,316,9	.667,588,4	.048,494,49	.006,287,240	.848,991,9	.214,683,1	.023,044,41	.383,277,3	.090,701,74	1.821,361,8
38 30	.622,514,6	.782,608,2	.671,951,8	.049,437,13	.006,490,886	.856,718,5	.217,391,8	.023,629,61	.387,524,5	.092,383,36	1.831,088,6
38 45	.625,923,5	.779,884,5	.676,315,1	.050,391,61	.006,699,685	.864,482,2	.220,115,5	.024,225,42	.391,780,2	.094,083,54	1.840,778,2
39 00	.629,320,4	.777,146,0	.680,678,4	.051,358,02	.006,913,731	.872,283,0	.222,854,0	.024,831,96	.396,044,2	.095,802,30	1.850,430,6
39 15	.632,705,3	.774,392,6	.685,041,7	.052,336,40	.007,133,116	.880,121,1	.225,607,4	.025,449,34	.400,316,0	.097,539,69	1.860,045,8
39 30	.636,078,2	.771,624,6	.689,405,1	.053,326,83	.007,357,937	.887,996,5	.228,375,4	.026,077,67	.404,595,5	.099,295,73	1.869,623,7
39 45	.639,439,0	.768,841,8	.693,768,4	.054,329,38	.007,588,289	.895,909,3	.231,158,2	.026,717,05	.408,882,2	.101,070,5	1.879,164,2
40 00	.642,787,6	.766,044,4	.698,131,7	.055,344,09	.007,824,270	.903,859,5	.233,955,6	.027,367,60	.413,175,9	.102,863,9	1.888,667,3
40 15	.646,124,0	.763,232,5	.702,495,0	.056,371,04	.008,065,977	.911,847,2	.236,767,5	.028,029,43	.417,476,2	.104,676,1	1.898,132,8
40 30	.649,448,0	.760,406,0	.706,858,3	.057,410,30	.008,313,509	.919,872,5	.239,594,0	.028,702,65	.421,782,8	.106,507,1	1.907,560,9
40 45	.652,759,8	.757,565,0	.711,221,7	.058,461,92	.008,566,966	.927,935,4	.242,435,0	.029,387,37	.426,095,3	.108,356,9	1.916,951,3
41 00	.656,059,0	.754,709,6	.715,585,0	.059,525,96	.008,826,449	.936,036,0	.245,290,4	.030,083,70	.430,413,4	.110,225,5	1.926,304,0
41 15	.659,345,8	.751,839,8	.719,948,3	.060,602,50	.009,092,060	.944,174,2	.248,160,2	.030,791,74	.434,736,9	.112,112,9	1.935,619,1
41 30	.662,620,0	.748,955,7	.724,311,6	.061,691,59	.009,363,901	.952,350,2	.251,044,3	.031,511,62	.439,065,3	.114,019,3	1.944,896,4
41 45	.665,881,7	.746,057,4	.728,675,0	.062,793,30	.009,642,076	.960,564,0	.253,942,6	.032,243,43	.443,398,4	.115,944,5	1.954,135,9
42 00	.669,130,6	.743,144,8	.733,038,3	.063,907,68	.009,926,690	.968,815,6	.256,855,2	.032,987,29	.447,735,8	.117,888,7	1.963,337,5
42 15	.672,366,8	.740,218,1	.737,401,6	.065,034,80	.010,217,85	.977,105,1	.259,781,9	.033,743,31	.452,077,1	.119,851,8	1.972,501,3
42 30	.675,590,2	.737,277,3	.741,764,9	.066,174,72	.010,515,66	.985,432,5	.262,722,7	.034,511,60	.456,422,1	.121,833,8	1.981,627,2
42 45	.678,800,7	.734,322,5	.746,128,3	.067,327,51	.010,820,23	.993,797,8	.265,677,5	.035,292,26	.460,770,5	.123,834,8	1.990,715,2
43 00	.681,998,4	.731,353,7	.750,491,6	.068,493,22	.011,131,66	1.002,201,1	.268,646,3	.036,085,42	.465,121,8	.125,854,8	1.999,765,2
43 15	.685,183,0	.728,371,0	.754,854,9	.069,671,91	.011,450,07	1.010,642,4	.271,629,0	.036,891,17	.469,475,7	.127,893,8	2.008,777,2
43 30	.688,354,6	.725,374,4	.759,218,2	.070,863,65	.011,775,57	1.019,121,7	.274,625,6	.037,709,62	.473,832,0	.129,951,7	2.017,751,2
43 45	.691,513,1	.722,364,0	.763,581,5	.072,068,49	.012,108,27	1.027,639,0	.277,636,0	.038,540,88	.478,190,3	.132,028,7	2.026,687,2
44 00	.694,658,4	.719,339,8	.767,944,9	.073,286,50	.012,448,27	1.036,194,3	.280,660,2	.039,385,07	.482,550,3	.134,124,7	2.035,585,2
44 15	.697,790,5	.716,301,9	.772,308,2	.074,517,73	.012,795,70	1.044,787,7	.283,698,1	.040,242,29	.486,911,5	.136,239,8	2.044,445,1
44 30	.700,909,3	.713,250,4	.776,671,5	.075,762,25	.013,150,67	1.053,419,2	.286,749,6	.041,112,65	.491,273,8	.138,373,8	2.053,266,9
44 45	.704,014,7	.710,185,4	.781,034,8	.077,020,12	.013,513,29	1.062,088,7	.289,814,6	.041,996,26	.495,636,7	.140,526,9	2.062,050,6
45 00	.707,106,8	.707,106,8	.785,398,2	.078,291,38	.013,883,68	1.070,796,3	.292,893,2	.042,893,22	.500,000,0	.142,699,1	2.070,796,3

TABLE 10—Continued

φ	SIN φ	COS φ	A_1	B_1	B_3		C_1	B_2		C_2	
° ′			(Radians)	(φ−SIN φ)	1ST TERM	2ND TERM	(VERS φ)	1ST TERM	2ND TERM	1ST TERM	2ND TERM
45 15	.710,185,4	.704,014,7	.789,761,5	.079,576,11	.014,261,96	1.079,542,0	.295,985,3	.043,803,64	.504,363,3	.144,890,3	2.079,503,9
45 30	.713,250,4	.700,909,3	.794,124,8	.080,874,36	.014,648,24	1.088,325,8	.299,090,7	.044,727,63	.508,726,2	.147,100,5	2.088,173,5
45 45	.716,301,9	.697,790,5	.798,488,1	.082,186,19	.015,042,64	1.097,147,6	.302,209,5	.045,665,30	.513,088,5	.149,329,7	2.096,804,9
46 00	.719,339,8	.694,658,4	.802,851,5	.083,511,66	.015,445,29	1.106,007,5	.305,341,6	.046,616,76	.517,449,7	.151,578,0	2.105,398,3
46 15	.722,364,0	.691,513,1	.807,214,8	.084,850,82	.015,856,30	1.114,905,4	.308,486,9	.047,582,10	.521,809,7	.153,845,3	2.113,953,7
46 30	.725,374,4	.688,354,6	.811,578,1	.086,203,73	.016,275,80	1.123,841,4	.311,645,4	.048,561,44	.526,168,0	.156,131,7	2.122,471,0
46 45	.728,371,0	.685,183,0	.815,941,4	.087,570,46	.016,703,90	1.132,815,5	.314,817,0	.049,554,87	.530,524,3	.158,437,0	2.130,950,2
47 00	.731,353,7	.681,998,4	.820,304,7	.088,951,05	.017,140,73	1.141,827,5	.318,001,6	.050,562,52	.534,878,2	.160,761,4	2.139,391,5
47 15	.734,322,5	.678,800,7	.824,668,1	.090,345,56	.017,586,42	1.150,877,5	.321,199,3	.051,584,48	.539,229,5	.163,104,7	2.147,794,8
47 30	.737,277,3	.675,590,2	.829,031,4	.091,754,06	.018,041,09	1.159,965,4	.324,409,8	.052,620,86	.543,577,9	.165,467,0	2.156,160,1
47 45	.740,218,1	.672,366,8	.833,394,7	.093,176,59	.018,504,87	1.169,091,3	.327,633,2	.053,671,75	.547,922,9	.167,848,3	2.164,487,5
48 00	.743,144,8	.669,130,6	.837,758,0	.094,613,22	.018,977,88	1.178,255,1	.330,869,4	.054,737,28	.552,264,2	.170,248,5	2.172,777,0
48 15	.746,057,4	.665,881,7	.842,121,4	.096,063,99	.019,460,26	1.187,456,8	.334,118,3	.055,817,53	.556,601,6	.172,667,7	2.181,028,7
48 30	.748,955,7	.662,620,0	.846,484,7	.097,528,97	.019,952,13	1.196,696,3	.337,380,0	.056,912,62	.560,934,7	.175,105,8	2.189,242,5
48 45	.751,839,8	.659,345,8	.850,848,0	.099,008,20	.020,453,62	1.205,973,6	.340,654,2	.058,022,64	.565,263,1	.177,562,8	2.197,418,5
49 00	.754,709,6	.656,059,0	.855,211,3	.100,501,8	.020,964,86	1.215,288,6	.343,941,0	.059,147,70	.569,586,6	.180,038,6	2.205,556,7
49 15	.757,565,0	.652,759,8	.859,574,7	.102,009,7	.021,485,98	1.224,641,4	.347,240,2	.060,287,89	.573,904,7	.182,533,4	2.213,657,2
49 30	.760,406,0	.649,448,0	.863,938,0	.103,532,0	.022,017,12	1.234,031,8	.350,552,0	.061,443,34	.578,217,2	.185,046,9	2.221,720,1
49 45	.763,232,5	.646,124,0	.868,301,3	.105,068,8	.022,558,42	1.243,459,8	.353,876,0	.062,614,12	.582,523,8	.187,579,3	2.229,745,4
50 00	.766,044,4	.642,787,6	.872,664,6	.106,620,2	.023,109,99	1.252,925,4	.357,212,4	.063,800,35	.586,824,1	.190,130,4	2.237,733,1
50 15	.768,841,8	.639,439,0	.877,027,9	.108,186,1	.023,671,99	1.262,428,4	.360,561,0	.065,002,12	.591,117,8	.192,700,2	2.245,683,4
50 30	.771,624,6	.636,078,2	.881,391,3	.109,766,7	.024,244,54	1.271,969,0	.363,921,8	.066,219,53	.595,404,5	.195,288,8	2.253,596,1
50 45	.774,392,6	.632,705,3	.885,754,6	.111,362,0	.024,827,78	1.281,546,8	.367,294,7	.067,452,69	.599,684,0	.197,896,1	2.261,471,5
51 00	.777,146,0	.629,320,4	.890,117,9	.112,972,0	.025,421,86	1.291,162,0	.370,679,6	.068,701,69	.603,955,8	.200,522,1	2.269,309,6
51 15	.779,884,5	.625,923,5	.894,481,2	.114,596,8	.026,026,90	1.300,814,5	.374,076,5	.069,966,62	.608,219,8	.203,166,6	2.277,110,5
51 30	.782,608,2	.622,514,6	.898,844,6	.116,236,4	.026,643,05	1.310,504,1	.377,485,4	.071,247,60	.612,475,5	.205,829,8	2.284,874,2
51 45	.785,316,9	.619,093,9	.903,207,9	.117,891,0	.027,270,45	1.320,230,8	.380,906,1	.072,544,71	.616,722,7	.208,511,5	2.292,600,7
52 00	.788,010,8	.615,661,5	.907,571,2	.119,560,5	.027,909,24	1.329,994,6	.384,338,5	.073,858,05	.620,960,9	.211,211,7	2.300,290,3
52 15	.790,689,6	.612,217,3	.911,934,5	.121,245,0	.028,559,56	1.339,795,2	.387,782,7	.075,187,72	.625,190,0	.213,930,4	2.307,942,9
52 30	.793,353,3	.608,761,4	.916,297,9	.122,944,5	.029,221,56	1.349,632,8	.391,238,6	.076,533,81	.629,409,5	.216,667,5	2.315,558,6
52 45	.796,002,0	.605,294,0	.920,661,2	.124,659,2	.029,895,38	1.359,507,1	.394,706,0	.077,896,42	.633,619,2	.219,423,0	2.323,137,6
53 00	.798,635,5	.601,815,0	.925,024,5	.126,389,0	.030,581,16	1.369,418,2	.398,185,0	.079,275,64	.637,818,7	.222,196,8	2.330,679,9
53 15	.801,253,8	.598,324,6	.929,387,8	.128,134,0	.031,279,05	1.379,365,8	.401,675,4	.080,671,56	.642,007,7	.224,989,0	2.338,185,5
53 30	.803,856,9	.594,822,8	.933,751,1	.129,894,3	.031,989,19	1.389,349,9	.405,177,2	.082,084,29	.646,185,9	.227,799,4	2.345,654,7
53 45	.806,444,6	.591,309,6	.938,114,5	.131,669,9	.032,711,74	1.399,370,5	.408,690,4	.083,513,90	.650,352,9	.230,628,0	2.353,087,4
54 00	.809,017,0	.587,785,3	.942,477,8	.133,460,8	.033,446,83	1.409,427,3	.412,214,7	.084,960,50	.654,508,5	.233,474,8	2.360,483,9
54 15	.811,574,0	.584,249,7	.946,841,1	.135,267,1	.034,194,63	1.419,520,4	.415,750,3	.086,424,17	.658,652,3	.236,339,6	2.367,844,1
54 30	.814,115,5	.580,703,0	.951,204,4	.137,088,9	.034,955,27	1.429,649,6	.419,297,0	.087,905,01	.662,784,1	.239,222,6	2.375,168,2
54 45	.816,641,6	.577,145,2	.955,567,8	.138,926,2	.035,728,91	1.439,814,8	.422,854,8	.089,403,10	.666,903,4	.242,123,5	2.382,456,3
55 00	.819,152,0	.573,576,4	.959,931,1	.140,779,0	.036,515,70	1.450,015,9	.426,423,6	.090,918,53	.671,010,1	.245,042,4	2.389,708,5
55 15	.821,646,9	.569,996,8	.964,294,4	.142,647,5	.037,315,79	1.460,252,7	.430,003,2	.092,451,39	.675,103,7	.247,979,2	2.396,924,9
55 30	.824,126,2	.566,406,2	.968,657,7	.144,531,5	.038,129,33	1.470,525,3	.433,593,8	.094,001,78	.679,184,0	.250,933,8	2.404,105,7
55 45	.826,589,7	.562,804,9	.973,021,1	.146,431,3	.038,956,48	1.480,833,3	.437,195,1	.095,569,77	.683,250,6	.253,906,1	2.411,250,9
56 00	.829,037,6	.559,192,9	.977,384,4	.148,346,8	.039,797,39	1.491,176,8	.440,807,1	.097,155,45	.687,303,3	.256,896,2	2.418,360,7
56 15	.831,469,6	.555,570,2	.981,747,7	.150,278,1	.040,652,22	1.501,555,6	.444,429,8	.098,758,91	.691,341,7	.259,904,0	2.425,435,2
56 30	.833,885,8	.551,937,0	.986,111,0	.152,225,2	.041,521,11	1.511,969,6	.448,063,0	.100,380,2	.695,365,6	.262,929,3	2.432,474,5
56 45	.836,286,2	.548,293,2	.990,474,4	.154,188,2	.042,404,23	1.522,418,7	.451,706,8	.102,019,5	.699,374,5	.265,972,2	2.439,478,7
57 00	.838,670,6	.544,639,0	.994,837,7	.156,167,1	.043,301,74	1.532,902,6	.455,361,0	.103,676,8	.703,368,3	.269,032,5	2.446,448,1
57 15	.841,039,0	.540,974,5	.999,201,0	.158,162,0	.044,213,79	1.543,421,4	.459,025,5	.105,352,2	.707,346,6	.272,110,2	2.453,382,6
57 30	.843,391,4	.537,299,6	1.003,564,3	.160,172,9	.045,140,54	1.553,974,7	.462,700,4	.107,045,8	.711,309,1	.275,205,2	2.460,282,5
57 45	.845,727,8	.533,614,5	1.007,927,6	.162,199,8	.046,082,14	1.564,562,6	.466,385,5	.108,757,7	.715,255,5	.278,317,5	2.467,147,9
58 00	.848,048,1	.529,919,3	1.012,291,0	.164,242,9	.047,038,77	1.575,184,9	.470,080,7	.110,488,0	.719,185,6	.281,447,0	2.473,979,0
58 15	.850,352,2	.526,213,9	1.016,654,3	.166,302,1	.048,010,58	1.585,841,4	.473,786,1	.112,236,6	.723,098,9	.284,593,6	2.480,775,8
58 30	.852,640,2	.522,498,6	1.021,017,6	.168,377,4	.048,997,72	1.596,532,0	.477,501,4	.114,003,8	.726,995,2	.287,757,2	2.487,538,5
58 45	.854,911,9	.518,773,3	1.025,380,9	.170,469,1	.050,000,37	1.607,256,5	.481,226,7	.115,789,6	.730,874,3	.290,937,8	2.494,267,3
59 00	.857,167,3	.515,038,1	1.029,744,3	.172,577,0	.051,018,69	1.618,014,7	.484,961,9	.117,594,0	.734,735,8	.294,135,2	2.500,962,3
59 15	.859,406,4	.511,293,1	1.034,107,6	.174,701,2	.052,052,83	1.628,806,6	.488,706,9	.119,417,2	.738,579,4	.297,349,5	2.507,623,7
59 30	.861,629,2	.507,538,4	1.038,470,9	.176,841,7	.053,102,96	1.639,632,0	.492,461,6	.121,259,2	.742,404,8	.300,580,5	2.514,251,7
59 45	.863,835,5	.503,774,0	1.042,834,2	.178,998,7	.054,169,26	1.650,490,6	.496,226,0	.123,120,1	.746,211,8	.303,828,2	2.520,846,3
60 00	.866,025,4	.500,000,0	1.047,197,6	.181,172,1	.055,251,87	1.661,382,4	.500,000,0	.125,000,0	.750,000,0	.307,092,4	2.527,407,8
60 15	.868,198,8	.496,216,5	1.051,560,9	.183,362,1	.056,350,97	1.672,307,2	.503,783,5	.126,898,9	.753,769,2	.310,373,1	2.533,936,3
60 30	.870,355,7	.492,423,6	1.055,924,2	.185,568,5	.057,466,73	1.683,264,7	.507,576,4	.128,816,9	.757,519,0	.313,670,3	2.540,432,0
60 45	.872,496,0	.488,621,2	1.060,287,5	.187,791,5	.058,599,31	1.694,255,0	.511,378,8	.130,754,1	.761,249,3	.316,983,7	2.546,895,1
61 00	.874,619,7	.484,809,6	1.064,650,8	.190,031,1	.059,748,88	1.705,277,6	.515,190,4	.132,710,6	.764,959,6	.320,313,4	2.553,325,7
61 15	.876,726,8	.480,988,8	1.069,014,2	.192,287,4	.060,915,60	1.716,332,6	.519,011,2	.134,686,3	.768,649,8	.323,659,2	2.559,724,1
61 30	.878,817,1	.477,158,8	1.073,377,5	.194,560,4	.062,099,65	1.727,419,7	.522,841,2	.136,681,5	.772,319,5	.327,021,1	2.566,090,3
61 45	.880,890,7	.473,319,7	1.077,740,8	.196,850,1	.063,301,20	1.738,538,7	.526,680,3	.138,696,1	.775,968,5	.330,399,0	2.572,424,5
62 00	.882,947,6	.469,471,6	1.082,104,1	.199,156,5	.064,520,41	1.749,689,5	.530,528,4	.140,730,2	.779,596,5	.333,792,7	2.578,727,1
62 15	.884,987,6	.465,614,5	1.086,467,5	.201,479,8	.065,757,46	1.760,871,8	.534,385,5	.142,783,9	.783,203,1	.337,202,2	2.584,998,0
62 30	.887,010,8	.461,748,6	1.090,830,8	.203,819,9	.067,012,52	1.772,085,5	.538,251,4	.144,857,3	.786,788,2	.340,627,4	2.591,237,6
62 45	.889,017,1	.457,873,9	1.095,194,1	.206,177,0	.068,285,76	1.783,330,5	.542,126,1	.146,950,3	.790,351,5	.344,068,2	2.597,446,0
63 00	.891,006,5	.453,990,5	1.099,557,4	.208,550,9	.069,577,34	1.794,606,4	.546,009,5	.149,063,2	.793,892,6	.347,524,5	2.603,623,4
63 15	.892,978,9	.450,098,4	1.103,920,8	.210,941,8	.070,887,46	1.805,913,1	.549,901,6	.151,195,9	.797,411,4	.350,996,2	2.609,769,9
63 30	.894,934,4	.446,197,8	1.108,284,1	.213,349,7	.072,216,27	1.817,250,4	.553,802,2	.153,348,4	.800,907,5	.354,483,2	2.615,885,9
63 45	.896,872,7	.442,288,7	1.112,647,4	.215,774,7	.073,563,95	1.828,618,1	.557,711,3	.155,521,0	.804,380,7	.357,985,4	2.621,971,5
64 00	.898,794,0	.438,371,1	1.117,010,7	.218,216,7	.074,930,68	1.840,016,1	.561,628,9	.157,713,5	.807,830,7	.361,502,7	2.628,026,8
64 15	.900,698,2	.434,445,3	1.121,374,0	.220,675,8	.076,316,63	1.851,444,0	.565,554,7	.159,926,1	.811,257,3	.365,035,0	2.634,052,2
64 30	.902,585,3	.430,511,1	1.125,737,4	.223,152,1	.077,721,97	1.862,901,8	.569,488,9	.162,158,8	.814,660,2	.368,582,2	2.640,047,7
64 45	.904,455,1	.426,568,7	1.130,100,7	.225,645,5	.079,146,89	1.874,389,1	.573,431,3	.164,411,7	.818,039,1	.372,144,2	2.646,013,7
65 00	.906,307,8	.422,618,3	1.134,464,0	.228,156,2	.080,591,56	1.885,905,8	.577,381,7	.166,684,8	.821,393,8	.375,720,9	2.651,950,2
65 15	.908,143,2	.418,659,7	1.138,827,3	.230,684,2	.082,056,15	1.897,451,7	.581,340,3	.168,978,3	.824,724,0	.379,312,2	2.657,857,7
65 30	.909,961,3	.414,693,2	1.143,190,7	.233,229,4	.083,540,85	1.909,026,5	.585,306,8	.171,292,0	.828,029,5	.382,917,9	2.663,736,1
65 45	.911,762,0	.410,718,9	1.147,554,0	.235,791,9	.085,045,82	1.920,630,1	.589,281,1	.173,626,1	.831,310,0	.386,538,1	2.669,585,8
66 00	.913,545,5	.406,736,6	1.151,917,3	.238,371,8	.086,571,25	1.932,262,2	.593,263,4	.175,980,7	.834,565,3	.390,172,4	2.675,407,0
66 15	.915,311,5	.402,746,7	1.156,280,6	.240,969,2	.088,117,32	1.943,922,6	.597,253,3	.178,355,8	.837,795,1	.393,821,0	2.681,199,9
66 30	.917,060,1	.398,749,1	1.160,644,0	.243,583,9	.089,684,21	1.955,611,1	.601,250,9	.180,751,3	.840,999,2	.397,483,6	2.686,964,8
66 45	.918,791,2	.394,743,9	1.165,007,3	.246,216,1	.091,272,09	1.967,327,4	.605,256,1	.183,167,5	.844,177,3	.401,160,0	2,692,701,7
67 00	.920,504,9	.390,731,1	1.169,370,6	.248,865,7	.092,881,14	1.979,071,3	.609,268,9	.185,604,3	.847,329,2	.404,850,3	2.698,411,1
67 15	.922,201,0	.386,711,0	1.173,733,9	.251,533,0	.094,511,55	1.990,842,6	.613,289,0	.188,061,7	.850,454,6	.408,554,3	2.704,093,1
67 30	.923,879,5	.382,683,4	1.178,097,2	.254,217,7	.096,163,50	2.002,641,1	.617,316,6	.190,539,9	.853,553,4	.412,271,9	2.709,747,9

TABLE 10—Continued

φ	SIN φ	COS φ	A_1	B_1	B_3		C_1	B_2		C_2	
° ′			(Radians)	(φ−SIN φ)	1ST TERM	2ND TERM	(VERS φ)	1ST TERM	2ND TERM	1ST TERM	2ND TERM
67 45	.925,540,5	.378,648,6	1.182,460,6	.256,920,1	.097,837,16	2.014,466,5	.621,351,4	.193,038,8	.856,625,2	.416,003,0	2.715,375,8
68 00	.927,183,9	.374,606,6	1.186,823,9	.259,640,0	.099,532,72	2.026,318,6	.625,393,4	.195,558,5	.859,669,9	.419,747,4	2.720,977,0
68 15	.928,809,6	.370,557,4	1.191,187,2	.262,377,7	.101,250,4	2.038,197,1	.629,442,6	.198,099,0	.862,687,2	.423,505,0	2.726,551,7
68 30	.930,417,6	.366,501,2	1.195,550,5	.265,133,0	.102,990,3	2.050,101,9	.633,498,8	.200,660,3	.865,676,9	.427,275,7	2.732,100,3
68 45	.932,007,9	.362,438,0	1.199,913,9	.267,906,0	.104,752,6	2.062,032,6	.637,562,0	.203,242,6	.868,638,7	.431,059,4	2.737,622,8
69 00	.933,580,4	.358,367,9	1.204,277,2	.270,696,8	.106,537,6	2.073,989,1	.641,632,1	.205,845,8	.871,572,4	.434,855,9	2.743,119,7
69 15	.935,135,2	.354,291,0	1.208,640,5	.273,505,3	.108,345,4	2.085,970,9	.645,709,0	.208,470,0	.874,477,9	.438,665,2	2.748,591,0
69 30	.936,672,2	.350,207,4	1.213,003,8	.276,331,6	.110,176,1	2.097,978,1	.649,792,6	.211,115,2	.877,354,8	.442,487,2	2.754,037,2
69 45	.938,191,3	.346,117,1	1.217,367,2	.279,175,8	.112,030,1	2.110,010,3	.653,882,9	.213,781,5	.880,203,0	.446,321,6	2.759,458,3
70 00	.939,692,6	.342,020,1	1.221,730,5	.282,037,9	.113,907,4	2.122,067,1	.657,979,9	.216,468,8	.883,022,2	.450,168,3	2.764,854,8
70 15	.941,176,0	.337,916,7	1.226,093,8	.284,917,8	.115,808,2	2.134,148,5	.662,083,3	.219,177,1	.885,812,3	.454,027,3	2.770,226,7
70 30	.942,641,5	.333,806,9	1.230,457,1	.287,815,6	.117,732,8	2.146,254,0	.666,193,1	.221,906,7	.888,573,0	.457,898,5	2.775,574,4
70 45	.944,089,0	.329,690,6	1.234,820,4	.290,731,4	.119,681,3	2.158,383,6	.670,309,4	.224,657,3	.891,304,1	.461,781,6	2.780,898,2
71 00	.945,518,6	.325,568,2	1.239,183,8	.293,665,2	.121,653,9	2.170,536,8	.674,431,8	.227,429,2	.894,005,4	.465,676,5	2.786,198,3
71 15	.946,930,1	.321,439,5	1.243,547,1	.296,617,0	.123,650,7	2.182,713,5	.678,560,5	.230,222,2	.896,676,7	.469,583,2	2.791,474,9
71 30	.948,323,7	.317,304,7	1.247,910,4	.299,586,8	.125,672,1	2.194,913,3	.682,695,3	.233,036,5	.899,317,8	.473,501,5	2.796,728,3
71 45	.949,699,1	.313,163,8	1.252,273,7	.302,574,6	.127,718,1	2.207,136,1	.686,836,2	.235,872,0	.901,928,4	.477,431,2	2.801,958,9
72 00	.951,056,5	.309,017,0	1.256,637,1	.305,580,5	.129,788,9	2.219,381,5	.690,983,0	.238,728,8	.904,508,5	.481,372,2	2.807,166,7
72 15	.952,395,8	.304,864,3	1.261,0004	.308,604,6	.131,884,7	2.231,649,3	.695,135,7	.241,606,8	.907,057,8	.485,324,5	2.812,352,2
72 30	.953,717,0	.300,705,8	1.265,363,7	.311,646,8	.134,005,8	2.243,939,2	.699,294,2	.244,506,2	.909,576,0	.489,287,7	2.817,515,6
72 45	.955,019,9	.296,541,6	1.269,727,0	.314,707,1	.136,152,2	2.256,250,9	.703,458,4	.247,426,9	.912,063,1	.493,262,0	2.822,657,2
73 00	.956,304,8	.292,371,7	1.274,090,4	.317,785,6	.138,324,2	2.268,584,3	.707,628,3	.250,368,9	.914,518,8	.497,247,0	2.827,777,2
73 15	.957,571,4	.288,196,3	1.278,453,7	.320,882,3	.140,522,0	2.280,938,9	.711,803,7	.253,332,3	.916,942,9	.501,242,6	2.832,875,8
73 30	.958,819,7	.284,015,3	1.282,817,0	.323,997,3	.142,745,8	2.293,314,5	.715,984,7	.256,317,0	.919,335,3	.505,248,7	2.837,953,5
73 45	.960,049,9	.279,829,0	1.287,180,3	.327,130,5	.144,995,7	2.305,710,8	.720,171,0	.259,323,1	.921,695,7	.509,265,3	2.843,010,5
74 00	.961,261,7	.275,637,4	1.291,543,6	.330,282,0	.147,271,9	2.318,127,7	.724,362,6	.262,350,6	.924,024,1	.513,292,0	2.848,046,9
74 15	.962,455,2	.271,440,4	1.295,907,0	.333,451,7	.149,574,6	2.330,564,7	.728,559,6	.265,399,5	.926,320,1	.517,328,8	2.853,063,2
74 30	.963,630,5	.267,238,4	1.300,270,3	.336,639,8	.151,904,1	2.343,021,5	.732,761,6	.268,469,8	.928,583,7	.521,375,6	2.858,059,6
74 45	.964,787,3	.263,031,2	1.304,633,6	.339,846,3	.154,260,4	2.355,498,1	.736,968,8	.271,561,5	.930,814,6	.525,432,2	2.863,036,4
75 00	.965,925,8	.258,819,0	1.308,996,9	.343,071,1	.156,643,8	2.367,993,9	.741,181,0	.274,674,6	.933,012,7	.529,498,5	2.867,993,9
75 15	.967,045,9	.254,601,9	1.313,360,3	.346,314,3	.159,054,4	2.380,508,7	.745,398,1	.277,809,1	.935,177,8	.533,574,2	2.872,932,3
75 30	.968,147,6	.250,380,0	1.317,723,6	.349,575,9	.161,492,5	2.393,042,4	.749,620,0	.280,965,1	.937,309,9	.537,659,4	2.877,852,0
75 45	.969,230,9	.246,153,3	1.322,086,9	.352,856,0	.163,958,2	2.405,594,4	.753,846,7	.284,142,4	.939,408,6	.541,753,8	2.882,753,2
76 00	.970,295,7	.241,921,9	1.326,450,2	.356,154,5	.166,451,8	2.418,164,7	.758,078,1	.287,341,2	.941,473,8	.545,857,2	2.887,636,2
76 15	.971,342,1	.237,685,9	1.330,813,6	.359,471,5	.168,973,3	2.430,752,8	.762,314,1	.290,561,4	.943,505,4	.549,969,6	2.892,501,4
76 30	.972,369,9	.233,445,4	1.335,176,9	.362,807,0	.171,523,1	2.443,358,5	.766,554,6	.293,803,0	.945,503,3	.554,090,8	2.897,349,0
76 45	.973,379,3	.229,200,4	1.339,540,2	.366,160,9	.174,101,2	2.455,981,5	.770,799,6	.297,066,0	.947,467,2	.558,220,6	2.902,179,3
77 00	.974,370,1	.224,951,1	1.343,903,5	.369,533,5	.176,707,9	2.468,621,5	.775,048,9	.300,350,4	.949,397,0	.562,359,0	2.906,992,6
77 15	.975,342,3	.220,697,4	1.348,266,8	.372,924,5	.179,343,4	2.481,278,1	.779,302,6	.303,656,2	.951,292,6	.566,505,6	2.911,789,2
77 30	.976,296,0	.216,439,6	1.352,630,2	.376,334,2	.182,007,8	2.493,951,2	.783,560,4	.306,983,4	.953,153,9	.570,660,5	2.916,569,5
77 45	.977,231,1	.212,177,7	1.356,993,5	.379,762,4	.184,701,3	2.506,640,4	.787,822,3	.310,332,0	.954,980,6	.574,823,4	2.921,333,6
78 00	.978,147,6	.207,911,7	1.361,356,8	.383,209,2	.187,424,2	2.519,345,3	.792,088,3	.313,701,9	.956,772,7	.578,994,2	2.926,082,0
78 15	.979,045,5	.203,641,8	1.365,720,1	.386,674,7	.190,176,5	2.532,065,7	.796,358,2	.317,093,2	.958,530,0	.583,172,8	2.930,814,8
78 30	.979,924,7	.199,367,9	1.370,083,5	.390,158,8	.192,958,6	2.544,801,4	.800,632,1	.320,505,9	.960,252,4	.587,358,9	2.935,532,5
78 45	.980,785,3	.195,090,3	1.374,446,8	.393,661,5	.195,770,5	2.557,551,9	.804,909,7	.323,939,8	.961,939,8	.591,552,5	2.940,235,3
79 00	.981,627,2	.190,809,0	1.378,810,1	.397,182,9	.198,612,5	2.570,316,9	.809,191,0	.327,395,0	.963,591,9	.595,753,4	2.944,923,5
79 15	.982,450,4	.186,524,0	1.383,173,4	.400,723,0	.201,484,7	2.583,096,3	.813,476,0	.330,871,6	.965,208,8	.599,961,4	2.949,597,5
79 30	.983,254,9	.182,235,5	1.387,536,8	.404,281,8	.204,387,3	2.595,889,5	.817,764,5	.334,369,4	.966,790,2	.604,176,4	2.954,257,5
79 45	.984,040,7	.177,943,5	1.391,900,1	.407,859,4	.207,320,6	2.608,696,5	.822,056,5	.337,888,4	.968,336,1	.608,398,2	2.958,903,8
80 00	.984,807,8	.173,648,2	1.396,263,4	.411,455,6	.210,284,6	2.621,516,7	.826,351,8	.341,428,7	.969,846,3	.612,626,7	2.963,536,9
80 15	.985,556,1	.169,349,5	1.400,626,7	.415,070,7	.213,279,7	2.634,350,0	.830,650,5	.344,990,1	.971,320,7	.616,861,6	2.968,156,9
80 30	.986,285,6	.165,047,6	1.404,990,0	.418,704,4	.216,305,9	2.647,196,0	.834,952,4	.348,572,8	.972,759,3	.621,103,0	2.972,764,2
80 45	.986,996,4	.160,742,6	1.409,353,4	.422,357,0	.219,363,5	2.660,054,4	.839,257,4	.352,176,5	.974,161,8	.625,350,5	2.977,359,1
81 00	.987,688,3	.156,434,5	1.413,716,7	.426,028,4	.222,452,6	2.672,924,9	.843,565,5	.355,801,4	.975,528,3	.629,604,1	2.981,941,9
81 15	.988,361,5	.152,123,4	1.418,080,0	.429,718,5	.225,573,5	2.685,807,1	.847,876,6	.359,447,4	.976,858,5	.633,863,6	2.986,512,9
81 30	.989,015,9	.147,809,4	1.422,443,3	.433,427,5	.228,726,2	2.698,700,8	.852,190,6	.363,114,4	.978,152,4	.638,128,7	2.991,072,5
81 45	.989,651,4	.143,492,6	1.426,806,7	.437,155,3	.231,911,1	2.711,605,7	.856,507,4	.366,802,4	.979,409,9	.642,399,5	2.995,621,0
82 00	.990,268,1	.139,173,1	1.431,170,0	.440,901,9	.235,128,2	2.724,521,3	.860,826,9	.370,511,5	.980,630,9	.646,675,7	3,000,158,7
82 15	.990,865,9	.134,850,9	1.435,533,3	.444,667,4	.238,377,8	2.737,447,4	.865,149,1	.374,241,5	.981,815,2	.650,957,1	3.004,685,8
82 30	.991,444,9	.130,526,2	1.439,896,6	.448,451,8	.241,660,0	2.750,383,7	.869,473,8	.377,992,4	.982,962,9	.655,243,6	3.009,202,8
82 45	.992,004,9	.126,199,0	1.444,260,0	.452,255,0	.244,975,0	2.763,329,9	.873,801,0	.381,764,1	.984,073,8	.659,535,0	3,013,709,9
83 00	.992,546,2	.121,869,3	1.448,623,3	.456,077,1	.248,323,1	2.776,285,6	.878,130,7	.385,556,7	.985,147,9	.663,831,2	3.018,207,5
83 15	.993,068,5	.117,537,4	1.452,986,6	.459,918,1	.251,704,3	2.789,250,5	.882,462,6	.389,370,1	.986,185,0	.668,132,0	3.022,695,9
83 30	.993,571,9	.113,203,2	1.457,349,9	.463,778,1	.255,118,9	2.802,224,3	.886,796,8	.393,204,3	.987,185,0	.672,437,2	3.027,175,4
83 45	.994,056,3	.108,866,9	1.461,713,2	.467,656,9	.258,567,1	2.815,206,7	.891,133,1	.397,059,1	.988,148,0	.676,746,7	3.031,646,3
84 00	.994,521,9	.104,528,5	1.466,076,6	.471,554,7	.262,049,0	2.828,197,3	.895,471,5	.400,934,6	.989,073,8	.681,060,4	3.036,109,0
84 15	.994,968,5	.100,188,1	1.470,439,9	.475,471,4	.265,564,8	2.841,195,8	.899,811,9	.404,830,8	.989,962,4	.685,378,0	3.040,563,8
84 30	.995,396,2	.095,845,75	1.474,803,2	.479,407,0	.269,114,7	2.854,201,9	.904,154,2	.408,747,5	.990,813,6	.689,699,4	3.045,010,9
84 45	.995,804,9	.091,501,62	1.479,166,5	.483,361,6	.272,698,8	2.867,215,3	.908,498,4	.412,684,7	.991,627,5	.694,024,4	3.049,450,8
85 00	.996,194,7	.087,155,74	1.483,529,9	.487,335,2	.276,317,4	2.880,235,6	.912,844,3	.416,642,3	.992,403,9	.698,352,9	3,053,883,8
85 15	.996,565,5	.082,808,21	1.487,893,2	.491,327,7	.279,970,7	2.893,262,6	.917,191,8	.420,620,4	.993,142,8	.702,684,7	3.058,310,2
85 30	.996,917,3	.078,459,10	1.492,256,5	.495,339,2	.283,658,7	2.906,295,8	.921,540,9	.424,618,8	.993,844,2	.707,019,6	3.062,730,3
85 45	.997,250,2	.074,108,49	1.496,619,8	.499,369,6	.287,381,7	2.919,335,0	.925,891,5	.428,637,5	.994,507,9	.711,357,6	3.067,144,4
86 00	.997,564,1	.069,756,47	1.500,983,2	.503,419,1	.291,139,9	2.932,379,8	.930,243,5	.432,676,5	.995,134,0	.715,698,3	3.071,552,9
86 15	.997,858,9	.065,403,13	1.505,346,5	.507,487,6	.294,933,4	2.945,429,9	.934,596,9	.436,735,7	.995,722,4	.720,041,7	3,075,956,1
86 30	.998,134,8	.061,048,54	1.509,709,8	.511,575,0	.298,762,4	2.958,484,9	.938,951,5	.440,814,9	.996,273,1	.724,387,6	3.080,354,3
86 45	.998,391,7	.056,692,79	1.514,073,1	.515,681,5	.302,627,2	2.971,544,6	.943,307,2	.444,914,2	.996,785,9	.728,735,8	3.084,747,9
87 00	.998,629,5	.052,335,96	1.518,436,4	.519,806,9	.306,527,7	2.984,608,7	.947,664,0	.449,033,6	.997,260,9	.733,086,1	3.089,137,1
87 15	.998,848,4	.047,978,13	1.522,799,8	.523,951,4	.310,464,3	2.997,676,7	.952,021,9	.453,172,8	.997,698,1	.737,438,4	3.093,522,4
87 30	.999,048,2	.043,619,39	1.527,163,1	.528,114,9	.314,437,1	3.010,748,3	.956,380,6	.457,331,9	.998,097,3	.741,792,6	3.097,904,1
87 45	.999,229,0	.039,259,82	1.531,526,4	.532,297,4	.318,446,3	3.023,823,3	.960,740,2	.461,510,9	.998,458,7	.746,148,4	3.102,282,4
88 00	.999,390,8	.034,899,50	1.535,889,7	.536,498,9	.322,492,1	3.036,901,2	.965,100,5	.465,709,5	.998,782,0	.750,505,8	3.106,657,7
88 15	.999,533,6	.030,538,51	1.540,253,1	.540,719,5	.326,574,6	3.049,981,9	.969,461,5	.469,927,8	.999,067,4	.754,864,4	3.111,030,4
88 30	.999,657,3	.026,176,95	1.544,616,4	.544,959,1	.330,693,9	3.063,064,8	.973,823,1	.474,165,7	.999,314,8	.759,224,2	3.115,400,8
88 45	.999,762,0	.021,814,89	1.548,979,7	.549,217,7	.334,850,4	3.076,149,7	.978,185,1	.478,423,1	.999,524,1	.763,585,0	3.119,769,1
89 00	.999,847,7	.017,452,41	1.553,343,0	.553,495,3	.339,044,0	3.089,236,3	.982,547,6	.482,699,9	.999,695,4	.767,946,6	3.124,135,8
89 15	.999,914,3	.013,089,60	1.557,706,4	.557,792,0	.343,275,1	3.102,324,2	.986,910,4	.486,996,1	.999,828,7	.772,308,9	3.128,501,2
89 30	.999,961,9	.008,726,535	1.562,069,7	.562,107,8	.347,543,8	3.115,413,2	.991,273,5	.491,311,5	.999,923,8	.776,671,7	3.132,865,6
89 45	.999,990,5	.004,363,309	1.566,433,0	.566,442,5	.351,850,2	3.128,502,7	.995,636,7	.495,646,2	.999,981,0	.781,034,9	3.137,229,3
90 00	1.000,000,0	.000,000,000	1.570,796,3	.570,796,3	.356,194,5	3.141,592,6	1.000,000,0	.500,000,0	1.000,000,0	.785,398,2	3.141,592,6

198. Functions Included.—Tabular quantities in table 10 represent the following trigonometric functions:

$$A_1 = \phi$$

$$B_1 = \phi - \sin\phi$$

$$B_3, 1st\ term = \phi - 2\sin\phi + \frac{\phi + \sin\phi\cos\phi}{2}$$

$$B_3, 2nd\ term = \frac{\phi + \sin\phi\cos\phi}{2} + 3\,\frac{\phi - \sin\phi\cos\phi}{2}$$

$$C_1 = vers\,\phi$$

$$B_2, 1st\ term = vers\,\phi - \frac{\sin^2\phi}{2}$$

$$B_2, 2nd\ term = \sin^2\phi$$

$$C_2, 1st\ term = \frac{\phi - \sin\phi\cos\phi}{2}$$

$$C_2, 2nd\ term = \frac{\phi - \sin\phi\cos\phi}{2} + 3\,\frac{\phi + \sin\phi\cos\phi}{2}$$

TRIGONOMETRIC FUNCTIONS AND INTEGRALS

199. Explanation.—Tables 11 to 15, inclusive, give trigonometric functions and integrals needed to evaluate load constants for use in analyzing circular arches. Quantities in the table are expressed in terms of ϕ, ϕ_1, and ϕ_0, in which ϕ is the angle from the arch point considered to the abutment; ϕ_1 is the angle under the loaded portion of the arch; and ϕ_0 is the angle between the point considered and the point where the loading begins. It is evident that $\phi = \phi_1 + \phi_0$, see table 4.

Integrals shown in tables 13, 14, and 15 can be obtained in trigonometric form. In many cases, however, the value obtained from the formulas is given as a small difference between two relatively large numbers. This required the use of trigonometric functions expressed to twelve or more places in order to obtain sufficient accuracy in results. Terms that required more than twelve places were evaluated by using series in which only enough terms were used to give the required accuracy. All series used were alternating, so that the error involved by stopping at any given term was less than the magnitude of the succeeding term.

200. Arrangement.—The arrangement in tables 11, 12, and 13 is self-explanatory. Tabular quantities are values of the functions of ϕ_1 given as column headings. In table 13 tabular quantities in five columns are also values of integrals of the functions of ϕ_1 given at the bottom of the table.

Tables 14 and 15 give quantities needed in evaluating the integrals of functions of ϕ_1 and ϕ, shown at the tops and bottoms of the tables. In each case the evaluation of the integral, after substituting $\phi_1 + \phi_0$ for ϕ, is a trigonometric function of ϕ_1 multiplied by $\pm \sin \phi_0$, plus a trigonometric function of ϕ_1 multiplied by $+ \cos \phi_0$. Values of trigonometric functions of ϕ_1 are given in the tables and the function of ϕ_0 by which each value is to be multiplied is given at the top or bottom of the column. For instance, to obtain the integral of $(\phi_1 - \sin \phi_1) \sin \phi \, d\phi_1$ for given values of ϕ_1 and ϕ_0, take the quantity opposite the given value of ϕ_1 in the second column of table 14, multiply by $\sin \phi_0$ and add the quantity in the third column multiplied by $\cos \phi_0$. If the integral of $(\phi_1 - \sin \phi_1) \cos \phi \, d\phi_1$ is desired, multiply the quantity in the second column of table 14 by $\cos \phi_0$ and subtract the quantity in the third column multiplied by $\sin \phi_0$. The same procedure is followed in obtaining values of other integrals in tables 14 and 15.

201. Formulas in Trigonometric Form.—Integrals expressed in trigonometric form in tables 13, 14, and 15 are as follows:

$$\int_0^{\phi_1} \phi_1 \, d\phi_1 = \frac{\phi_1^2}{2}$$

$$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1 = \frac{\phi_1^3}{6}$$

$$\int_0^{\phi_1} vers \, \phi_1 \, d\phi_1 = \phi_1 - \sin \phi_1$$

$$\int_0^{\phi_1} [\phi_1 - \sin \phi_1] \, d\phi_1 = \frac{\phi_1^2}{2} - vers \, \phi_1$$

$$\int_0^{\phi_1} \left[\frac{\phi_1^2}{2} - vers \, \phi_1 \right] d\phi_1 = \sin \phi_1 - \phi_1 + \frac{\phi_1^3}{6}$$

$$\int_0^{\phi_1} [\phi_1 - \sin \phi_1] \sin \phi \, d\phi_1 = \sin \phi_0 \left[\phi_1 \sin \phi_1 - vers \, \phi_1 - \frac{\sin^2 \phi_1}{2} \right]$$

$$+ \cos \phi_0 \left[\sin \phi_1 - \frac{\phi_1 - \sin \phi_1 \cos \phi_1}{2} - \phi_1 \cos \phi_1 \right]$$

$$\int_0^{\phi_1} vers\ \phi_1 \sin\phi\, d\phi_1 = \sin\phi_0 \left[\sin\phi_1 - \frac{\phi_1 + \sin\phi_1 \cos\phi_1}{2}\right] + \cos\phi_0 \left[vers\ \phi_1 - \frac{\sin^2\phi_1}{2}\right]$$

$$\int_0^{\phi_1} \sin\phi_1 \sin\phi\, d\phi_1 = \sin\phi_0 \left[\frac{\sin^2\phi_1}{2}\right] + \cos\phi_0 \left[\frac{\phi_1 - \sin\phi_1 \cos\phi_1}{2}\right]$$

$$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi\, d\phi_1 = \sin\phi_0 \left[\frac{\phi_1^2}{2}\sin\phi_1 + \phi_1 \cos\phi_1 - \sin\phi_1\right] + \cos\phi_0 \left[\phi_1 \sin\phi_1 - \frac{\phi_1^2}{2}\cos\phi_1 - vers\ \phi_1\right]$$

$$\int_0^{\phi_1} \phi_1 \sin\phi\, d\phi_1 = \sin\phi_0\ [\phi_1 \sin\phi_1 - vers\ \phi_1] + \cos\phi_0\ [\sin\phi_1 - \phi_1 \cos\phi_1]$$

$$\int_0^{\phi_1} \left[\frac{\phi_1^2}{2} - vers\ \phi_1\right] \sin\phi\, d\phi_1 = \sin\phi_0 \left[\frac{\phi_1^2}{2}\sin\phi_1 + \phi_1 \cos\phi_1 - 2\sin\phi_1 + \frac{\phi_1 + \sin\phi_1 \cos\phi_1}{2}\right] + \cos\phi_0 \left[\phi_1 \sin\phi_1 - \frac{\phi_1^2}{2}\cos\phi_1 - 2\ vers\ \phi_1 + \frac{\sin^2\phi_1}{2}\right]$$

202. Formulas in Power Series.—The integrals expressed in terms of power series are given below. Developments for *vers* ϕ_1, *sin* ϕ_1, $\int_0^{\phi_1} \phi_1\, d\phi_1$, and $\int_0^{\phi_1} \frac{\phi_1^2}{2} d\phi_1$ are added for the sake of completeness.

$$\int_0^{\phi_1} \phi_1\, d\phi_1 = \frac{\phi_1^2}{2} = 0.500{,}000{,}000{,}000\ \phi_1^2$$

$$\int_0^{\phi_1} \frac{\phi_1^2}{2} d\phi_1 = \frac{\phi_1^3}{6} = 0.166{,}666{,}666{,}666\ \phi_1^3$$

$$vers\ \phi_1 = +0.500,000,000,000\ \phi_1^2 - 0.041,666,666,666\ \phi_1^4 + 0.001,388,888,888\ \phi_1^6 - 0.000,024,801,587\ \phi_1^8 + 0.000,000,275,573\ \phi_1^{10} - 0.000,000,002,087\ \phi_1^{12} + 0.000,000,000,011\ \phi_1^{14} - \ldots\ldots$$

$$\sin \phi_1 = +1.000,000,000,000\ \phi_1 - 0.166,666,666,666\ \phi_1^3 + 0.008,333,333,333\ \phi_1^5 - 0.000,198,412,698\ \phi_1^7 + 0.000,002,755,731\ \phi_1^9 - 0.000,000,025,052\ \phi_1^{11} + 0.000,000,000,160\ \phi_1^{13} - 0.000,000,000,000,7\ \phi_1^{15} + \ldots$$

$$\int_0^{\phi_1} vers\ \phi_1\, d\phi_1 = +0.166,666,666,666\ \phi_1^3 - 0.008,333,333,333\ \phi_1^5 + 0.000,198,412,698\ \phi_1^7 - 0.000,002,755,731\ \phi_1^9 + 0.000,000,025,052\ \phi_1^{11} - 0.000,000,000,160\ \phi_1^{13} + 0.000,000,000,000,7\ \phi_1^{15} - \ldots\ldots$$

$$\int_0^{\phi_1} [\phi_1 - \sin \phi_1]\, d\phi_1 = +0.041,666,666,666\ \phi_1^4 - 0.001,388,888,888\ \phi_1^6 + 0.000,024,801,587\ \phi_1^8 - 0.000,000,275,573\ \phi_1^{10} + 0.000,000,002,087\ \phi_1^{12} - 0.000,000,000,011\ \phi_1^{14} + \ldots\ldots$$

$$\int_0^{\phi_1} \left[\frac{\phi_1^2}{2} - vers\ \phi_1\right] d\phi_1 = +0.008,333,333,333\ \phi_1^5 - 0.000,198,412,698\ \phi_1^7 + 0.000,002,755,731\ \phi_1^9 - 0.000,000,025,052\ \phi_1^{11} + 0.000,000,000,160\ \phi_1^{13} - 0.000,000,000,000,7\ \phi_1^{15} + \ldots\ldots$$

$$\int_0^{\phi_1} [\phi_1 - \sin \phi_1] \sin \phi\, d\phi_1 = \sin \phi_0\, [+0.041,666,666,66\ \phi_1^4 - 0.015,277,777,77\ \phi_1^6 + 0.001,413,690,47\ \phi_1^8 - 0.000,068,066,57\ \phi_1^{10} + 0.000,002,114,81\ \phi_1^{12} - 0.000,000,046,80\ \phi_1^{14} + \ldots\ldots]$$

$$+ \cos \phi_0\, [+0.033,333,333,33\ \phi_1^5 - 0.005,158,730,15\ \phi_1^7 + 0.000,330,687,83\ \phi_1^9 - 0.000,012,576,11\ \phi_1^{11} + 0.000,000,326,96\ \phi_1^{13} - 0.000,000,006,2\ \phi_1^{15} + \ldots\ldots]$$

$$\int_0^{\phi_1} vers\ \phi_1 \sin \phi\, d\phi_1 = \sin \phi_0\, [0.166,666,666,66\ \phi_1^3 - 0.058,333,333,33\ \phi_1^5 + 0.006,150,793,65\ \phi_1^7 - 0.000,349,977,95\ \phi_1^9 + 0.000,012,801,62\ \phi_1^{11} - 0.000,000,328,72\ \phi_1^{13} + 0.000,000,006,2\ \phi_1^{15} - \ldots\ldots]$$

$$+ \cos \phi_0\, [+0.125,000,000,00\ \phi_1^4 - 0.020,833,333,33\ \phi_1^6 + 0.001,562,500,00\ \phi_1^8 - 0.000,070,271,16\ \phi_1^{10} + 0.000,002,135,69\ \phi_1^{12} - 0.000,000,046,97\ \phi_1^{14} + \ldots\ldots]$$

$$\int_0^{\phi_1} \sin\phi_1 \sin\phi \, d\phi_1 = \sin\phi_0 \, [+ 0.500{,}000{,}000{,}000\ \phi_1^2 - 0.166{,}666{,}666{,}666\ \phi_1^4 + 0.022{,}222{,}222{,}222\ \phi_1^6 - 0.001{,}587{,}301{,}587\ \phi_1^8 + 0.000{,}070{,}546{,}737\ \phi_1^{10} - 0.000{,}002{,}137{,}779\ \phi_1^{12} + 0.000{,}000{,}046{,}984\ \phi_1^{14} - 0.000{,}000{,}000{,}783\ \phi_1^{16} + \ldots .]$$

$$+ \cos\phi_0 \, [+0.333{,}333{,}333{,}333\ \phi_1^3 - 0.066{,}666{,}666{,}666\ \phi_1^5 + 0.006{,}349{,}206{,}336\ \phi_1^7 - 0.000{,}352{,}733{,}686\ \phi_1^9 + 0.000{,}012{,}826{,}679\ \phi_1^{11} - 0.000{,}000{,}328{,}889\ \phi_1^{13} + 0.000{,}000{,}006{,}264\ \phi_1^{15} - 0.000{,}000{,}000{,}092\ \phi_1^{17} + \ldots .]$$

$$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1 = \sin\phi_0 \, [+ 0.166{,}666{,}666{,}666\ \phi_1^3 - 0.050{,}000{,}000{,}000\ \phi_1^5 + 0.002{,}976{,}190{,}470\ \phi_1^7 - 0.000{,}077{,}160{,}493\ \phi_1^9 + 0.000{,}001{,}127{,}344\ \phi_1^{11} - 0.000{,}000{,}010{,}598\ \phi_1^{13} + 0.000{,}000{,}000{,}069\ \phi_1^{15} - \ldots .]$$

$$+ \cos\phi_0 \, [+0.125{,}000{,}000{,}000\ \phi_1^4 - 0.013{,}888{,}888{,}888\ \phi_1^6 + 0.000{,}520{,}833{,}333\ \phi_1^8 - 0.000{,}009{,}920{,}634\ \phi_1^{10} + 0.000{,}000{,}114{,}822\ \phi_1^{12} - 0.000{,}000{,}000{,}894\ \phi_1^{14} + 0.000{,}000{,}000{,}005\ \phi_1^{16} - \ldots .]$$

$$\int_0^{\phi_1} \phi_1 \sin\phi \, d\phi_1 = \sin\phi_0 \, [+ 0.500{,}000{,}000{,}000\ \phi_1^2 - 0.125{,}000{,}000{,}000\ \phi_1^4 + 0.006{,}944{,}444{,}444\ \phi_1^6 - 0.000{,}173{,}611{,}111\ \phi_1^8 + 0.000{,}002{,}480{,}158\ \phi_1^{10} - 0.000{,}000{,}022{,}964\ \phi_1^{12} + 0.000{,}000{,}000{,}149\ \phi_1^{14} - \ldots .]$$

$$+ \cos\phi_0 \, [+0.333{,}333{,}333{,}333\ \phi_1^3 - 0.033{,}333{,}333{,}333\ \phi_1^5 + 0.001{,}190{,}476{,}188\ \phi_1^7 - 0.000{,}022{,}045{,}855\ \phi_1^9 + 0.000{,}000{,}250{,}521\ \phi_1^{11} - 0.000{,}000{,}001{,}927\ \phi_1^{13} + 0.000{,}000{,}000{,}010\ \phi_1^{15} - \ldots .]$$

$$\int_0^{\phi_1} \left[\frac{\phi_1^2}{2} - \mathrm{vers}\,\phi_1\right] \sin\phi \, d\phi_1 = \sin\phi_0 \, [+ 0.008{,}333{,}333{,}333\ \phi_1^5 - 0.003{,}174{,}603{,}168\ \phi_1^7 + 0.000{,}272{,}817{,}460\ \phi_1^9 - 0.000{,}011{,}674{,}282\ \phi_1^{11} + 0.000{,}000{,}318{,}129\ \phi_1^{13} - 0.000{,}000{,}006{,}194\ \phi_1^{15} + 0.000{,}000{,}000{,}091\ \phi_1^{17} - \ldots .]$$

$$+ \cos\phi_0 \, [+0.006{,}944{,}444{,}444\ \phi_1^6 - 0.001{,}041{,}666{,}666\ \phi_1^8 + 0.000{,}060{,}350{,}529\ \phi_1^{10} - 0.000{,}002{,}020{,}870\ \phi_1^{12} + 0.000{,}000{,}046{,}077\ \phi_1^{14} - 0.000{,}000{,}000{,}778\ \phi_1^{16} + 0.000{,}000{,}000{,}010\ \phi_1^{18} - \ldots .]$$

TABLE 11—TRIGONOMETRIC FUNCTIONS FOR ARCH LOAD CONSTANTS

ϕ_1 ° ′	ϕ_1 RADIANS	SIN ϕ_1	COS ϕ_1	VERS ϕ_1	ϕ_1 SIN ϕ_1	ϕ_1 COS ϕ_1
0 15	.004,363,323,130	.004,363,309,285	.999,990,480,721	.0³009,519,279	.0³019,038,528	.004,363,281,594
0 30	.008,726,646,260	.008,726,535,498	.999,961,923,064	.0³038,076,936	.0³076,153,388	.008,726,313,976
0 45	.013,089,969,390	.013,089,595,571	.999,914,327,574	.0³085,672,426	.0³171,342,405	.013,088,847,941
1 00	.017,453,292,520	.017,452,406,437	.999,847,695,156	.0³152,304,844	.0³304,601,955	.017,450,634,299
1 15	.021,816,615,650	.021,814,885,035	.999,762,027,080	.0³237,972,920	.0³475,926,962	.021,811,423,886
1 30	.026,179,938,780	.026,176,948,308	.999,657,324,976	.0³342,675,024	.0³685,310,904	.026,170,967,569
1 45	.030,543,261,910	.030,538,513,210	.999,533,590,837	.0³466,409,163	.0³932,745,807	.030,529,016,253
2 00	.034,906,585,040	.034,899,496,703	.999,390,827,019	.0³609,172,981	.001,218,222,250	.034,885,320,892
2 15	.039,269,908,170	.039,259,815,759	.999,229,036,241	.0³770,963,759	.001,541,729,360	.039,239,632,494
2 30	.043,633,231,300	.043,619,387,365	.999,048,221,582	.0³951,778,418	.001,903,254,818	.043,591,702,132
2 45	.047,996,554,430	.047,978,128,521	.998,848,386,485	.001,151,613,515	.002,302,784,857	.047,941,280,949
3 00	.052,359,877,560	.052,335,956,243	.998,629,534,755	.001,370,465,245	.002,740,304,261	.052,288,120,167
3 15	.056,723,200,690	.056,692,787,563	.998,391,670,557	.001,608,329,443	.003,215,796,367	.056,631,971,096
3 30	.061,086,523,820	.061,048,539,535	.998,134,798,422	.001,865,201,578	.003,729,243,064	.060,972,585,139
3 45	.065,449,846,950	.065,403,129,230	.997,858,923,239	.002,141,076,761	.004,280,624,798	.065,309,713,803
4 00	.069,813,170,080	.069,756,473,744	.997,564,050,260	.002,435,949,740	.004,869,920,566	.069,643,108,706
4 15	.074,176,493,210	.074,108,490,195	.997,250,185,099	.002,749,814,901	.005,497,107,920	.073,972,521,583
4 30	.078,539,816,340	.078,459,095,728	.996,917,333,733	.003,082,666,267	.006,162,162,969	.078,297,704,297
4 45	.082,903,139,470	.082,808,207,512	.996,565,502,498	.003,434,497,502	.006,865,060,377	.082,618,408,844
5 00	.087,266,462,600	.087,155,742,748	.996,194,698,092	.003,805,301,908	.007,605,773,365	.086,934,387,363
5 15	.091,629,785,730	.091,501,618,663	.995,804,927,575	.004,195,072,425	.008,384,273,712	.091,245,392,142
5 30	.095,993,108,860	.095,845,752,520	.995,396,198,367	.004,603,801,633	.009,200,531,755	.095,551,175,628
5 45	.100,356,431,990	.100,188,061,612	.994,968,518,251	.005,031,481,749	.010,054,516,391	.099,851,490,434
6 00	.104,719,755,120	.104,528,463,268	.994,521,895,368	.005,478,104,632	.010,946,195,076	.104,146,089,344
6 15	.109,083,078,250	.108,866,874,852	.994,056,338,222	.005,943,661,778	.011,875,533,828	.108,434,725,327
6 30	.113,446,401,380	.113,203,213,768	.993,571,855,677	.006,428,144,323	.012,842,497,227	.112,717,151,539
6 45	.117,809,724,510	.117,537,397,458	.993,068,456,955	.006,931,543,045	.013,847,048,414	.116,993,121,333
7 00	.122,173,047,640	.121,869,343,405	.992,546,151,641	.007,453,848,359	.014,889,149,098	.121,262,388,269
7 15	.126,536,370,770	.126,198,969,136	.992,004,949,680	.007,995,050,320	.015,968,759,549	.125,524,706,118
7 30	.130,899,693,900	.130,526,192,220	.991,444,861,374	.008,555,138,626	.017,085 838 608	.129,779,828,873
7 45	.135,263,017,030	.134,850,930,274	.990,865,897,387	.009,134,102,613	.018,240,343,678	.134,027,510,753
8 00	.139,626,340,160	.139,173,100,960	.990,268,068,742	.009,731,931,258	.019,432,230,736	.138,267,506,216
8 15	.143,989,663,290	.143,492,621,991	.989,651,386,820	.010,348,613,180	.020,661,454,325	.142,499,569,963
8 30	.148,352,986,419	.147,809,411,130	.989,015,863,362	.010,984,136,638	.021,927,967,562	.146,723,456,946
8 45	.152,716,309,549	.152,123,386,190	.988,361,510,468	.011,638,489,532	.023,231,722,135	.150,938,922,379
9 00	.157,079,632,679	.156,434,465,040	.987,688,340,595	.012,311,659,405	.024,572,668,307	.155,145,721,742
9 15	.161,442,955,809	.160,742,565,604	.986,996,366,560	.013,003,633,440	.025,950,754,915	.159,343,610,790
9 30	.165,806,278,939	.165,047,605,861	.986,285,601,537	.013,714,398,463	.027,365,929,376	.163,532,345,562
9 45	.170,169,602,069	.169,349,503,849	.985,556,059,058	.014,443,940,942	.028,818,137,681	.167,711,682,387
10 00	.174,532,925,199	.173,648,177,667	.984,807,753,012	.015,192,246,988	.030,307,324,404	.171,881,377,892
10 15	.178,896,248,329	.177,943,545,474	.984,040,697,646	.015,959,302,354	.031,833,432,700	.176,041,189,012
10 30	.183,259,571,459	.182,235,525,492	.983,254,907,564	.016,745,092,436	.033,396,404,306	.180,190,872,995
10 45	.187,622,894,589	.186,524,036,009	.982,450,397,726	.017,549,602,274	.034,996,179,546	.184,330,187,411
11 00	.191,986,217,719	.190,808,995,377	.981,627,183,448	.018,372,816,552	.036,632,697,329	.188,458,890,160
11 15	.196,349,540,849	.195,090,322,016	.980,785,280,403	.019,214,719,597	.038,305,895,152	.192,576,739,479
11 30	.200,712,863,979	.199,367,934,417	.979,924,704,621	.020,075,295,379	.040,015,709,102	.196,683,493,948
11 45	.205,076,187,109	.203,641,751,140	.979,045,472,485	.020,954,527,515	.041,762,073,860	.200,778,912,504
12 00	.209,439,510,239	.207,911,690,818	.978,147,600,734	.021,852,399,266	.043,544,922,698	.204,862,754,439
12 15	.213,802,833,369	.212,177,672,156	.977,231,106,463	.022,768,893,537	.045,364,187,485	.208,934,779,418
12 30	.218,166,156,499	.216,439,613,938	.976,296,007,120	.023,703,992,880	.047,219,798,687	.212,994,747,479
12 45	.222,529,479,629	.220,697,435,022	.975,342,320,509	.024,657,679,491	.049,111,685,371	.217,042,419,043
13 00	.226,892,802,759	.224,951,054,344	.974,370,064,785	.025,629,935,215	.051,039,775,204	.221,077,554,924
13 15	.231,256,125,889	.229,200,390,922	.973,379,258,460	.026,620,741,540	.053,003,994,457	.225,099,916,332
13 30	.235,619,449,019	.233,445,363,856	.972,369,920,398	.027,630,079,602	.055,004,268,008	.229,109,264,887
13 45	.239,982,772,149	.237,685,892,326	.971,342,069,813	.028,657,930,187	.057,040,519,341	.233,105,362,619
14 00	.244,346,095,279	.241,921,895,600	.970,295,726,276	.029,704,273,724	.059,112,670,552	.237,087,971,981
14 15	.248,709,418,409	.246,153,293,029	.969,230,909,707	.030,769,090,293	.061,220,642,349	.241,056,855,857
14 30	.253,072,741,539	.250,380,004,054	.968,147,640,378	.031,852,359,622	.063,364,354,052	.245,011,777,565
14 45	.257,436,064,669	.254,601,948,206	.967,045,938,914	.032,954,061,086	.065,543,723,603	.248,952,500,868
15 00	.261,799,387,799	.258,819,045,103	.965,925,826,289	.034,074,173,711	.067,758,667,559	.252,878,789,982
15 15	.266,162,710,929	.263,031,214,458	.964,787,323,829	.035,212,676,171	.070,009,101,099	.256,790,409,580
15 30	.270,526,034,059	.267,238,376,078	.963,630,453,209	.036,369,546,791	.072,294,938,029	.260,687,124,805
15 45	.274,889,357,189	.271,440,449,865	.962,455,236,454	.037,544,763,546	.074,616,090,778	.264,568,701,272
16 00	.279,252,680,319	.275,637,355,817	.961,261,695,938	.038,738,304,062	.076,972,470,408	.268,434,905,079
16 15	.283,616,003,449	.279,829,014,031	.960,049,854,386	.039,950,145,614	.079,363,986,609	.272,285,502,813
16 30	.287,979,326,579	.284,015,344,704	.958,819,734,868	.041,180,265,132	.081,790,547,706	.276,120,261,558
16 45	.292,342,649,709	.288,196,268,134	.957,571,360,805	.042,428,639,195	.084,252,060,663	.279,938,948,903
17 00	.296,705,972,839	.292,371,704,723	.956,304,755,963	.043,695,244,037	.086,748,431,080	.283,741,332,949
17 15	.301,069,295,969	.296,541,574,976	.955,019,944,457	.044,980,055,543	.089,279,563,204	.287,527,182,314
17 30	.305,432,619,099	.300,705,799,504	.953,716,950,748	.046,283,049,252	.091,845,359,921	.291,296,266,146
17 45	.309,795,942,229	.304,864,299,028	.952,395,799,643	.047,604,200,357	.094,445,722,769	.295,048,354,125
18 00	.314,159,265,359	.309,016,994,375	.951,056,516,295	.048,943,483,705	.097,080,551,936	.298,783,216,474
18 15	.318,522,588,489	.313,163,806,484	.949,699,126,202	.050,300,873,798	.099,749,746,262	.302,500,623,964
18 30	.322,885,911,619	.317,304,656,405	.948,323,655,206	.051,676,344,794	.102,453,203,244	.306,200,347,921
18 45	.327,249,234,749	.321,439,465,303	.946,930,129,495	.053,069,870,505	.105,190,819,039	.309,882,160,238
19 00	.331,612,557,879	.325,568,154,457	.945,518,575,599	.054,481,424,401	.107,962,488,463	.313,545,833,376
19 15	.335,975,881,009	.329,690,645,263	.944,089,020,393	.055,910,979,607	.110,768,105,003	.317,191,140,377
19 30	.340,339,204,139	.333,806,859,234	.942,641,491,092	.057,358,508,908	.113,607,560,808	.320,817,854,867
19 45	.344,702,527,269	.337,916,718,003	.941,176,015,256	.058,823,984,744	.116,480,746,702	.324,425,751,064
20 00	.349,065,850,399	.342,020,143,326	.939,692,620,786	.060,307,379,214	.119,387,552,184	.328,014,603,788
20 15	.353,429,173,529	.346,117,057,077	.938,191,335,922	.061,808,664,078	.122,327,865,427	.331,584,188,467
20 30	.357,792,496,659	.350,207,381,259	.936,672,189,248	.063,327,810,752	.125,301,573,289	.335,134,281,142
20 45	.362,155,819,789	.354,291,037,998	.935,135,209,686	.064,864,790,314	.128,308,561,310	.338,664,658,477
21 00	.366,519,142,919	.358,367,949,545	.933,580,426,497	.066,419,573,503	.131,348,713,717	.342,175,097,766
21 15	.370,882,466,049	.362,438,038,284	.932,007,869,283	.067,992,130,717	.134,421,913,429	.345,665,376,937
21 30	.375,245,789,179	.366,501,226,724	.930,417,567,982	.069,582,432,018	.137,528,042,057	.349,135,274,563
21 45	.379,609,112,309	.370,557,437,510	.928,809,552,872	.071,190,447,128	.140,666,979,913	.352,584,569,870
22 00	.383,972,435,439	.374,606,593,416	.927 183 854 567	.072 816 145 433	.143 838 606 005	.356 013 042 738
22 15	.388,335,758,569	.378,648,617,352	.925,540,504,018	.074,459,495,982	.147,042,798,050	.359,420,473,714
22 30	.392,699,081,699	.382,683,432,365	.923,879,532,511	.076,120,467,489	.150,279,432,471	.362,806,644,018

TABLE 11—Continued

ϕ_1 ° ′	ϕ_1 RADIANS	SIN ϕ_1	COS ϕ_1	VERS ϕ_1	ϕ_1 SIN ϕ_1	ϕ_1 COS ϕ_1
22 45	.397,062,404,829	.386,710,961,637	.922,200,971,670	.077,799,028,330	.153,548,384,401	.366,171,335,547
23 00	.401,425,727,959	.390,731,128,489	.920,504,853,452	.079,495,146,548	.156,849,527,690	.369,514,330,887
23 15	.405,789,051,089	.394,743,856,384	.918,791,210,149	.081,208,789,851	.160,182,734,905	.372,835,413,315
23 30	.410,152,374,219	.398,749,068,925	.917,060,074,385	.082,939,925,615	.163,547,877,337	.376,134,366,810
23 45	.414,515,697,349	.402,746,689,859	.915,311,479,119	.084,688,520,881	.166,944,825,002	.379,410,976,059
24 00	.418,879,020,479	.406,736,643,076	.913,545,457,643	.086,454,542,357	.170,373,446,645	.382,665,026,461
24 15	.423,242,343,609	.410,718,852,613	.911,762,043,577	.088,237,956,423	.173,833,609,744	.385,896,304,137
24 30	.427,605,666,739	.414,693,242,656	.909,961,270,877	.090,038,729,123	.177,325,180,518	.389,104,595,940
24 45	.431,968,989,869	.418,659,737,537	.908,143,173,825	.091,856,826,175	.180,848,023,923	.392,289,689,454
25 00	.436,332,312,999	.422,618,261,741	.906,307,787,037	.093,692,212,963	.184,402,003,661	.395,451,373,007
25 15	.440,695,636,128	.426,568,739,901	.904,455,145,454	.095,544,854,546	.187,986,982,183	.398,589,435,675
25 30	.445,058,959,258	.430,511,096,808	.902,585,284,350	.097,414,715,650	.191,602,820,694	.401,703,667,294
25 45	.449,422,282,388	.434,445,257,404	.900,698,239,323	.099,301,760,677	.195,249,379,155	.404,793,858,459
26 00	.453,785,605,518	.438,371,146,789	.898,794,046,299	.101,205,953,701	.198,926,516,287	.407,859,800,536
26 15	.458,148,928,648	.442,288,690,219	.896,872,741,533	.103,127,258,467	.202,634,089,577	.410,901,285,667
26 30	.462,512,251,778	.446,197,813,110	.894,934,361,602	.105,065,638,398	.206,371,955,280	.413,918,106,778
26 45	.466,875,574,908	.450,098,441,037	.892,978,943,411	.107,021,056,589	.210,139,968,424	.416,910,057,586
27 00	.471,238,898,038	.453,990,499,740	.891,006,524,188	.108,993,475,812	.213,937,982,817	.419,876,932,603
27 15	.475,602,221,168	.457,873,915,117	.889,017,141,486	.110,982,858,514	.217,765,851,045	.422,818,527,147
27 30	.479,965,544,298	.461,748,613,235	.887,010,833,178	.112,989,166,822	.221,623,424,480	.425,734,637,345
27 45	.484,328,867,428	.465,614,520,325	.884,987,637,463	.115,012,362,537	.225,510,553,287	.428,625,060,140
28 00	.488,692,190,558	.469,471,562,786	.882,947,592,859	.117,052,407,141	.229,427,086,423	.431,489,593,302
28 15	.493,055,513,688	.473,319,667,185	.880,890,738,205	.119,109,261,795	.233,372,871,643	.434,328,035,429
28 30	.497,418,836,818	.477,158,760,260	.878,817,112,662	.121,182,887,338	.237,347,755,506	.437,140,185,956
28 45	.501,782,159,948	.480,988,768,919	.876,726,755,708	.123,273,244,292	.241,351,583,379	.439,925,845,163
29 00	.506,145,483,078	.484,809,620,246	.874,619,707,139	.125,380,292,861	.245,384,199,440	.442,684,814,179
29 15	.510,508,806,208	.488,621,241,497	.872,496,007,073	.127,503,992,927	.249,445,446,685	.445,416,894,992
29 30	.514,872,129,338	.492,423,560,103	.870,355,695,940	.129,644,304,060	.253,535,166,926	.448,121,890,450
29 45	.519,235,452,468	.496,216,503,675	.868,198,814,489	.131,801,185,511	.257,653,200,808	.450,799,604,273
30 00	.523,598,775,598	.500,000,000,000	.866,025,403,784	.133,974,596,216	.261,799,387,799	.453,449,841,058
30 15	.527,962,098,728	.503,773,977,046	.863,835,505,204	.136,164,494,796	.265,973,566,206	.456,072,406,283
30 30	.532,325,421,858	.507,538,362,961	.861,629,160,442	.138,370,839,558	.270,175,573,172	.458,667,106,317
30 45	.536,688,744,988	.511,293,086,077	.859,406,411,501	.140,593,588,499	.274,405,244,688	.461,233,748,423
31 00	.541,052,068,118	.515,038,074,910	.857,167,300,702	.142,832,699,298	.278,662,415,590	.463,772,140,768
31 15	.545,415,391,248	.518,773,258,161	.854,911,870,673	.145,088,129,327	.282,946,919,569	.466,282,092,426
31 30	.549,778,714,378	.522,498,564,716	.852,640,164,354	.147,359,835,646	.287,258,589,174	.468,763,413,386
31 45	.554,142,037,508	.526,213,923,652	.850,352,224,996	.149,647,775,004	.291,597,255,818	.471,215,914,559
32 00	.558,505,360,638	.529,919,264,233	.848,048,096,156	.151,951,903,844	.295,962,749,779	.473,639,407,782
32 15	.562,868,683,768	.533,614,515,916	.845,727,821,704	.154,272,178,296	.300,354,900,213	.476,033,705,829
32 30	.567,232,006,898	.537,299,608,347	.843,391,445,813	.156,608,554,187	.304,773,535,148	.478,398,622,409
32 45	.571,595,330,028	.540,974,471,368	.841,039,012,964	.158,960,987,036	.309,218,481,498	.480,733,972,182
33 00	.575,958,653,158	.544,639,035 015	.838,670,567,945	.161,329,432,055	.313,689,565,065	.483,039,570,757
33 15	.580,321,976,288	.548,293,229,520	.836,286,155,848	.163,713,844,152	.318,186,610,540	.485,315,234,704
33 30	.584,685,299,418	.551,936,985,312	.833,885,822,067	.166,114,177,933	.322,709,441,517	.487,560,781,556
33 45	.589,048,622,548	.555,570,233,020	.831,469,612,303	.168,530,387,697	.327,257,880,489	.489,776,029,818
34 00	.593,411,945,678	.559,192,903,471	.829,037,572,555	.170,962,427,445	.331,831,748,858	.491,960,798,970
34 15	.597,775,268,808	.562,804,927,695	.826,589,749,127	.173,410,250,873	.336,430,866,939	.494,114,909,478
34 30	.602,138,591,938	.566,406,236,925	.824,126,188,622	.175,873,811,378	.341,055,053,967	.496,238,182,796
34 45	.606,501,915,068	.569,996,762,596	.821,646,937,942	.178,353,062,058	.345,704,128,097	.498,330,441,372
35 00	.610,865,238,198	.573,576,436,351	.819,152,044,289	.180,847,955,711	.350,377,906,416	.500,391,508,655
35 15	.615,228,561,328	.577,145,190,037	.816,641,555,162	.183,358,444,838	.355,076,204,944	.502,421,209,103
35 30	.619,591,884,458	.580,702,955,711	.814,115,518,356	.185,884,481,644	.359,798,838,639	.504,419,368,185
35 45	.623,955,207,588	.584,249,665,637	.811,573,981,965	.188,426,018,035	.364,545,621,406	.506,385,812,390
36 00	.628,318,530,718	.587,785,252,292	.809,016,994,375	.190,983,005,625	.369,316,366,098	.508,320,369,232
36 15	.632,681,853,848	.591,309,648,364	.806,444,604,267	.193,555,395,733	.374,110,884,525	.510,222,867,253
36 30	.637,045,176,978	.594,822,786,751	.803,856,860,617	.196,143,139,383	.378,928,987,456	.512,093,136,037
36 45	.641,408,500,108	.598,324,600,571	.801,253,812,691	.198,746,187,309	.383,770,484,630	.513,931,006,204
37 00	.645,771,823,238	.601,815,023,152	.798,635,510,047	.201,364,489,953	.388,635,184,753	.515,736,309,426
37 15	.650,135,146,368	.605,293,988,043	.796,002,002,535	.203,997,997,465	.393,522,895,512	.517,508,878,427
37 30	.654,498,469,498	.608,761,429,009	.793,353,340,291	.206,646,659,709	.398,433,423,576	.519,248,546,992
37 45	.658,861,792,628	.612,217,280,034	.790,689,573,744	.209,310,426,256	.403,366,574,601	.520,955,149,969
38 00	.663,225,115,758	.615,661,475,326	.788,010,753,607	.211,989,246,393	.408,322,153,241	.522,628,523,280
38 15	.667,588,438,888	.619,093,949,310	.785,316,930,881	.214,683,069,119	.413,299,963,145	.524,268,503,919
38 30	.671,951,762,018	.622,514,636,638	.782,608,156,852	.217,391,843,148	.418,299,806,971	.525,874,929,966
38 45	.676,315,085,148	.625,923,472,184	.779,884,483,093	.220,115,516,907	.423,321,486,386	.527,447,640,589
39 00	.680,678,408,278	.629,320,391,050	.777,145,961,457	.222,854,038,543	.428,364,802,077	.528,986,476,044
39 15	.685,041,731,408	.632,705,328,563	.774,392,644,082	.225,607,355,918	.433,429,553,750	.530,491,277,692
39 30	.689,405,054,538	.636,078,220,278	.771,624,583,388	.228,375,416,612	.438,515,540,141	.531,961,887,993
39 45	.693,768,377,668	.639,439,001,981	.768,841,832,073	.231,158,167,927	.443,622,559,022	.533,398,150,521
40 00	.698,131,700,798	.642,787,609,687	.766,044,443,119	.233,955,556,881	.448,750,407,203	.534,799,909,962
40 15	.702,495,023,928	.646,123,979,643	.763,232,469,783	.236,767,530,217	.453,898,880,540	.536,167,012,123
40 30	.706,858,347,058	.649,448,048,330	.760,405,965,600	.239,594,034,400	.459,067,773,943	.537,499,303,937
40 45	.711,221,670,188	.652,759,752,463	.757,564,984,384	.242,435,015,616	.464,256,881,378	.538,796,633,470
41 00	.715,584,993,318	.656,059,028,991	.754,709,580,223	.245,290,419,777	.469,465,995,877	.540,058,849,921
41 15	.719,948,316,448	.659,345,815,100	.751,839,807,479	.248,160,192,251	.474,694,909,538	.541,285,803,633
41 30	.724,311,639,578	.662,620,048,216	.748,955,720,789	.251,044,279,211	.479,943,413,541	.542,477,346,096
41 45	.728,674,962,707	.665,881,666,001	.746,057,375,062	.253,942,624,938	.485,211,298,141	.543,633,329,951
42 00	.733,038,285,837	.669,130,606,359	.743,144,825,477	.256,855,174,523	.490,498,352,686	.544,753,608,996
42 15	.737,401,608,967	.672,366,807,435	.740,218,127,487	.259,781,872,513	.495,804,365,619	.545,838,038,195
42 30	.741,764,932,097	.675,590,207,616	.737,277,336,810	.262,722,663,190	.501,129,124,478	.546,886,473,676
42 45	.746,128,255,227	.678,800,745,533	.734,322,509,436	.265,677,490,564	.506,472,415,911	.547,898,772,739
43 00	.750,491,578,357	.681,998,360,062	.731,353,701,619	.268,646,298,381	.511,834,025,680	.548,874,793,865
43 15	.754,854,901,487	.685,182,990,326	.728,370,969,882	.271,629,030,118	.517,213,738,663	.549,814,396,716
43 30	.759,218,224,617	.688,354,575,694	.725,374,371,012	.274,625,628,988	.522,611,338,865	.550,717,442,142
43 45	.763,581,547,747	.691,513,055,782	.722,363,962,060	.277,636,037,940	.528,026,609,421	.551,583,792,186
44 00	.767,944,870,877	.694,658,370,459	.719,339,800,339	.280,660,199,661	.533,459,332,606	.552,413,310,088
44 15	.772,308,194,007	.697,790,459,842	.716,301,943,425	.283,698,056,575	.538,909,289,836	.553,205,860,290
44 30	.776,671,517,137	.700,909,264,300	.713,250,449,154	.286,749,550,846	.544,376,261,679	.553,961,308,443
44 45	.781,034,840,267	.704,014,724,456	.710,185,375,623	.289,814,624,377	.549,860,027,861	.554,679,521,410
45 00	.785,398,163,397	.707,106,781,187	.707,106,781,187	.292,893,218,813	.555,360,367,270	.555,360,367,270

TABLE 11—Continued

ϕ_1 ° ′	ϕ_1 RADIANS	SIN ϕ_1	COS ϕ_1	VERS ϕ_1	ϕ_1 SIN ϕ_1	ϕ_1 COS ϕ_1
45 15	.789,761,486,527	.710,185,375,623	.704,014,724,456	.295,985,275,544	.560,877,057,962	.556,003,715,323
45 30	.794,124,809,657	.713,250,449,154	.700,909,264,300	.299,090,735,700	.566,409,877,172	.556,609,436,099
45 45	.798,488,132,787	.716,301,943,425	.697,790,459,842	.302,209,540,158	.571,958,601,317	.557,177,401,356
46 00	.802,851,455,917	.719,339,800,339	.694,658,370,459	.305,341,629,541	.577,523,006,001	.557,707,484,088
46 15	.807,214,779,047	.722,363,962,060	.691,513,055,782	.308,486,944,218	.583,102,866,026	.558,199,558,531
46 30	.811,578,102,177	.725,374,371,012	.688,354,575,694	.311,645,424,306	.588,697,955,394	.558,653,500,167
46 45	.815,941,425,307	.728,370,969,882	.685,182,990,326	.314,817,009,674	.594,308,047,318	.559,069,185,723
47 00	.820,304,748,437	.731,353,701,619	.681,998,360,062	.318,001,639,938	.599,932,914,225	.559,446,493,185
47 15	.824,668,071,567	.734,322,509,436	.678,800,745,533	.321,199,254,467	.605,572,327,765	.559,785,301,797
47 30	.829,031,394,697	.737,277,336,810	.675,590,207,616	.324,409,792,384	.611,226,058,814	.560,085,492,064
47 45	.833,394,717,827	.740,218,127,487	.672,366,807,435	.327,633,192,565	.616,893,877,487	.560,346,945,759
48 00	.837,758,040,957	.743,144,825,477	.669,130,606,359	.330,869,393,641	.622,575,553,139	.560,569,545,928
48 15	.842,121,364,087	.746,057,375,062	.665,881,666,001	.334,118,333,999	.628,270,854,374	.560,753,176,893
48 30	.846,484,687,217	.748,955,720,789	.622,620,048,216	.337,379,951,784	.633,979,549,051	.560,897,724,258
48 45	.850,848,010,347	.751,839,807,479	.659,345,815,100	.340,654,184,900	.639,701,404,293	.561,003,074,908
49 00	.855,211,333,477	.754,709,580,223	.656,059,028,991	.343,940,971,009	.645,436,186,490	.561,069,117,023
49 15	.859,574,656,607	.757,564,984,384	.652,759,752,463	.347,240,247,537	.651,183,661,309	.561,095,740,070
49 30	.863,937,979,737	.760,405,965,600	.649,448,048,330	.350,551,951,670	.656,943,593,700	.561,082,834,818
49 45	.868,301,302,867	.763,232,469,783	.646,123,979,643	.353,876,020,357	.662,715,747,903	.561,030,293,338
50 00	.872,664,625,997	.766,044,443,119	.642,787,609,687	.357,212,390,313	.668,499,887,452	.560,938,009,003
50 15	.877,027,949,127	.768,841,832,073	.639,439,001,981	.360,560,998,019	.674,295,775,186	.560,805,876,499
50 30	.881,391,272,257	.771,624,583,388	.636,078,220,278	.363,921,779,722	.680,103,173,257	.560,633,791,826
50 45	.885,754,595,387	.774,392,644,082	.632,705,328,563	.367,294,671,437	.685,921,843,130	.560,421,652,301
51 00	.890,117,918,517	.777,145,961,457	.629,320,391,050	.370,679,608,950	.691,751,545,596	.560,169,356,562
51 15	.894,481,241,647	.779,884,483,093	.625,923,472,184	.374,076,527,816	.697,592,040,778	.559,876,804,575
51 30	.898,844,564,777	.782,608,156,852	.622,514,636,638	.377,485,363,362	.703,443,088,137	.559,543,897,636
51 45	.903,207,887,907	.785,316,930,881	.619,093,949,310	.380,906,050,690	.709,304,446,479	.559,170,538,372
52 00	.907,571,211,037	.788,010,753,607	.615,661,475,326	.384,338,524,674	.715,175,873,961	.558,756,630,750
52 15	.911,934,534,167	.790,689,573,744	.612,217,280,034	.387,782,719,966	.721,057,128,103	.558,302,080,077
52 30	.916,297,857,297	.793,353,340,291	.608,761,429,009	.391,238,570,991	.726,947,965,788	.557,806,793,006
52 45	.920,661,180,427	.796,002,002,535	.605,293,988,043	.394,706,011,957	.732,848,143,276	.557,270,677,537
53 00	.925,024,503,557	.798,635,510,047	.601,815,023,152	.398,184,976,848	.738,757,416,204	.556,693,643,024
53 15	.929,387,826,687	.801,253,812,691	.598,324,600,571	.401,675,399,429	.744,675,539,601	.556,075,600,178
53 30	.933,751,149,817	.803,856,860,617	.594,822,786,751	.405,177,213,249	.750,602,267,889	.555,416,461,066
53 45	.938,114,472,947	.806,444,604,267	.591,309,648,364	.408,690,351,636	.756,537,354,893	.554,716,139,123
54 00	.942,477,796,077	.809,016,994,375	.587,785,252,292	.412,214,747,708	.762,480,553,847	.553,974,549,147
54 15	.946,841,119,207	.811,573,981,965	.584,249,665,637	.415,750,334,363	.768,431,617,403	.553,191,607,308
54 30	.951,204,442,337	.814,115,518,356	.580,702,955,711	.419,297,044,289	.774,390,297,636	.552,367,231,151
54 45	.955,567,765,467	.816,641,555,162	.577,145,190,037	.422,854,809,963	.780,356,346,054	.551,501,339,594
55 00	.959,931,088,597	.819,152,044,289	.573,576,436,351	.426,423,563,649	.786,329,513,601	.550,593,852,940
55 15	.964,294,411,727	.821,646,937,942	.569,996,762,596	.430,003,237,404	.792,309,550,670	.549,644,692,874
55 30	.968,657,734,857	.824,126,188,622	.566,406,236,925	.433,593,763,075	.798,296,207,107	.548,653,782,469
55 45	.973,021,057,987	.826,589,749,127	.562,804,927,695	.437,195,072,305	.804,289,232,217	.547,621,046,186
56 00	.977,384,381,117	.829,037,572,555	.559,192,903,471	.440,807,096,529	.810,288,374,774	.546,546,409,884
56 15	.981,747,704,247	.831,469,612,303	.555,570,233,020	.444,429,766,980	.816,293,383,030	.545,429,800,815
56 30	.986,111,027,377	.833,885,822,067	.551,936,985,312	.448,063,014,688	.822,304,004,714	.544,271,147,633
56 45	.990,474,350,507	.836,286,155,848	.548,293,229,520	.451,706,770,480	.828,319,987,052	.543,070,380,396
57 00	.994,837,673,637	.838,670,567,945	.544,639,035,015	.455,360,964,985	.834,341,076,762	.541,827,430,566
57 15	.999,200,996,767	.841,039,012,964	.540,974,471,368	.459,025,528,632	.840,367,020,074	.540,542,231,016
57 30	1.003,564,319,897	.843,391,445,813	.537,299,608,347	.462,700,391,653	.846,397,562,724	.539,214,716,032
57 45	1.007,927,643,027	.845,727,821,704	.533,614,515,916	.466,385,484,084	.852,432,449,972	.537,844,821,312
58 00	1.012,290,966,157	.848,048,096,156	.529,919,264,233	.470,080,735,767	.858,471,426,605	.536,432,483,976
58 15	1.016,654,289,287	.850,352,224,996	.526,213,923,652	.473,786,076,348	.864,514,236,947	.534,977,642,563
58 30	1.021,017,612,416	.852,640,164,354	.522,498,564,716	.477,501,435,284	.870,560,624,859	.533,480,237,037
58 45	1.025,380,935,546	.854,911,870,673	.518,773,258,161	.481,226,741,839	.876,610,333,760	.531,940,208,789
59 00	1.029,744,258,676	.857,167,300,702	.515,038,074,910	.484,961,925,090	.882,663,106,623	.530,357,500,638
59 15	1.034,107,581,806	.859,406,411,501	.511,293,086,077	.488,706,913,923	.888,718,685,986	.528,732,056,837
59 30	1.038,470,904,936	.861,629,160,442	.507,538,362,961	.492,461,637,039	.894,776,813,963	.527,063,823,074
59 45	1.042,834,228,066	.863,835,505,204	.503,773,977,046	.496,226,022,954	.900,837,232,245	.525,352,746,473
60 00	1.047,197,551,196	.866,025,403,784	.500,000,000,000	.500,000,000,000	.906,899,682,116	.523,598,775,598
60 15	1.051,560,874,326	.868,198,814,489	.496,216,503,675	.503,783,496,325	.912,963,904,453	.521,801,860,459
60 30	1.055,924,197,456	.870,355,695,940	.492,423,560,103	.507,576,439,897	.919,029,639,737	.519,961,952,510
60 45	1.060,287,520,586	.872,496,007,073	.488,621,241,497	.511,378,758,503	.925,096,628,061	.518,079,004,653
61 00	1.064,650,843,716	.874,619,707,139	.484,809,620,246	.515,190,379,754	.931,164,609,136	.516,152,971,237
61 15	1.069,014,166,846	.876,726,755,708	.480,988,768,919	.519,011,231,081	.937,233,322,305	.514,183,808,068
61 30	1.073,377,489,976	.878,817,112,662	.477,158,760,260	.522,841,239,740	.943,302,506,537	.512,171,472,408
61 45	1.077,740,813,106	.880,890,738,205	.473,319,667,185	.526,680,332,815	.949,371,900,451	.510,115,922,971
62 00	1.082,104,136,236	.882,947,592,859	.469,471,562,786	.530,528,437,214	.955,441,242,312	.508,017,119,936
62 15	1.086,467,459,366	.884,987,637,463	.465,614,520,325	.534,385,479,675	.961,510,270,045	.505,875,024,941
62 30	1.090,830,782,496	.887,010,833,178	.461,748,613,235	.538,251,386,765	.967,578,721,238	.503,689,601,092
62 45	1.095,194,105,626	.889,017,141,486	.457,873,915,117	.542,126,084,883	.973,646,333,156	.501,460,812,956
63 00	1.099,557,428,756	.891,006,524,188	.453,990,499,740	.546,009,500,260	.979,712,842,741	.499,188,626,574
63 15	1.103,920,751,886	.892,978,943,411	.450,098,441,037	.549,901,558,963	.985,777,986,629	.496,873,009,452
63 30	1.108,284,075,016	.894,934,361,602	.446,197,813,110	.553,802,186,890	.991,841,501,148	.494,513,930,577
63 45	1.112,647,398,146	.896,872,741,533	.442,288,690,219	.557,711,309,781	.997,903,122,335	.492,111,360,402
64 00	1.117,010,721,276	.898,794,046,299	.438,371,146,789	.561,628,853,211	1.003,962,585,935	.489,665,270,861
64 15	1.121,374,044,406	.900,698,239,323	.434,445,257,404	.565,554,742,596	1.010,019,627,419	.487,175,635,368
64 30	1.125,737,367,536	.902,585,284,350	.430,511,096,808	.569,488,903,192	1.016,073,981,981	.484,642,428,816
64 45	1.130,100,690,666	.904,455,145,454	.426,568,739,901	.573,431,260,099	1.022,125,384,554	.482,065,627,579
65 00	1.134,464,013,796	.906,307,787,037	.422,618,261,741	.577,381,738,259	1.028,173,569,817	.479,445,209,518
65 15	1.138,827,336,926	.908,143,173,825	.418,659,737,537	.581,340,262,463	1.034,218,272,195	.476,781,153,977
65 30	1.143,190,660,056	.909,961,270,877	.414,693,242,656	.585,306,757,344	1.040,259,225,879	.474,073,441,793
65 45	1.147,553,983,186	.911,762,043,577	.410,718,852,613	.589,281,147,387	1.046,296,164,825	.471,322,055,286
66 00	1.151,917,306,316	.913,545,457,643	.406,736,643,076	.593,263,356,924	1.052,328,822,765	.468,526,978,272
66 15	1.156,280,629,446	.915,311,479,119	.402,746,689,859	.597,253,310,141	1.058,356,933,215	.465,688,196,057
66 30	1.160,643,952,576	.917,060,074,385	.398,749,068,925	.601,250,931,075	1.064,380,229,484	.462,805,695,443
66 45	1.165,007,275,706	.918,791,210,149	.394,743,856,384	.605,256,143,616	1.070,398,444,678	.459,879,464,728
67 00	1.169,370,598,836	.920,504,853,452	.390,731,128,489	.609,268,871,511	1.076,411,311,713	.456,909,493,705
67 15	1.173,733,921,966	.922,200,971,670	.386,710,961,637	.613,289,038,363	1.082,418,563,319	.453,895,773,669
67 30	1.178,097,245,096	.923,879,532,511	.382,683,432,365	.617,316,567,635	1.088,419,932,052	.450,838,297,413

TABLE 11—Continued

ϕ_1 ° ′	ϕ_1 RADIANS	SIN ϕ_1	COS ϕ_1	VERS ϕ_1	ϕ_1 SIN ϕ_1	ϕ_1 COS ϕ_1
67 45	1.182,460,568,226	.925,540,504,018	.378,648,617,352	.621,351,382,648	1.094,415,150,297	.447,737,059,232
68 00	1.186,823,891,356	.927,183,854,567	.374,606,593,416	.625,393,406,584	1.100,403,950,280	.444,592,054,926
68 15	1.191,187,214,486	.928,809,552,872	.370,557,437,510	.629,442,562,490	1.106,386,064,074	.441,403,281,795
68 30	1.195,550,537,616	.930,417,567,982	.366,501,226,724	.633,498,773,276	1.112,361,223,608	.438,170,738,647
68 45	1.199,913,860,746	.932,007,869,283	.362,438,038,284	.637,561,961,716	1.118,329,160,677	.434,894,425,799
69 00	1.204,277,183,876	.933,580,426,497	.358,367,949,545	.641,632,050,455	1.124,289,606,944	.431,574,345,069
69 15	1.208,640,507,006	.935,135,209,686	.354,291,037,998	.645,708,962,002	1.130,242,293,954	.428,210,499,794
69 30	1.213,003,830,136	.936,672,189,248	.350,207,381,259	.649,792,618,741	1.136,186,953,140	.424,802,894,809
69 45	1.217,367,153,266	.938,191,335,922	.346,117,057,077	.653,882,942,923	1.142,123,315,830	.421,351,536,471
70 00	1.221,730,476,376	.939,692,620,786	.342,020,143,326	.657,979,856,674	1.148,051,113,259	.417,856,432,643
70 15	1.226,093,799,526	.941,176,015,256	.337,916,718,003	.662,083,281,997	1.153,970,076,568	.414,317,592,700
70 30	1.230,457,122,656	.942,641,491,092	.333,806,859,234	.666,193,140,766	1.159,879,936,825	.410,735,027,536
70 45	1.234,820,445,786	.944,089,020,393	.329,690,645,263	.670,309,354,737	1.165,780,425,023	.407,108,749,555
71 00	1.239,183,768,916	.945,518,575,599	.325,568,154,457	.674,431,845,543	1.171,671,272,091	.403,438,772,679
71 15	1.243,547,092,046	.946,930,129,495	.321,439,465,303	.678,560,534,697	1.177,552,208,904	.399,725,112,346
71 30	1.247,910,415,176	.948,323,655,206	.317,304,656,405	.682,695,343,595	1.183,422,966,289	.395,967,785,512
71 45	1.252,273,738,306	.949,699,126,202	.313,163,806,484	.686,836,193,516	1.189,283,275,035	.392,166,810,648
72 00	1.256,637,061,436	.951,056,516,295	.309,016,994,375	.690,983,005,625	1.195,132,865,897	.388,322,207,745
72 15	1.261,000,384,566	.952,395,799,643	.304,864,299,028	.695,135,700,972	1.200,971,469,609	.384,433,998,315
72 30	1.265,363,707,696	.953,716,950,748	.300,705,799,504	.699,294,200,496	1.206,798,816,891	.380,502,205,386
72 45	1.269,727,030,826	.955,019,944,457	.296,541,574,976	.703,458,425,024	1.212,614,638,455	.376,526,853,511
73 00	1.274,090,353,956	.956,304,755,963	.292,371,704,723	.707,628,295,277	1.218,418,665,015	.372,507,968,757
73 15	1.278,453,677,086	.957,571,360,805	.288,196,268,134	.711,803,731,866	1.224,210,627,293	.368,445,578,718
73 30	1.282,817,000,216	.958,819,734,868	.284,015,344,704	.715,984,655,296	1.229,990,256,031	.364,339,712,508
73 45	1.287,180,323,346	.960,049,854,386	.279,829,014,031	.720,170,985,969	1.235,757,281,997	.360,190,400,762
74 00	1.291,543,646,476	.961,261,695,938	.275,637,355,817	.724,362,644,183	1.241,511,435,989	.355,997,675,637
74 15	1.295,906,969,606	.962,455,236,454	.271,440,449,865	.728,559,550,135	1.247,252,448,855	.351,761,570,813
74 30	1.300,270,292,736	.963,630,453,209	.267,238,376,078	.732,761,623,922	1.252,980,051,483	.347,482,121,493
74 45	1.304,633,615,866	.964,787,323,829	.263,031,214,458	.736,968,785,542	1.258,693,974,829	.343,159,364,404
75 00	1.308,996,938,996	.965,925,826,289	.258,819,045,103	.741,180,954,897	1.264,393,949,909	.338,793,337,794
75 15	1.313,360,262,125	.967,045,938,914	.254,601,948,206	.745,398,051,794	1.270,079,707,819	.334,384,081,433
75 30	1.317,723,585,255	.968,147,640,378	.250,380,004,054	.749,619,995,946	1.275,750,979,735	.329,931,636,618
75 45	1.322,086,908,385	.969,230,909,707	.246,153,293,029	.753,846,706,971	1.281,407,496,926	.325,436,046,169
76 00	1.326,450,231,515	.970,295,726,276	.241,921,895,600	.758,078,104,400	1.287,048,990,757	.320,897,354,427
76 15	1.330,813,554,645	.971,342,069,813	.237,685,892,326	.762,314,107,674	1.292,675,192,704	.316,315,607,255
76 30	1.335,176,877,775	.972,369,920,398	.233,445,363,856	.766,554,636,144	1.298,285,834,359	.311,690,852,044
76 45	1.339,540,200,905	.973,379,258,460	.229,200,390,922	.770,799,609,078	1.303,880,647,434	.307,023,137,703
77 00	1.343,903,524,035	.974,370,064,785	.224,951,054,344	.775,048,945,656	1.309,459,363,779	.302,312,514,668
77 15	1.348,266,847,165	.975,342,320,509	.220,697,435,022	.779,302,564,978	1.315,021,715,379	.297,559,034,895
77 30	1.352,630,170,295	.976,296,007,120	.216,439,613,938	.783,560,386,062	1.320,567,434,369	.292,762,751,860
77 45	1.356,993,493,425	.977,231,106,463	.212,177,672,156	.787,822,327,844	1.326,096,253,043	.287,923,720,566
78 00	1.361,356,816,555	.978,147,600,734	.207,911,690,818	.792,088,309,182	1.331,607,903,856	.283,041,997,537
78 15	1.365,720,139,685	.979,045,472,485	.203,641,751,140	.796,358,248,860	1.337,102,119,440	.278,117,640,813
78 30	1.370,083,462,815	.979,924,704,621	.199,367,934,417	.800,632,065,583	1.342,578,632,605	.273,150,709,960
78 45	1.374,446,785,945	.980,785,280,403	.195,090,322,016	.804,909,677,984	1.348,037,176,352	.268,141,266,064
79 00	1.378,810,109,075	.981,627,183,448	.190,808,995,377	.809,191,004,623	1.353,477,483,881	.263,089,371,728
79 15	1.383,173,432,205	.982,450,397,726	.186,524,036,009	.813,475,963,991	1.358,899,288,594	.257,995,091,075
79 30	1.387,536,755,335	.983,254,907,564	.182,235,525,492	.817,764,474,508	1.364,302,324,109	.252,858,489,748
79 45	1.391,900,078,465	.984,040,697,646	.177,943,545,474	.822,056,454,526	1.369,686,324,266	.247,679,634,908
80 00	1.396,263,401,595	.984,807,753,012	.173,648,177,667	.826,351,822,333	1.375,051,023,138	.242,458,595,230
80 15	1.400,626,724,725	.985,556,059,058	.169,349,503,849	.830,650,496,151	1.380,396,155,031	.237,195,440,910
80 30	1.404,990,047,855	.986,285,601,537	.165,047,605,861	.834,952,394,139	1.385,721,454,502	.231,890,243,657
80 45	1.409,353,370,985	.986,996,366,560	.160,742,565,604	.839,257,434,396	1.391,026,656,361	.226,543,076,695
81 00	1.413,716,694,115	.987,688,340,595	.156,434,465,040	.843,565,534,960	1.396,311,495,682	.221,154,014,762
81 15	1.418,080,017,245	.988,361,510,468	.152,123,386,190	.847,876,613,810	1.401,575,707,809	.215,723,134,112
81 30	1.422,443,340,375	.989,015,863,362	.147,809,411,130	.852,190,588,870	1.406,819,028,365	.210,250,512,507
81 45	1.426,806,663,505	.989,651,386,820	.143,492,621,991	.856,507,378,009	1.412,041,193,262	.204,736,229,221
82 00	1.431,169,986,635	.990,268,068,742	.139,173,100,960	.860,826,899,040	1.417,241,938,707	.199,180,365,041
82 15	1.435,533,309,765	.990,865,897,387	.134,850,930,274	.865,149,069,726	1.422,421,001,209	.193,583,002,261
82 30	1.439,896,632,895	.991,444,861,374	.130,526,192,220	.869,473,807,780	1.427,578,117,593	.187,944,224,682
82 45	1.444,259,956,025	.992,004,949,680	.126,198,969,136	.873,801,030,864	1.432,713,025,001	.182,264,117,615
83 00	1.448,623,279,155	.992,546,151,641	.121,869,343,405	.878,130,656,595	1.437,825,460,903	.176,542,767,872
83 15	1.452,986,602,285	.993,068,456,955	.117,537,397,458	.882,462,602,542	1.442,915,163,107	.170,780,263,774
83 30	1.457,349,925,415	.993,571,855,677	.113,203,213,768	.886,796,786,232	1.447,981,869,765	.164,976,695,142
83 45	1.461,713,248,545	.994,056,338,222	.108,866,874,852	.891,133,125,148	1.453,025,319,379	.159,132,153,299
84 00	1.466,076,571,675	.994,521,895,368	.104,528,463,268	.895,471,536,732	1.458,045,250,817	.153,246,731,070
84 15	1.470,439,894,805	.994,968,518,251	.100,188,061,612	.899,811,938,388	1.463,041,403,311	.147,320,522,777
84 30	1.474,803,217,935	.995,396,198,367	.095,845,752,520	.904,154,247,480	1.468,013,516,472	.141,353,624,242
84 45	1.479,166,541,065	.995,804,927,575	.091,501,618,663	.908,498,381,337	1.472,961,330,297	.135,346,132,780
85 00	1.483,529,864,195	.996,194,698,092	.087,155,742,748	.912,844,257,252	1.477,884,585,172	.129,298,147,203
85 15	1.487,893,187,325	.996,565,502,498	.082,808,207,512	.917,191,792,488	1.482,783,021,890	.123,209,767,812
85 30	1.492,256,510,455	.996,917,333,733	.078,459,095,728	.921,540,904,272	1.487,656,381,649	.117,081,096,405
85 45	1.496,619,833,585	.997,250,185,099	.074,108,490,195	.925,891,509,805	1.492,504,406,065	.110,912,236,263
86 00	1.500,983,156,715	.997,564,050,260	.069,756,473,744	.930,243,526,256	1.497,326,837,185	.104,703,292,162
86 15	1.505,346,479,845	.997,858,923,239	.065,403,129,230	.934,596,870,770	1.502,123,417,480	.098,454,370,357
86 30	1.509,709,802,975	.998,134,798,422	.061,048,539,535	.938,951,460,465	1.506,893,889,868	.092,165,578,593
86 45	1.514,073,126,105	.998,391,670,557	.056,692,787,563	.943,307,212,437	1.511,637,997,717	.085,837,026,093
87 00	1.518,436,449,235	.998,629,534,755	.052,335,956,243	.947,664,043,757	1.516,355,484,855	.079,468,823,565
87 15	1.522,799,772,365	.998,848,386,485	.047,978,128,521	.952,021,871,479	1.521,046,095,567	.073,061,083,190
87 30	1.527,163,095,495	.999,048,221,582	.043,619,387,365	.956,380,612,635	1.525,709,574,620	.066,613,918,632
87 45	1.531,526,418,625	.999,229,036,241	.039,259,815,759	.960,740,184,241	1.530,345,667,260	.060,127,445,025
88 00	1.535,889,741,755	.999,390,827,019	.034,899,496,703	.965,100,503,297	1.534,954,119,223	.053,601,778,979
88 15	1.540,253,064,885	.999,533,590,837	.030,538,513,210	.969,461,486,790	1.539,534,676,742	.047,037,038,569
88 30	1.544,616,388,015	.999,657,324,976	.026,176,948,308	.973,823,051,692	1.544,087,086,557	.040,433,343,345
88 45	1.548,979,711,145	.999,762,027,080	.021,814,885,035	.978,185,114,965	1.548,611,095,920	.033,790,814,320
89 00	1.553,343,034,275	.999,847,695,156	.017,452,406,437	.982,547,593,563	1.553,106,452,606	.027,109,573,970
89 15	1.557,706,357,405	.999,914,327,574	.013,089,595,571	.986,910,404,429	1.557,572,904,922	.020,389,746,237
89 30	1.562,069,680,535	.999,961,923,064	.008,726,535,498	.991,273,464,502	1.562,010,201,708	.013,631,456,518
89 45	1.566,433,003,665	.999,990,480,721	.004,363,309,285	.995,636,690,715	1.566,418,092,352	.006,834,831,669
90 00	1.570,796,326,795	1.0	0	1.0	1.570,796,326,795	0

TABLE 12—TRIGONOMETRIC FUNCTIONS FOR ARCH LOAD CONSTANTS—Continued

Φ_1 ° ′	$\frac{\sin \Phi_1}{\Phi_1}$	$\frac{\sin^2 \Phi_1}{\Phi_1}$	$\frac{\Phi_1^2}{2} \sin \Phi_1$	$\frac{\Phi_1^2}{2} \cos \Phi_1$	$\frac{\Phi_1 + \sin \Phi_1 \cos \Phi_1}{2}$	$\frac{\Phi_1 - \sin \Phi_1 \cos \Phi_1}{2}$
0 15	.999,996,826,906	.004,363,295,440	.0^6 ,041,535,626	.0^{5}9,519,203,752	.004,363,295,440	.0^6 ,027,690
0 30	.999,987,307,656	.008,726,424,738	.0^6 ,332,281,841	.0^3 ,038,075,728	.008,726,424,739	.0^6 ,221,521
0 45	.999,971,442,361	.013,089,221,763	.0^{5}1,121,433,421	.0^3 ,085,666,309	.013,089,221,772	.0^6 ,747,618
1 00	.999,949,231,203	.017,451,520,400	.0^{5}2,658,153,509	.0^3 ,152,285,513	.017,451,520,436	.0^3 ,001,772,084
1 15	.999,920,674,435	.021,813,154,556	.0^{5}5,191,557,806	.0^3 ,237,925,726	.021,813,154,666	.0^3 ,003,460,984
1 30	.999,885,772,383	.026,173,958,177	.0^{5}8,970,698,758	.0^3 ,342,577,164	.026,173,958,451	.0^3 ,005,980,329
1 45	.999,844,525,444	.030,533,765,248	.0^3 ,014,244,550	.0^3 ,466,227,870	.030,533,765,839	.0^3 ,009,496,071
2 00	.999,796,934,092	.034,892,409,804	.0^3 ,021,261,989	.0^3 ,608,863,710	.034,892,410,956	.0^3 ,014,174,084
2 15	.999,742,998,869	.039,249,725,942	.0^3 ,030,271,785	.0^3 ,770,468,382	.039,249,728,017	.0^3 ,020,180,153
2 30	.999,682,720,393	.043,605,547,823	.0^3 ,041,522,579	.0^3 ,951,023,411	.043,605,551,337	.0^3 ,027,679,963
2 45	.999,616,099,350	.047,959,709,686	.0^3 ,055,262,869	.001,150,508,150	.047,959,715,345	.0^3 ,036,839,085
3 00	.999,543,136,501	.052,312,045,855	.0^3 ,071,740,998	.001,368,899,785	.052,312,054,597	.0^3 ,047,822,963
3 15	.999,463,832,681	.056,662,390,743	.0^3 ,091,205,131	.001,606,173,331	.056,662,403,787	.0^3 ,060,796,903
3 30	.999,378,188,796	.061,010,578,869	.0^3 ,113,903,248	.001,862,301,637	.061,010,597,761	.0^3 ,075,926,059
3 45	.999,286,205,823	.065,356,444,857	.0^3 ,140,083,119	.002,137,255,386	.065,356,471,530	.0^3 ,093,375,420
4 00	.999,187,884,813	.069,699,823,452	.0^3 ,169,992,296	.002,431,003,096	.069,699,860,280	.0^3 ,113,309,800
4 15	.999,083,226,891	.074,040,549,524	.0^3 ,203,878,094	.002,743,511,122	.074,040,599,387	.0^3 ,135,893,822
4 30	.998,972,233,249	.078,378,458,078	.0^3 ,241,987,574	.003,074,743,658	.078,378,524,430	.0^3 ,161,291,910
4 45	.998,854,905,157	.082,713,384,261	.0^3 ,284,567,529	.003,424,662,736	.082,713,471,200	.0^3 ,189,668,270
5 00	.998,731,243,954	.087,045,163,372	.0^3 ,331,864,468	.003,793,228,232	.087,045,275,717	.0^3 ,221,186,883
5 15	.998,601,251,053	.091,373,630,871	.0^3 ,384,124,602	.004,180,397,865	.091,373,774,238	.0^3 ,256,011,492
5 30	.998,464,927,939	.095,698,622,383	.0^3 ,441,593,823	.004,586,127,202	.095,698,803,274	.0^3 ,294,305,586
5 45	.998,322,276,165	.100,019,973,713	.0^3 ,504,517,695	.005,010,369,655	.100,020,199,599	.0^3 ,336,232,391
6 00	.998,173,297,371	.104,337,520,849	.0^3 ,573,141,434	.005,453,076,486	.104,337,800,265	.0^3 ,381,954,855
6 15	.998,017,993,245	.108,651,099,971	.0^3 ,647,709,893	.005,914,196,814	.108,651,442,610	.0^3 ,431,635,640
6 30	.997,856,365,570	.112,960,547,461	.0^3 ,728,467,548	.006,393,677,608	.112,960,964,276	.0^3 ,485,437,104
6 45	.997,688,416,189	.117,265,699,913	.0^3 ,815,658,480	.006,891,463,697	.117,266,203 219	.0^3 ,543 521 291
7 00	.997,514,147,016	.121,566,394,134	.0^3 ,909,526,361	.007,407,497,770	.121,566,997,720	.0^3 ,606,049,920
7 15	.997,333,560,051	.125,862,467,163	.001,010,314,440	.007,941,720,377	.125,863,186,399	.0^3 ,673,184,371
7 30	.997,146,657,346	.130,153,756,268	.001,118,265,522	.008,494,069,937	.130,154,608,226	.0^3 ,745,085,674
7 45	.996,953,441,044	.134,440,098,965	.001,233,621,959	.009,064,482,735	.134,441,102,535	.0^3 ,821,914,495
8 00	.996,753,913,341	.138,721,333,014	.001,356,625,629	.009,652,892,928	.138,722,509,034	.0^3 ,903,831,126
8 15	.996,548,076,524	.142,997,296,441	.001,487,517,926	.010,259,232,549	.142,998,667,821	.0^3 ,990,995,469
8 30	.996,335,932,952	.147,267,827,537	.001,626,539,737	.010,883,431,508	.147,269,419,390	.001,083,567,029
8 45	.996,117,485,023	.151,532,764,865	.001,773,931,434	.011,525,417,596	.151,534,604,651	.001,181,704,898
9 00	.995,892,735,245	.155,791,947,275	.001,929,932,856	.012,185,116,491	.155,794,064,933	.001,285,567,746
9 15	.995,661,686,188	.160,045,213,911	.002,094,783,289	.012,862,451,757	.160,047,642,006	.001,395,313,803
9 30	.995,424,340,484	.164,292,404,213	.002,268,721,460	.013,557,344,852	.164,295,178,084	.001,511,100,855
9 45	.995,180,700,842	.168,533,357,928	.002,451,985,511	.014,269,715,127	.168,536,515,843	.001,633,086,226
10 00	.994,930,770,048	.172,767,915,124	.002,644,812,992	.014,999,479,835	.172,771,498,431	.001,761,426,768
10 15	.994,674,550,954	.176,995,916,190	.002,847,440,841	.015,746,554,133	.176,999,969,479	.001,896,278,850
10 30	.994,412,046,482	.181,217,201,846	.003,060,105,371	.016,510,851,083	.181,221,773,116	.002,037,798,343
10 45	.994,143,259,636	.185,431,613,158	.003,283,042,253	.017,292,281,661	.185,436,753,976	.002,186,140,613
11 00	.993,868,193,478	.189,638,991,535	.003,516,486,503	.018,090,754,759	.189,644,757,214	.002,341,460,505
11 15	.993,586,851,145	.193,839,178,741	.003,760,672,462	.018,906,177,188	.193,845,628,516	.002,503,912,333
11 30	.993,299,235,857	.198,032,016,911	.004,015,833,789	.019,738,453,684	.198,039,214,112	.002,673,649,867
11 45	.993,005,350,893	.202,217,348,547	.004,282,203,437	.020,587,486,914	.202,225,360,786	.002,850,826,323
12 00	.992,705,199,610	.206,395,016,535	.004,560,013,642	.021,453,177,478	.206,403,915,889	.003,035,594,350
12 15	.992,398,785,426	.210,564,864,142	.004,849,495,909	.022,335,423,914	.210,574,727,328	.003,228,106,021
12 30	.992,086,111,848	.214,726,735,042	.005,150,880,995	.023,234,122,706	.214,737,643,685	.003,428,512,814
12 45	.991,767,182,442	.218,880,473,304	.005,464,398,895	.024,149,168,284	.218,892,514,017	.003,636,965,612
13 00	.991,442,000,842	.223,025,923,410	.005,790,278,824	.025,080,453,032	.223,039,188,077	.003,853,614,682
13 15	.991,110,570,762	.227,162,930,266	.006,128,749,207	.026,027,867,295	.227,177,516,222	.004,078,609,667
13 30	.990,772,895,989	.231,291,339,203	.006,480,037,661	.026,991,299,379	.231,307,349,444	.004,312,099,575
13 45	.990,428,980,370	.235,410,995,985	.006,844,370,978	.027,970,635,562	.235,428,539,383	.004,554,232,766
14 00	.990,078,827,835	.239,521,746,823	.007,221,975,115	.028,965,760,096	.239,540,938,336	.004,805,156,943
14 15	.989,722,442,373	.243,623,438,375	.007,613,075,177	.029,976,555,212	.243,644,399,269	.005,065,019,140
14 30	.989,359,828,053	.247,715,917,759	.008,017,895,398	.031,002,901,129	.247,738,775,831	.005,333,965,708
14 45	.988,990,989,018	.251,799,032,562	.008,436,659,134	.032,044,676,056	.251,823,922,361	.005,612,142,308
15 00	.988,615,929,468	.255,872,630,839	.008,869,588,842	.033,101,756,202	.255,899,693,900	.005,899,693,900
15 15	.988,234,653,682	.259,936,561,127	.009,316,906,069	.034,174,015,778	.259,965,946,205	.006,196,764,724
15 30	.987,847,166,013	.263,990,672,459	.009,778,831,434	.035,261,327,002	.264,022,535,757	.006,503,498,302
15 45	.987,453,470,882	.268,034,814,357	.010,255,584,615	.036,363,560,113	.268,069,319,773	.006,820,037,416
16 00	.987,053,572,779	.272,068,836,851	.010,747,384,336	.037,480,583,368	.272,106,156,218	.007,146,524,101
16 15	.986,647,476,264	.276,092,590,479	.011,254,448,350	.038,612,263,052	.276,132,903,811	.007,483,099,638
16 30	.986,235,185,969	.280,105,926,302	.011,776,993,424	.039,758,463,489	.280,149,422,043	.007,829,904,536
16 45	.985,816,706,597	.284,108,695,905	.012,315,235,329	.040,919,047,039	.284,155,571,182	.008,187,078,527
17 00	.985,392,042,922	.288,100,751,410	.012,869,388,818	.042,093,874,113	.288,151,212,287	.008,554,760,552
17 15	.984,961,199,785	.292,081,945,474	.013,439,667,619	.043,282,803,176	.292,136,207,216	.008,933,088,753
17 30	.984,524,182,096	.296,052,131,308	.014,026,284,416	.044,485,690,752	.296,110,418,637	.009,322,200,462
17 45	.984,080,994,846	.300,011,162,681	.014,629,450,837	.045,702,391,435	.300,073,710,042	.009,722,232,187
18 00	.983,631,643,084	.303,958,893,913	.015,249,377,438	.046,932,757,894	.304,025,945,753	.010,133,319,606
18 15	.983,176,131,933	.307,895,179,920	.015,886,273,690	.048,176,640,882	.307,966,990,932	.010,555,597,557
18 30	.982,714,466,587	.311,819,876,165	.016,540,347,964	.049,433,889,238	.311,896,711,597	.010,989,200,022
18 45	.982,246,652,309	.315,732,838,714	.017,211,807,517	.050,704,349,900	.315,814,974,627	.011,434,260,122
19 00	.981,772,694,434	.319,633,924,223	.017,900,858,477	.051,987,867,910	.319,721,647,771	.011,890,910,108
19 15	.981,292,598,364	.323,522,989,946	.018,607,705,833	.053,284,286,418	.323,616,599,664	.012,359,281,345
19 30	.980,806,369,570	.327,399,893,743	.019,332,553,415	.054,593,446,699	.327,499,699,832	.012,839,504,307
19 45	.980,314,013,594	.331,264,494,086	.020,075,603,883	.055,915,188,152	.331,370,818,704	.013,331,708,565
20 00	.979,815,536,052	.335,116,650,074	.020,837,058,715	.057,249,348,307	.335,229,827,621	.013,836,022,778
20 15	.979,310,942,617	.338,956,221,422	.021,617,118,189	.058,595,762,843	.339,076,598,847	.014,352,574,682
20 30	.978,800,239,047	.342,783,068,492	.022,415,981,371	.059,954,265,583	.342,911,005,577	.014,881,491,082
20 45	.978,283,431,161	.346,597,052,282	.023,233,846,104	.061,324,688,513	.346,732,921,949	.015,422,897,840
21 00	.977,760,524,842	.350,398,034,434	.024,070,908,987	.062,706,861,781	.350,542,223,049	.015,976,919,870
21 15	.977,231,526,055	.354,185,877,253	.024,927,365,372	.064,100,613,713	.354,338,784,929	.016,543,681,120
21 30	.976,696,440,820	.357,960,443,697	.025,803,409,338	.065,505,770,817	.358,122,484,605	.017,123,304,574
21 45	.976,155,275,241	.361,721,597,405	.026,699,233,688	.066,922,157,791	.361,893,200,078	.017,715,912,231
22 00	.975,608,035,477	.365,469,202,679	.027,615,029,929	.068,349,597,534	.365,650,810,334	.018,321,625,105
22 15	.975,054,727,763	.369,203,124,510	.028,550,988,262	.069,787,911,153	.369,395,195,359	.018,940,563,210
22 30	.974,495,358,404	.372,923,228,578	.029,507,297,565	.071,236,917,970	.373,126,236,146	.019,572,845,553

TABLE 12—Continued

ϕ_1 ° ′	$\frac{\sin\phi_1}{\phi_1}$	$\frac{\sin^2\phi_1}{\phi_1}$	$\frac{\phi_1^2}{2}\sin\phi_1$	$\frac{\phi_1^2}{2}\cos\phi_1$	$\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}$	$\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}$
22 45	.973,929,933,768	.376,629,381,254	.030,484,145,384	.072,696,435,536	.376,843,814,703	.020,218,590,126
23 00	.973,358,460,295	.380,321,449,615	.031,481,717,917	.074,166,279,634	.380,547,814,064	.020,877,913,895
23 15	.972,780,944,495	.383,999,301,447	.032,500,199,999	.075,646,264,291	.384,238,118,297	.021,550,932,792
23 30	.972,197,392,943	.387,662,805,247	.033,539,775,094	.077,136,201,787	.387,914,612,514	.022,237,761,705
23 45	.971,607,812,285	.391,311,830,239	.034,600,625,277	.078,635,902,661	.391,577,182,877	.022,938,514,472
24 00	.971,012,209,231	.394,946,246,368	.035,682,931,223	.080,145,175,728	.395,225,716,609	.023,653,303,870
24 15	.970,410,590,563	.398,565,924,320	.036,786,872,193	.081,663,828,077	.398,860,102,002	.024,382,241,607
24 30	.969,802,963,133	.402,170,735,519	.037,912,626,023	.083,191,665,089	.402,480,228,425	.025,125,438,314
24 45	.969,189,333,855	.405,760,552,135	.039,060,369,107	.084,728,490,444	.406,085,986,334	.025,883,003,535
25 00	.968,569,709,716	.409,335,247,095	.040,230,276,390	.086,274,106,132	.409,677,267,280	.026,655,045,719
25 15	.967,944,097,765	.412,894,694,078	.041,422,521,348	.087,828,312,454	.413,253,963,911	.027,441,672,217
25 30	.967,312,505,125	.416,438,767,537	.042,637,275,985	.089,390,908,048	.416,815,969,993	.028,242,989,265
25 45	.966,674,938,981	.419,967,342,692	.043,874,710,807	.090,961,689,882	.420,363,180,407	.029,059,101,981
26 00	.966,031,406,590	.423,480,295,541	.045,134,994,824	.092,540,453,277	.423,895,491,161	.029,890,114,357
26 15	.965,381,915,274	.426,977,502,868	.046,418,295,524	.094,126,991,905	.427,412,799,397	.030,736,129,251
26 30	.964,726,472,423	.430,458,842,244	.047,724,778,870	.095,721,097,808	.430,915,003,401	.031,597,248,377
26 45	.964,065,085,490	.433,924,192,037	.049,054,609,285	.097,322,561,410	.434,402,002,608	.032,473,572,300
27 00	.963,397,762,006	.437,373,431,422	.050,407,949,636	.098,931,171,516	.437,873,697,613	.033,365,200,425
27 15	.962,724,509,555	.440,806,440,369	.051,784,961,226	.100,546,715,331	.441,329,990,173	.034,272,230,995
27 30	.962,045,335,797	.444,223,099,673	.053,185,803,780	.102,168,978,470	.444,770,783,221	.035,194,761,077
27 45	.961,360,248,456	.447,623,290,944	.054,610,635,433	.103,797,744,964	.448,195,980,869	.036,132,886,559
28 00	.960,669,255,324	.451,006,896,617	.056,059,612,719	.105,432,797,277	.451,605,488,418	.037,086,702,140
28 15	.959,972,364,257	.454,373,799,957	.057,532,890,554	.107,073,916,309	.454,999,212,361	.038,056,301,327
28 30	.959,269,583,179	.457,723,885,065	.059,030,622,233	.108,720,881,413	.458,377,060,396	.039,041,776,422
28 45	.958,560,920,079	.461,057,036,883	.060,552,959,407	.110,373,470,401	.461,738,941,427	.040,043,218,521
29 00	.957,846,383,016	.464,373,141,204	.062,100,052,083	.112,031,459,562	.465,084,765,578	.041,060,717,500
29 15	.957,125,980,111	.467,672,084,671	.063,672,048,600	.113,694,623,664	.468,414,444,193	.042,094,362,015
29 30	.956,399,719,550	.470,953,754,782	.065,269,095,629	.115,362,735,970	.471,727,889,844	.043,144,239,494
29 45	.955,667,609,591	.474,218,039,907	.066,891,338,151	.117,035,568,249	.475,025,016,344	.044,210,436,124
30 00	.954,929,658,552	.477,464,829,276	.068,538,919,452	.118,712,890,787	.478,305,738,745	.045,293,036,853
30 15	.954,185,874,819	.480,694,012,998	.070,211,981,110	.120,394,472,397	.481,569,973,349	.046,392,125,379
30 30	.953,436,266,841	.483,905,482,060	.071,910,662,982	.122,080,080,431	.484,817,637,714	.047,507,784,144
30 45	.952,680,843,136	.487,099,128,333	.073,635,103,195	.123,769,480,793	.488,048,650,659	.048,640,094,329
31 00	.951,919,612,287	.490,274,844,581	.075,385,438,131	.125,462,437,949	.491,262,932,274	.049,789,135,844
31 15	.951,152,582,940	.493,432,524,460	.077,161,802,419	.127,158,714,936	.494,460,403,919	.050,954,987,329
31 30	.950,379,763,806	.496,572,062,524	.078,964,328,925	.128,858,073,380	.497,640,988,236	.052,137,726,142
31 45	.949,601,163,663	.499,693,354,236	.080,793,148,735	.130,560,273,500	.500,804,609,155	.053,337,428,353
32 00	.948,816,791,351	.502,796,295,965	.082,648,391,151	.132,265,074,128	.503,951,191,894	.054,554,168,744
32 15	.948,026,655,780	.505,880,785,000	.084,530,183,673	.133,972,232,714	.507,080,662,972	.055,788,020,796
32 30	.947,230,765,918	.508,946,719,542	.086,438,651,996	.135,681,505,343	.510,192,950,208	.057,039,056,690
32 45	.946,429,130,800	.511,993,998,722	.088,373,919,991	.137,392,646,742	.513,287,982,733	.058,307,347,295
33 00	.945,621,759,529	.515,022,522,599	.090,336,109,702	.139,105,410,298	.516,365,690,990	.059,592,962,168
33 15	.944,808,661,266	.518,032,192,164	.092,325,341,328	.140,819,548,063	.519,426,006,740	.060,895,969,548
33 30	.943,989,845,241	.521,022,909,347	.094,341,733,219	.142,534,810,774	.522,468,863,072	.062,216,436,346
33 45	.943,165,320,745	.523,994,577,023	.096,385,401,860	.144,250,947,861	.525,494,194,402	.063,554,428,146
34 00	.942,335,097,134	.526,947,099,009	.098,456,461,864	.145,967,707,457	.528,501,936,481	.064,910,009,197
34 15	.941,499,183,828	.529,880,380,079	.100,555,025,960	.147,684,836,418	.531,492,026,399	.066,283,242,409
34 30	.940,657,590,310	.532,794,325,962	.102,681,204,985	.149,402,080,328	.534,464,402,593	.067,674,189,345
34 45	.939,810,326,126	.535,688,843,346	.104,835,107,869	.151,119,183,515	.537,419,004,846	.069,082,910,222
35 00	.938,957,400,888	.538,563,839,887	.107,016,841,631	.152,835,889,063	.540,355,774,295	.070,509,463,903
35 15	.938,098,824,267	.541,419,224,205	.109,226,511,365	.154,551,938,829	.543,274,653,437	.071,953,907,891
35 30	.937,234,606,000	.544,254,905,899	.111,464,220,229	.156,267,073,445	.546,175,586,129	.073,416,298,329
35 45	.936,364,755,886	.547,070,795,541	.113,730,069,440	.157,981,032,345	.549,058,517,595	.074,896,689,993
36 00	.935,489,283,788	.549,866,804,688	.116,024,158,258	.159,693,553,765	.551,923,394,433	.076,395,136,285
36 15	.934,608,199,631	.552,642,845,882	.118,346,583,983	.161,404,374,765	.554,770,164,611	.077,911,689,237
36 30	.933,721,513,398	.555,398,832,649	.120,697,441,938	.163,113,231,237	.557,598,777,480	.079,446,399,498
36 45	.932,829,235,145	.558,134,679,519	.123,076,825,466	.164,819,857,924	.560,409,183,771	.080,999,316,337
37 00	.931,931,374,978	.560,850,302,008	.125,484,825,916	.166,523,988,424	.563,201,335,603	.082,570,487,635
37 15	.931,027,943,074	.563,545,616,643	.127,921,532,636	.168,225,355,211	.565,975,186,486	.084,159,959,882
37 30	.930,118,949,668	.566,220,540,948	.130,387,032,964	.169,923,689,648	.568,730,691,321	.085,767,778,177
37 45	.929,204,405,057	.568,874,993,460	.132,881,412,214	.171,618,721,993	.571,467,806,408	.087,393,986,220
38 00	.928,284,319,604	.571,508,893,729	.135,404,753,675	.173,310,181,425	.574,186,489,448	.089,038,626,310
38 15	.927,358,703,727	.574,122,162,317	.137,957,138,594	.174,997,796,044	.576,886,699,544	.090,701,739,344
38 30	.926,427,567,908	.576,714,720,808	.140,538,646,173	.176,681,292,896	.579,568,397,205	.092,383,364,813
38 45	.925,490,922,692	.579,286,491,806	.143,149,353,555	.178,360,397,978	.582,231,544,354	.094,083,540,794
39 00	.924,548,778,684	.581,837,398,946	.145,789,335,820	.180,034,836,257	.584,876,104,323	.095,802,303,955
39 15	.923,601,146,550	.584,367,366,889	.148,458,665,972	.181,704,331,683	.587,502,041,859	.097,539,689,549
39 30	.922,648,037,015	.586,876,321,327	.151,157,414,933	.183,368,607,202	.590,109,323,131	.099,295,731,407
39 45	.921,689,460,869	.589,364,188,994	.153,885,651,535	.185,027,384,769	.592,697,915,725	.101,070,461,943
40 00	.920,725,428,959	.591,830,897,659	.156,643,442,507	.186,680,385,364	.595,267,788,652	.102,863,912,146
40 15	.919,755,952,192	.594,276,376,131	.159,430,852,473	.188,327,329,005	.597,818,912,348	.104,676,111,580
40 30	.918,781,041,538	.596,700,554,269	.162,247,943,938	.189,967,934,763	.600,351,258,678	.106,507,088,380
40 45	.917,800,708,027	.599,103,362,982	.165,094,777,285	.191,601,920,774	.602,864,800,935	.108,356,869,253
41 00	.916,814,962,747	.601,484,734,224	.167,971,410,761	.193,229,004,256	.605,359,513,845	.110,225,479,473
41 15	.915,823,816,844	.603,844,601,005	.170,877,900,474	.194,848,901,521	.607,835,373,567	.112,112,942,881
41 30	.914,827,281,530	.606,182,897,397	.173,814,300,383	.196,461,327,992	.610,292,357,699	.114,019,281,879
41 45	.913,825,368,073	.608,499,558,526	.176,780,662,289	.198,065,998,214	.612,730,445,273	.115,944,517,434
42 00	.912,818,087,796	.610,794,520,582	.179,777,035,829	.199,662,625,871	.615,149,616,760	.117,888,669,077
42 15	.911,805,452,089	.613,067,720,823	.182,803,468,470	.201,250,923,801	.617,549,854,075	.119,851,754,892
42 30	.910,787,472,395	.615,319,097,569	.185,860,005,495	.202,830,604,005	.619,931,140,572	.121,833,791,525
42 45	.909,764,160,220	.617,548,590,217	.188,946,690,002	.204,401,377,672	.622,293,461,047	.123,834,794,180
43 00	.908,735,527,126	.619,756,139,230	.192,063,562,894	.205,962,955,184	.624,636,801,743	.125,854,776,614
43 15	.907,701,584,737	.621,941,686,154	.195,210,662,873	.207,515,046,134	.626,961,150,349	.127,893,751,138
43 30	.906,662,344,731	.624,105,173,605	.198,388,026,429	.209,057,359,344	.629,266,495,997	.129,951,728,620
43 45	.905,617,818,846	.626,246,545,281	.201,595,687,837	.210,589,602,875	.631,552,829,269	.132,028,718,478
44 00	.904,568,018,881	.628,365,745,965	.204,833,679,148	.212,111,484,043	.633,820,142,193	.134,124,728,684
44 15	.903,512,956,688	.630,462,721,521	.208,102,030,183	.213,622,709,437	.636,068,428,248	.136,239,765,759
44 30	.902,452,644,181	.632,537,418,898	.211,400,768,526	.215,122,984,932	.638,297,682,358	.138,373,834,779
44 45	.901,387,093,328	.634,589,786,138	.214,729,919,515	.216,612,015,701	.640,507,900,899	.140,526,939,368
45 00	.900,316,316,158	.636,619,772,369	.218,089,506,239	.218,089,506,239	.642,699,081,699	.142,699,081,699

TABLE 12—Continued

ϕ_1 ° ′	$\frac{\sin\phi_1}{\phi_1}$	$\frac{\sin^2\phi_1}{\phi_1}$	$\frac{\phi_1^2}{2}\sin\phi_1$	$\frac{\phi_1^2}{2}\cos\phi_1$	$\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}$	$\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}$
45 15	.899,240,324,754	.638,627,327,811	.221,479,549,528	.219,555,160,364	.644,871,224,029	.144,890,262,498
45 30	.898,159,131,261	.640,612,403,784	.224,900,067,948	.221,008,681,247	.647,024,328,618	.147,100,481,039
45 45	.897,072,747,875	.642,574,952,696	.228,351,077,799	.222,449,771,420	.649,158,397,638	.149,329,735,149
46 00	.895,981,186,853	.644,514,928,058	.231,832,593,097	.223,878,132,788	.651,273,434,713	.151,578,021,204
46 15	.894,884,460,506	.646,432,284,477	.235,344,625,580	.225,293,466,652	.653,369,444,919	.153,845,334,128
46 30	.893,782,581,204	.648,326,977,662	.238,887,184,697	.226,695,473,720	.655,446,434,777	.156,131,667,400
46 45	.892,675,561,371	.650,198,964,426	.242,460,277,600	.228,083,854,122	.657,504,412,259	.158,437,013,048
47 00	.891,563,413,491	.652,048,202,685	.246,063,909,141	.229,458,307,428	.659,543,386,783	.160,761,361,654
47 15	.890,446,150,099	.653,874,651,458	.249,698,081,866	.230,818,532,662	.661,563,369,217	.163,104,702,350
47 30	.889,323,783,787	.655,678,270,872	.253,362,796,007	.232,164,228,318	.663,564,371,872	.165,467,022,825
47 45	.888,196,327,206	.657,459,022,165	.257,058,049,479	.233,495,092,373	.665,546,408,505	.167,848,309,322
48 00	.887,063,793,059	.659,216,867,680	.260,783,837,873	.234,810,822,308	.667,509,494,320	.170,248,546,637
48 15	.885,926,194,107	.660,951,770,874	.264,540,154,451	.236,111,115,121	.669,453,645,963	.172,667,718,124
48 30	.884,783,543,163	.662,663,696,311	.268,326,990,140	.237,395,667,339	.671,378,881,519	.175,105,805,698
48 45	.883,635,853,097	.664,352,609,674	.272,144,333,530	.238,664,175,042	.673,285,220,517	.177,562,789,830
49 00	.882,483,136,834	.666,018,477,754	.275,992,170,862	.239,916,333,871	.675,172,683,924	.180,038,649,553
49 15	.881,325,407,352	.667,661,268,458	.279,870,486,029	.241,151,839,048	.677,041,294,144	.182,533,362,463
49 30	.880,162,677,686	.669,280,950,811	.283,779,260,571	.242,370,385,389	.678,891,075,017	.185,046,904,720
49 45	.878,994,960,923	.670,877,494,952	.287,718,473,667	.243,571,667,326	.680,722,051,818	.187,579,251,049
50 00	.877,822,270,204	.672,450,872,136	.291,688,102,131	.244,755,378,917	.682,534,251,252	.190,130,374,745
50 15	.876,644,618,724	.674,001,054,737	.295,688,120,408	.245,921,213,862	.684,327,701,455	.192,700,247,672
50 30	.875,462,019,736	.675,528,016,251	.299,718,500,571	.247,068,865,524	.686,102,431,991	.195,288,840,266
50 45	.874,274,486,540	.677,031,731,285	.303,779,212,314	.248,198,026,940	.687,858,473,849	.197,896,121,538
51 00	.873,082,032,493	.678,512,175,573	.307,870,222,949	.249,308,390,840	.689,595,859,442	.200,522,059,075
51 15	.871,884,671,004	.679,969,325,963	.311,991,497,399	.250,399,649,663	.691,314,622,603	.203,166,619,044
51 30	.870,682,415,537	.681,403,160,427	.316,142,998,201	.251,471,495,572	.693,014,798,585	.205,829,766,192
51 45	.869,475,279,607	.682,813,658,058	.320,324,685,494	.252,523,620,472	.694,696,424,053	.208,511,463,854
52 00	.868,263,276,781	.684,200,799,065	.324,536,517,018	.253,555,716,023	.696,359,537,088	.211,211,673,949
52 15	.867,046,420,680	.685,564,564,784	.328,778,448,112	.254,567,473,660	.698,004,177,178	.213,930,356,989
52 30	.865,824,724,977	.686,904,937,667	.333,050,431,709	.255,558,584,609	.699,630,385,221	.216,667,472,076
52 45	.864,598,203,397	.688,221,901,292	.337,352,418,331	.256,528,739,900	.701,238,203,516	.219,422,976,911
53 00	.863,366,869,716	.689,515,440,353	.341,684,356,086	.257,477,630,386	.702,827,675,763	.222,196,827,794
53 15	.862,130,737,764	.690,785,540,672	.346,046,190,669	.258,404,946,762	.704,398,847,061	.224,988,979,626
53 30	.860,889,821,420	.692,032,189,184	.350,437,865,348	.259,310,379,574	.705,951,763,899	.227,799,385,918
53 45	.859,644,134,616	.693,255,373,951	.354,859,320,975	.260,193,619,244	.707,486,474,161	.230,627,998,786
54 00	.858,393,691,334	.694,455,084,153	.359,310,495,971	.261,054,356,081	.709,003,027,112	.233,474,768,965
54 15	.857,138,505,608	.695,631,310,092	.363,791,326,328	.261,892,280,300	.710,501,473,405	.236,339,645,802
54 30	.855,878,591,521	.696,784,043,186	.368,301,745,607	.262,707,082,036	.711,981,865,068	.239,222,577,269
54 45	.854,613,963,211	.697,913,275,980	.372,841,684,933	.263,498,451,364	.713,444,255,507	.242,123,509,960
55 00	.853,344,634,859	.699,019,002,128	.377,411,072,994	.264,266,078,314	.714,888,699,495	.245,042,389,102
55 15	.852,070,620,704	.700,101,216,412	.382,009,836,035	.265,009,652,887	.716,315,253,175	.247,979,158,552
55 30	.850,791,935,031	.701,159,914,727	.386,637,897,860	.265,728,865,073	.717,723,974,053	.250,933,760,804
55 45	.849,508,592,175	.702,195,094,087	.391,295,179,829	.266,423,404,868	.719,114,920,989	.253,906,136,998
56 00	.848,220,606,521	.703,206,752,621	.395,981,600,853	.267,092,962,288	.720,488,154,200	.256,896,226,917
56 15	.846,927,992,504	.704,194,889,576	.400,697,077,391	.267,737,227,389	.721,843,735,252	.259,903,968,995
56 30	.845,630,764,606	.705,159,505,309	.405,441,523,452	.268,355,890,282	.723,181,727,052	.262,929,300,325
56 45	.844,328,937,362	.706,100,601,298	.410,214,850,593	.268,948,641,151	.724,502,193,850	.265,972,156,657
57 00	.843,022,525,352	.707,018,180,127	.415,016,967,913	.269,515,170,269	.725,805,201,229	.269,032,472,408
57 15	.841,711,543,208	.707,912,245,500	.419,847,782,054	.270,055,168,013	.727,090,816,103	.272,110,180,664
57 30	.840,396,005,609	.708,782,802,226	.424,707,197,199	.270,568,324,886	.728,359,106,708	.275,205,213,189
57 45	.839,075,927,280	.709,629,856,223	.429,595,115,070	.271,054,331,530	.729,610,142,601	.278,317,500,426
58 00	.837,751,322,997	.710,453,414,520	.434,511,434,929	.271,512,878,741	.730,843,994,653	.281,446,971,504
58 15	.836,422,207,585	.711,253,485,256	.439,456,053,571	.271,943,657,492	.732,060,735,044	.284,593,554,243
58 30	.835,088,595,912	.712,030,077,669	.444,428,865,328	.272,346,358,945	.733,260,437,255	.287,757,175,161
58 45	.833,750,502,898	.712,783,202,107	.449,429,762,070	.272,720,674,472	.734,443,176,068	.290,937,759,478
59 00	.832,407,943,506	.713,512,870 018	.454,458,633,195	.273,066,295,664	.735,609,027,553	.294,135,231,123
59 15	.831,060,932,751	.714,219,093,954	.459,515,365,635	.273,382,914,359	.736,758,069,068	.297,349,512,738
59 30	.829,709,485,693	.714,901,887,568	.464,599,843,856	.273,670,222,653	.737,890,379,253	.300,580,525,683
59 45	.828,353,617,435	.715,561,265,605	.469,711,949,851	.273,927,912,915	.739,006,038,018	.303,828,190,048
60 00	.826,993,343,133	.716,197,243,913	.474,851,563,146	.274,155,677,808	.740,105,126,544	.307,092,424,652
60 15	.825,628,677,983	.716,809,839,433	.480,018,560,798	.274,353,210,305	.741,187,727,273	.310,373,147,053
60 30	.824,259,637,232	.717,399,070,198	.485,212,817,389	.274,520,203,706	.742,253,923,903	.313,670,273,553
60 45	.822,886,236,170	.717,964,955,334	.490,434,205,034	.274,656,351,655	.743,303,801,382	.316,983,719,204
61 00	.821,508,490,132	.718,507,515,051	.495,682,593,378	.274,761,348,157	.744,337,445,897	.320,313,397,819
61 15	.820,126,414,502	.719,026,770,657	.500,957,849,592	.274,834,887,594	.745,354,944,876	.323,659,221,970
61 30	.818,740,024,706	.719,522,744,533	.506,259,838,378	.274,876,664,745	.746,356,386,975	.327,021,103,002
61 45	.817,349,336,215	.719,995,460,150	.511,588,421,966	.274,886,374,800	.747,341,862,070	.330,398,951,036
62 00	.815,954,364,549	.720,444,942,061	.516,943,460,119	.274,863,713,381	.748,311,461,257	.333,792,674,979
62 15	.814,555,125,268	.720,871,215,894	.522,324,810,125	.274,808,376,553	.749,265,276,838	.337,202,182,528
62 30	.813,151,633,976	.721,274,308,353	.527,732,326,807	.274,720,060,847	.750,203,402,320	.340,627,380,176
62 45	.811,743,906,326	.721,654,247,221	.533,165,862,518	.274,598,463,276	.751,125,932,402	.344,068,173,224
63 00	.810,331,958,009	.722,011,061,344	.538,625,267,142	.274,443,281,350	.752,032,962,972	.347,524,465,784
63 15	.808,915,804,767	.722,344,780,649	.544,110,388,096	.274,254,213,093	.752,924,591,097	.350,996,160,789
63 30	.807,495,462,378	.722,655,436,120	.549,621,070,331	.274,030,957,066	.753,800,915,020	.354,483,159,996
63 45	.806,070,946,669	.722,943,059,809	.555,157,156,334	.273,773,212,375	.754,662,034,146	.357,985,364,000
64 00	.804,642,273,507	.723,207,684,829	.560,718,486,125	.273,480,678,694	.755,508,049,040	.361,502,672,236
64 15	.803,209,458,803	.723,449,345,351	.566,304,897,264	.273,153,056,285	.756,339,061,416	.365,034,982,990
64 30	.801,772,518,510	.723,668,076,603	.571,916,224,848	.272,790,046,005	.757,155,174,132	.368,582,193,404
64 45	.800,331,468,624	.723,863,914,866	.577,552,301,516	.272,391,349,337	.757,956,491,180	.372,144,199,486
65 00	.798,886,325,186	.724,036,897,473	.583,212,957,446	.271,956,668,393	.758,743,117,678	.375,720,896,118
65 15	.797,437,104,273	.724,187,062,800	.588,898,020,362	.271,485,705,940	.759,515,159,863	.379,312,177,063
65 30	.795,983,822,010	.724,314,450,274	.594,607,315,531	.270,978,165,419	.760,272,725,084	.382,917,934,972
65 45	.794,526,494,558	.724,419,100,354	.600,340,665,768	.270,433,750,953	.761,015,921,790	.386,538,061,396
66 00	.793,065,138,126	.724,501,054,550	.606,097,891,439	.269,852,167,374	.761,744,859,527	.390,172,446,789
66 15	.791,599,768,957	.724,560,355,394	.611,878,810,458	.269,233,120,231	.762,459,648,926	.393,820,980,520
66 30	.790,130,403,342	.724,597,046,463	.617,683,238,296	.268,576,315,817	.763,160,401,693	.397,483,550,883
66 45	.788,657,057,607	.724,611,172,351	.623,510,987,977	.267,881,461,178	.763,847,230,606	.401,160,045,100
67 00	.787,179,748,121	.724,602,778,685	.629,361,870,086	.267,148,264,134	.764,520,249,502	.404,850,349,334
67 15	.785,698,491,295	.724,571,912,112	.635,235,692,767	.266,376,433,296	.765,179,573,272	.408,554,348,694
67 30	.784,213,303,576	.724,518,620,297	.641,132,261,729	.265,565,678,083	.765,825,317,845	.412,271,927,251

TABLE 12—Continued

ϕ_1 ° ′	$\frac{\sin\phi_1}{\phi_1}$	$\frac{\sin^2\phi_1}{\phi_1}$	$\frac{\phi_1^2}{2}\sin\phi_1$	$\frac{\phi_1^2}{2}\cos\phi_1$	$\frac{\phi_1+\sin\phi_1\cos\phi_1}{2}$	$\frac{\phi_1-\sin\phi_1\cos\phi_1}{2}$
67 45	.782,724,201,456	.724,442,951,923	.647,051,380,248	.264,715,708,738	.766,457,600,188	.416,002,968,038
68 00	.781,231,201,461	.724,344,956,679	.652,992,849,167	.263,826,236,346	.767,076,538,293	.419,747,353,063
68 15	.779,734,320,161	.724,224,685,268	.658,956,466,905	.262,896,972,853	.767,682,251,167	.423,504,963,319
68 30	.778,233,574,164	.724,082,189,395	.664,942,029,454	.261,927,631,078	.768,274,858,823	.427,275,678,793
68 45	.776,728,980,115	.723,917,521,767	.670,949,330,386	.260,917,924,738	.768,854,482,277	.431,059,378,469
69 00	.775,220,554,700	.723,730,736,086	.676,978,160,855	.259,867,568,457	.769,421,243,528	.434,855,940,348
69 15	.773,708,314,644	.723,521,887,050	.683,028,309,602	.258,776,277,788	.769,975,265,557	.438,665,241,449
69 30	.772,192,276,708	.723,291,030,344	.689,099,562,955	.257,643,769,228	.770,516,672,315	.442,487,157,821
69 45	.770,672,457,693	.723,038,222,641	.695,191,704,836	.256,469,760,239	.771,045,588,715	.446,321,564,551
70 00	.769,148,874,438	.722,763,521,595	.701,304,516,764	.255,253,969,259	.771,562,140,620	.450,168,335,776
70 15	.767,621,543,817	.722,466,985,834	.707,437,777,859	.253,996,115,722	.772,066,454,832	.454,027,344,694
70 30	.766,090,482,745	.722,148,674,966	.713,591,264,847	.252,695,920,078	.772,558,659,091	.457,898,463,565
70 45	.764,555,708,172	.721,808,649,564	.719,764,752,058	.251,353,103,805	.773,038,882,053	.461,781,563,733
71 00	.763,017,237,085	.721,446,971,166	.725,958,011,440	.249,967,389,427	.773,507,253,290	.465,676,515,626
71 15	.761,475,086,510	.721,063,702,276	.732,170,812,558	.248,538,500,538	.773,963,903,275	.469,583,188,771
71 30	.759,929,273,507	.720,658,906,350	.738,402,922,596	.247,066,161,807	.774,408,963,376	.473,501,451,800
71 45	.758,379,815,173	.720,232,647,799	.744,654,106,367	.245,550,099,005	.774,842,565,841	.477,431,172,465
72 00	.756,826,728,640	.719,784,991,979	.750,924,126,313	.243,990,039,016	.775,264,843,791	.481,372,217,645
72 15	.755,270,031,080	.719,316,005,197	.757,212,742,515	.242,385,709,858	.775,675,931,211	.485,324,453,355
72 30	.753,709,739,696	.718,825,754,692	.763,519,712,692	.240,736,840,697	.776,075,962,936	.489,287,744,760
72 45	.752,145,871,728	.718,314,308,641	.769,844,792,211	.239,043,161,867	.776,465,074,644	.493,261,956,182
73 00	.750,578,444,452	.717,781,736,152	.776,187,734,088	.237,304,404,883	.776,843,402,846	.497,246,951,110
73 15	.749,007,475,177	.717,228,107,258	.782,548,288,996	.235,520,302,459	.777,211,084,871	.501,242,592,215
73 30	.747,432,981,249	.716,653,492,913	.788,926,205,269	.233,690,588,530	.777,568,258,862	.505,248,741,354
73 45	.745,854,980,047	.716,057,964,987	.795,321,228,909	.231,814,998,260	.777,915,063,760	.509,265,259,586
74 00	.744,273,488,984	.715,441,596,262	.801,733,103,589	.229,893,268,064	.778,251,639,297	.513,292,007,179
74 15	.742,688,525,509	.714,804,460,430	.808,161,570,665	.227,925,135,628	.778,578,125,982	.517,328,843,624
74 30	.741,100,107,103	.714,146,632,081	.814,606,369,167	.225,910,339,917	.778,894,665,095	.521,375,627,641
74 45	.739,508,251,279	.713,468,186,701	.821,067,235,825	.223,848,621,200	.779,201,398,673	.525,432,217,193
75 00	.737,912,975,587	.712,769,200,673	.827,543,905,058	.221,739,721,062	.779,498,469,498	.529,498,469,498
75 15	.736,314,297,609	.712,049,751,267	.834,036,108,991	.219,583,382,421	.779,786,021,089	.533,574,241,036
75 30	.734,712,234,957	.711,309,916,630	.840,543,577,455	.217,379,349,547	.780,064,197,689	.537,659,387,566
75 45	.733,106,805,279	.710,549,775,793	.847,066,037,996	.215,127,368,079	.780,333,144,257	.541,753,764,128
76 00	.731,498,026,253	.709,769,408,653	.853,603,215,880	.212,827,185,036	.780,593,006,454	.545,857,225,061
76 15	.729,885,915,591	.708,968,895,978	.860,154,834,103	.210,478,548,841	.780,843,930,631	.549,969,624,014
76 30	.728,270,491,037	.708,148,319,398	.866,720,613,390	.208,081,209,332	.781,086,063,822	.554,090,813,953
76 45	.726,651,770,363	.707,307,761,394	.873,300,272,211	.205,634,917,781	.781,319,553,730	.558,220,647,175
77 00	.725,029,771,378	.706,447,305,309	.879,893,526,782	.203,139,426,911	.781,544,548,715	.562,358,975,320
77 15	.723,404,511,918	.705,567,035,321	.886,500,091,074	.200,594,490,911	.781,761,197,785	.566,505,649,380
77 30	.721,776,009,851	.704,667,036,453	.893,119,676,818	.197,999,865,452	.781,969,650,583	.570,660,519,712
77 45	.720,144,283,077	.703,747,394,564	.899,751,993,517	.195,355,307,705	.782,170,057,376	.574,823,436,049
78 00	.718,509,349,525	.702,808,196,343	.906,396,748,446	.192,660,576,359	.782,362,569,046	.578,994,247,509
78 15	.716,871,227,154	.701,849,529,300	.913,053,646,668	.189,915,431,630	.782,547,337,074	.583,172,802,611
78 30	.715,229,933,954	.700,871,481,766	.919,722,391,031	.187,119,635,286	.782,724,513,530	.587,358,949,285
78 45	.713,585,487,945	.699,874,142,886	.926,402,682,186	.184,272,950,660	.782,894,251,064	.591,552,534,881
79 00	.711,937,907,176	.698,857,602,611	.933,094,218,590	.181,375,142,665	.783,056,702,892	.595,753,406,183
79 15	.710,287,209,725	.697,821,951,694	.939,796,696,513	.178,425,977,807	.783,212,022,784	.599,961,409,421
79 30	.708,633,413,698	.696,767,281,682	.946,509,810,045	.175,425,224,212	.783,360,365,054	.604,176,390,281
79 45	.706,976,537,232	.695,693,684,917	.953,233,251,109	.172,372,651,631	.783,501,884,547	.608,398,193,918
80 00	.705,316,598,492	.694,601,254,523	.959,966,709,467	.169,268,031,461	.783,636,736,629	.612,626,664,966
80 15	.703,653,615,671	.693,490,084,403	.966,709,872,722	.166,111,136,761	.783,765,077,171	.616,861,647,554
80 30	.701,987,606,989	.692,360,269,231	.973,462,426,338	.162,901,742,266	.783,887,062,542	.621,102,985,313
80 45	.700,318,590,695	.691,211,904,450	.980,224,053,636	.159,639,624,406	.784,002,849,594	.625,350,521,391
81 00	.698,646,585,067	.690,045,086,267	.986,994,435,815	.156,324,561,320	.784,112,595,651	.629,604,098,464
81 15	.696,971,608,406	.688,859,911,637	.993,773,251,950	.152,956,332,871	.784,216,458,499	.633,863,558,746
81 30	.695,293,679,045	.687,656,478,271	1.000,560,179,005	.149,534,720,663	.784,314,596,368	.638,128,744,007
81 45	.693,612,815,340	.686,434,884,617	1.007,354,891,845	.146,059,508,056	.784,407,167,928	.642,399,495,577
82 00	.691,929,035,677	.685,195,229,866	1.014,157,063,238	.142,530,480,187	.784,494,332,271	.646,675,654,364
82 15	.690,242,358,465	.683,937,613,935	1.020,966,363,872	.138,947,423,975	.784,576,248,902	.650,957,060,863
82 30	.688,552,802,142	.682,662,137,468	1.027,782,462,359	.135,310,128,146	.784,653,077,723	.655,243,555,172
82 45	.686,860,385,169	.681,368,901,827	1.034,605,025,242	.131,618,383,246	.784,724,979,026	.659,534,976,999
83 00	.685,165,126,036	.680,058,009,086	1.041,433,717,013	.127,871,981,653	.784,792,113,477	.663,831,165,678
83 15	.683,467,043,256	.678,729,562,026	1.048,268,200,115	.124,070,717,599	.784,854,642,107	.668,131,960,178
83 30	.681,766,155,369	.677,383,664,127	1.055,108,134,953	.120,214,387,180	.784,912,726,294	.672,437,199,121
83 45	.680,062,480,936	.676,020,419,561	1.061,953,179,904	.116,302,788,373	.784,966,527,757	.676,746,720,788
84 00	.678,356,038,547	.674,639,933,190	1.068,802,991,332	.112,335,721,054	.785,016,208,542	.681,060,363,133
84 15	.676,646,846,815	.673,242,310,555	1.075,657,223,590	.108,312,987,008	.785,061,931,007	.685,377,963,798
84 30	.674,934,924,376	.671,827,657,869	1.082,515,529,033	.104,234,389,949	.785,103,857,812	.689,699,360,123
84 45	.673,220,289,892	.670,396,082,018	1.089,377,558,029	.100,099,735,535	.785,142,151,905	.694,024,389,160
85 00	.671,502,962,047	.668,947,690,544	1.096,242,958,968	.095,908,831,380	.785,176,976,515	.698,352,887,680
85 15	.669,782,959,548	.667,482,591,647	1.103,111,378,275	.091,661,487,069	.785,208,495,128	.702,684,692,197
85 30	.668,060,301,127	.666,000,894,172	1.109,982,460,418	.087,357,514,180	.785,236,871,488	.707,019,638,967
85 45	.666,335,005,537	.664,502,707,610	1.116,855,847,915	.082,996,726,289	.785,262,269,575	.711,357,564,010
86 00	.664,607,091,557	.662,988,142,085	1.123,731,181,356	.078,578,938,994	.785,284,853,597	.715,698,303,118
86 15	.662,876,577,983	.661,457,308,347	1.130,608,099,398	.074,103,969,921	.785,304,787,977	.720,041,691,868
86 30	.661,143,483,638	.659,910,317,769	1.137,486,238,788	.069,571,638,750	.785,322,237,339	.724,387,565,636
86 45	.659,407,827,365	.658,347,282,341	1.144,365,234,372	.064,981,767,216	.785,337,366,494	.728,735,759,611
87 00	.657,669,628,029	.656,768,314,661	1.151,244,719,101	.060,334,179,139	.785,350,340,435	.733,086,108,800
87 15	.655,928,904,516	.655,173,527,925	1.158,124,324,042	.055,628,700,425	.785,361,324,312	.737,438,448,053
87 30	.654,185,675,734	.653,563,035,926	1.165,003,678,402	.050,865,159,090	.785,370,483,434	.741,792,612,061
87 45	.652,439,960,610	.651,936,953,045	1.171,882,409,519	.046,043,385,270	.785,377,983,244	.746,148,435,381
88 00	.650,691,778,094	.650,295,394,244	1.178,760,142,889	.041,163,211,236	.785,383,989,314	.750,505,752,441
88 15	.648,941,147,156	.648,638,475,059	1.185,636,502,175	.036,224,471,409	.785,388,667,326	.754,864,397,559
88 30	.647,188,086,785	.646,966,311,592	1.192,511,109,209	.031,227,002,376	.785,392,183,068	.759,224,204,947
88 45	.645,432,615,990	.645,279,020,506	1.199,383,584,017	.026,170,642,902	.785,394,702,414	.763,585,008,731
89 00	.643,674,753,801	.643,576,719,018	1.206,253,544,822	.021,055,233,944	.785,396,391,313	.767,946,642,962
89 15	.641,914,519,268	.641,859,524,894	1.213,120,608,060	.015,880,618,669	.785,397,415,779	.772,308,941,626
89 30	.640,151,931,456	.640,127,556,432	1.219,984,388,387	.010,646,642,464	.785,397,941,877	.776,671,738,658
89 45	.638,387,009,455	.638,380,932,471	1.226,844,498,699	.005,353,152,951	.785,398,135,707	.781,034,867,958
90 00	.636,619,772,368	.636,619,772,368	1.233,700,550,136	0	.785,398,163,397	.785,398,163,397

TABLE 13—TRIGONOMETRIC FUNCTIONS AND INTEGRALS FOR ARCH LOAD CONSTANTS

ϕ_1 ° ′	$\frac{\phi_1^2}{2}$	$\frac{\phi_1^3}{6}$	$\frac{\sin^2\phi_1}{2}$	$\phi_1 - \sin\phi_1$	$\frac{\phi_1^2}{2} - \text{vers}\,\phi_1$	$\sin\phi_1 - \phi_1 + \frac{\phi_1^3}{6}$
0 15	.0⁵9,519,294,368	.0⁷13,845,252,433	.0⁵9,519,233,958	.0⁷13,845,239	.0¹⁰15,102,818	.0¹³13,179,697
0 30	.0⁴38,077,177,473	.0⁶110,762,019,463	.0⁴38,076,210,90	.0⁶110,761,598	.0⁹241,644,627	.0¹²421,749,742
0 45	.0⁴85,673,649,315	.0⁶373,821,815,69	.0⁴85,668,756,11	.0⁶373,818,613	.0⁸1,223,322	.0¹13,202,654,8
1 00	.0³152,308,709,89	.0⁶886,096,155,70	.0³152,293,245,22	.0⁶886,082,660	.0⁸3,866,284	.0¹0¹3,495,918
1 15	.0³237,982,359,21	.0⁵1,730,656,554	.0³237,944,604,54	.0⁵1,730,615	.0⁸9,439,117	.0¹0⁴1,186,106
1 30	.0³342,694,597,26	.0⁵2,990,574,525	.0³342,616,311,36	.0⁵2,990,472	.0⁷19,572,817	.0⁹102,483,701
1 45	.0³466,445,424,05	.0⁵4,748,921,584	.0³466,300,394,53	.0⁵4,748,700	.0⁷36,260,761	.0⁹221,506,354
2 00	.0³609,234,839,57	.0⁵7,088,769,246	.0³608,987,435,04	.0⁵7,088,337	.0⁷61,858,669	.0⁹431,859,990
2 15	.0³771,062,843,84	.0⁴10,093,189,024	.0³770,666,566,72	.0⁴10,092,411	.0⁷99,084,559	.0⁹778,219,728
2 30	.0³951,929,436,84	.0⁴13,845,252,433	.0³951,325,477,05	.0⁴13,843,935	.0⁶151,018,691	.0⁸1,317,910,6
2 45	.001,151,834,619	.0⁴18,428,030,988	.001,150,950,408	.0⁴18,425,909	.0⁶221,103,519	.0⁸2,122,488
3 00	.001,370,778,389	.0⁴23,924,596,204	.001,369,526,158	.0⁴23,921,317	.0⁶313,143,614	.0⁸3,279,317,9
3 15	.001,608,760,748	.0⁴30,418,019,595	.001,607,036,081	.0⁴30,413,127	.0⁶431,305,597	.0⁸4,893,157
3 30	.001,865,781,696	.0⁴37,991,372,676	.001,863,462,090	.0⁴37,984,285	.0⁶580,118,061	.0⁸7,087,731
3 45	.002,141,841,233	.0⁴46,727,726,961	.002,138,784,657	.0⁴46,717,720	.0⁶764,471,479	.0⁷10,007,317
4 00	.002,436,939,358	.0⁴56,710,153,965	.002,432,982,815	.0⁴56,696,336	.0⁶989,618,118	.0⁷13,818,317
4 15	.002,751,076,072	.0⁴68,021,725,202	.002,746,034,159	.0⁴68,003,014	.0⁵1,261,172	.0⁷18,710,843
4 30	.003,084,251,375	.0⁴80,745,512,188	.003,077,914,851	.0⁴80,720,612	.0⁵1,585,109	.0⁷24,900,288
4 45	.003,436,465,267	.0⁴94,964,586,437	.003,428,599,616	.0⁴94,931,958	.0⁵1,967,765	.0⁷32,628,910
5 00	.003,807,717,747	.0³110,762,019,46	.003,798,061,747	.0³110,719,852	.0⁵2,415,839	.0⁷42,167,404
5 15	.004,198,008,816	.0³128,220,882,78	.004,186,273,109	.0³128,167,067	.0⁵2,936,391	.0⁷53,816,480
5 30	.004,607,338,474	.0³147,424,247,90	.004,593,204,138	.0³147,356,340	.0⁵3,536,841	.0⁷67,908,441
5 45	.005,035,706,721	.0³168,455,186,35	.005,018,823,845	.0³168,370,378	.0⁵4,224,972	.0⁷84,808,753
6 00	.005,483,113,556	.0³191,396,769,63	.005,463,099,817	.0³191,291,852	.0⁵5,008,924	.0⁶104,917,625
6 15	.005,949,558,980	.0³216,332,069,26	.005,925,998,220	.0³216,203,398	.0⁵5,897,202	.0⁶128,671,582
6 30	.006,435,042,993	.0³243,344,156,76	.006,407,483,804	.0³243,187,612	.0⁵6,898,670	.0⁶156,545,035
6 45	.006,939,565,595	.0³272,516,103,66	.006,907,519,901	.0³272,327,052	.0⁵8,022,550	.0⁶189,051,856
7 00	.007,463,126,785	.0³303,930,981,42	.007,426,068,431	.0³303,704,235	.0⁵9,278,426	.0⁶226,746,950
7 15	.008,005,726,564	.0³337,671,861,59	.007,963,089,905	.0³337,401,634	.0⁴10,676,244	.0⁶270,227,825
7 30	.008,567,364,932	.0³373,821,815,71	.008,518,543,428	.0³373,501,680	.0⁴12,226,306	.0⁶320,136,163
7 45	.009,148,041,888	.0³412,463,915,23	.009,092,386,698	.0³412,086,756	.0⁴13,939,275	.0⁶377,159,389
8 00	.009,747,757,433	.0³453,681,231,71	.009,684,576,015	.0³453,239,200	.0⁴15,826,175	.0⁶442,032,238
8 15	.010,366,511,567	.0³497,556,836,67	.010,295,066,283	.0³497,041,299	.0⁴17,898,387	.0⁶515,538,326
8 30	.011,004,304,290	.0³544,173,801,63	.010,923,811,009	.0³543,575,289	.0⁴20,167,652	.0⁶598,511,713
8 45	.011,661,135,601	.0³593,615,198,05	.011,570,762,313	.0³592,923,359	.0⁴22,646,069	.0⁶691,838,471
9 00	.012,337,005,501	.0³645,964,097,49	.012,235,870,926	.0³645,167,639	.0⁴25,346,096	.0⁶796,458,247
9 15	.013,031,913,990	.0³701,303,571,46	.012,919,086,198	.0³700,390,205	.0⁴28,280,550	.0⁶913,365,831
9 30	.013,745,861,068	.0³759,716,691,50	.013,620,356,100	.0³758,673,078	.0⁴31,462,605	.0⁵1,043,613
9 45	.014,478,846,734	.0³821,286,529,05	.014,339,627,227	.0³820,098,220	.0⁴34,905,792	.0⁵1,188,309
10 00	.015,230,870,989	.0³886,096,155,68	.015,076,844,804	.0³884,747,532	.0⁴38,624,001	.0⁵1,348,623
10 15	.016,001,933,833	.0³954,228,642,92	.015,831,952,688	.0³952,702,855	.0⁴42,631,479	.0⁵1,525,787
10 30	.016,792,035,266	.001,025,767,062	.016,604,893,376	.001,024,045,967	.0⁴46,942,830	.0⁵1,721,095
10 45	.017,601,175,287	.001,100,794,485	.017,395,608,005	.001,098,858,580	.0⁴51,573,013	.0⁵1,935,905
11 00	.018,429,353,897	.001,179,393,983	.018,204,036,358	.001,177,222,342	.0⁴56,537,345	.0⁵2,171,640
11 15	.019,276,571,096	.001,261,648,628	.019,030,116,872	.001,259,218,833	.0⁴61,851,499	.0⁵2,429,795
11 30	.020,142,826,883	.001,347,641,491	.019,873,786,637	.001,344,929,562	.0⁴67,531,504	.0⁵2,711,929
11 45	.021,028,121,260	.001,437,455,643	.020,734,981,404	.001,434,435,969	.0⁴73,593,745	.0⁵3,019,674
12 00	.021,932,454,225	.001,531,174,157	.021,613,635,589	.001,527,819,421	.0⁴80,054,959	.0⁵3,354,735
12 15	.022,855,825,778	.001,628,880,103	.022,509,682,281	.001,625,161,213	.0⁴86,932,241	.0⁵3,718,891
12 30	.023,798,235,921	.001,730,656,554	.023,423,053,241	.001,726,542,561	.0⁴94,243,041	.0⁵4,113,993
12 45	.024,759,684,652	.001,836,586,580	.024,353,678,913	.001,832,044,607	.0³3,102,005,161	.0⁵4,541,973
13 00	.025,740,171,972	.001,946,753,254	.025,301,488,425	.001,941,748,415	.0³3,110,236,757	.0⁵5,004,839
13 15	.026,739,697,881	.002,061,239,646	.026,266,409,599	.002,055,734,967	.0³3,118,956,341	.0⁵5,504 680
13 30	.027,758,262,378	.002,180,128,829	.027,248,368,953	.002,174,085,163	.0³3,128,182,776	.0⁵6,043,666
13 45	.028,795,865,464	.002,303,503,873	.028,247,291,705	.002,296,879,823	.0³3,137,935,277	.0⁵6,624,050
14 00	.029,852,507,139	.002,431,447,851	.029,263,101,785	.002,424,199,679	.0³3,148,233,415	.0⁵7,248,172
14 15	.030,928,187,403	.002,564,043,834	.030,295,721,835	.002,556,125,380	.0³3,159,097,110	.0⁵7,918,454
14 30	.032,022,906,255	.002,701,374,893	.031,345,073,215	.002,692,737,485	.0³3,170,546,633	.0⁵8,637,408
14 45	.033,136,663,696	.002,843,524,099	.032,411,076,015	.002,834,116,463	.0³3,182,602,610	.0⁵9,407,636
15 00	.034,269,459,726	.002,990,574,525	.033,493,649,054	.002,980,342,696	.0³3,195,286,015	.0⁴10,231,829
15 15	.035,421,294,345	.003,142,609,242	.034,592,709,890	.003,131,496,471	.0³3,208,618,174	.0⁴11,112,771
15 30	.036,592,167,552	.003,299,711,322	.035,708,174,824	.003,287,657,981	.0³3,222,620,761	.0⁴12,053,341
15 45	.037,782,079,348	.003,461,963,835	.036,839,958,911	.003,448,907,324	.0³3,237,315,802	.0⁴13,056,511
16 00	.038,991,029,733	.003,629,449,854	.037,987,975,961	.003,615,324,502	.0³3,252,725,671	.0⁴14,125,352
16 15	.040,219,018,706	.003,802,252,449	.039,152,138,547	.003,786,989,418	.0³3,268,873,092	.0⁴15,263,031
16 30	.041,466,046,268	.003,980,454,693	.040,332,358,014	.003,963,981,875	.0³3,285,781,136	.0⁴16,472,818
16 45	.042,732,112,419	.004,164,139,657	.041,528,544,483	.004,146,381,575	.0³3,303,473,224	.0⁴17,758,082
17 00	.044,017,217,159	.004,353,390,413	.042,740,606,861	.004,334,268,116	.0³3,321,973,122	.0⁴19,122,297
17 15	.045,321,360,488	.004,548,290,031	.043,968,452,845	.004,527,720,993	.0³3,341,304,945	.0⁴20,569,038
17 30	.046,644,542,405	.004,748,921,584	.045,211,988,928	.004,726,819,595	.0³3,361,493,153	.0⁴22,101,989
17 45	.047,986,762,911	.004,955,368,144	.046,471,120,411	.004,931,643,201	.0³3,382,562,554	.0⁴23,724,943
18 00	.049,348,022,005	.005,167,712,780	.047,745,751,406	.005,142,270,984	.0³3,404,538,300	.0⁴25,441,796
18 15	.050,728,319,689	.005,386,038,566	.049,035,784,846	.005,358,782,005	.0³3,427,445,891	.0⁴27,256,561
18 30	.052,127,655,961	.005,610,428,572	.050,341,122,488	.005,581,255,214	.0³3,451,311,167	.0⁴29,173,358
18 45	.053,546,030,822	.005,840,965,870	.051,661,664,927	.005,809,769,446	.0³3,476,160,317	.0⁴31,196,424
19 00	.054,983,444,272	.006,077,733,532	.052,997,311,598	.006,044,403,422	.0³3,502,019,871	.0⁴33,330,110
19 15	.056,439,896,310	.006,320,814,629	.054,347,960,787	.006,285,235,746	.0³3,528,916,703	.0⁴35,578,883
19 30	.057,915,386,937	.006,570,292,233	.055,713,509,636	.006,532,344,905	.0³3,556,878,029	.0⁴37,947,328
19 45	.059,409,916,153	.006,826,249,414	.057,093,854,153	.006,785,809,266	.0³3,585,931,409	.0⁴40,440,148
20 00	.060,923,483,957	.007,088,769,246	.058,488,889,220	.007,045,707,073	.0³3,616,104,743	.0⁴43,062,173
20 15	.062,456,090,351	.007,357,934,798	.059,898,508,600	.007,312,116,452	.0³3,647,426,273	.0⁴45,818,347
20 30	.064,007,735,333	.007,633,829,143	.061,322,604,944	.007,585,115,400	.0³3,679,924,581	.0⁴48,713,744
20 45	.065,578,418,904	.007,916,535,353	.062,761,069,803	.007,864,781,791	.0³3,713,628,590	.0⁴51,753,563
21 00	.067,168,141,063	.008,206,136,498	.064,213,793,631	.008,151,193,374	.0³3,748,567,560	.0⁴54,943,125
21 15	.068,776,901,811	.008,502,715,650	.065,680,665,798	.008,444,427,765	.0³3,784,771,094	.0⁴58,287,886
21 30	.070,404,701,148	.008,806,355,881	.067,161,574,595	.008,744,562,455	.0³3,822,269,130	.0⁴61,793,427
21 45	.072,051,539,074	.009,117,140,263	.068,656,407,247	.009,051,674,799	.0³3,861,091,946	.0⁴65,465,465
22 00	.073,717,415,588	.009,435,151,866	.070,165,049,915	.009,365,842,023	.0³3,901,270,155	.0⁴69,309,844
22 15	.075,402,330,692	.009,760,473,762	.071,687,387,711	.009,687,141,217	.0³3,942,834,710	.0⁴73,332,545
22 30	.077,106,284,384	.010,093,189,024	.073,223,304,703	.010,015,649,334	.0³3,985,816,895	.0⁴77,539,690
° ϕ_1 ′	$\int \phi_1 d\phi_1$	$\int \frac{\phi_1^2}{2} d\phi_1$		$\int \text{vers}\,\phi_1 d\phi_1$	$\int (\phi_1 - \sin\phi_1) d\phi_1$	$\int (\frac{\phi_1^2}{2} - \text{vers}\,\phi_1) d\phi_1$

TABLE 13—Continued

ϕ_1 ° ′	$\frac{\phi_1^2}{2}$	$\frac{\phi_1^3}{6}$	$\frac{\text{SIN}^2\phi_1}{2}$	$\phi_1 - \text{SIN}\phi_1$	$\frac{\phi_1^2}{2} - \text{VERS}\,\phi_1$	$\text{SIN}\,\phi_1 - \phi_1 + \frac{\phi_1^3}{6}$
22 45	.078,829,276,664	.010,433,380,721	.074,772,683,925	.010,351,443,192	.001,030,248,334	.0481,937,529
23 00	.080,571,307,534	.010,781,131,926	.076,335,407,385	.010,694,599,470	.001,076,160,986	.0486,532,456
23 15	.082,332,376,992	.011,136,525,711	.077,911,356,076	.011,045,194,705	.001,123,587,141	.0491,331,006
23 30	.084,112,485,039	.011,499,645,147	.079,500,409,984	.011,403,305,294	.001,172,559,424	.0496,339,853
23 45	.085,911,631,674	.011,870,573,305	.081,102,448,096	.011,769,007,490	.001,223,110,793	.03,101,565,815
24 00	.087,729,816,899	.012,249,393,256	.082,717,348,410	.012,142,377,403	.001,275,274,542	.03,107,015,853
24 15	.089,567,040,712	.012,636,188,074	.084,344,987,946	.012,523,490,996	.001,329,084,289	.03,112,697,078
24 30	.091,423,303,114	.013,031,040,828	.085,985,242,752	.012,912,424,083	.001,384,573,991	.03,118,616,745
24 45	.093,298,604,104	.013,434,034,590	.087,637,987,917	.013,309,252,332	.001,441,777,929	.03,124,782,258
25 00	.095,192,943,684	.013,845,252,433	.089,303,097,578	.013,714,051,258	.001,500,730,721	.03,131,201,175
25 15	.097,106,321,851	.014,264,777,427	.090,980,444,930	.014,126,896,227	.001,561,467,305	.03,137,881,200
25 30	.099,038,738,608	.014,692,692,644	.092,669,902,237	.014,547,862,450	.001,624,022,958	.03,144,830,194
25 45	.100,990,193,953	.015,129,081,155	.094,371,340,840	.014,977,024,984	.001,688,433,276	.03,152,056,171
26 00	.102,960,687,888	.015,574,026,033	.096,084,631,169	.015,414,458,729	.001,754,734,187	.03,159,567,304
26 15	.104,950,220,411	.016,027,610,348	.097,809,642,748	.015,860,238,429	.001,822,961,944	.03,167,371,919
26 30	.106,958,791,522	.016,489,917,171	.099,546,244,212	.016,314,438,668	.001,893,153,124	.03,175,478,503
26 45	.108,986,401,223	.016,961,029,576	.101,294,303,312	.016,777,133,871	.001,965,344,634	.03,183,895,705
27 00	.111,033,049,512	.017,441,030,633	.103,053,686,927	.017,248,398,298	.002,039,573,700	.03,192,632,335
27 15	.113,098,736,390	.017,930,003,413	.104,824,261,072	.017,728,306,051	.002,115,877,876	.03,201,697,362
27 30	.115,183,461,857	.018,428,030,988	.106,605,890,912	.018,216,931,063	.002,194,295,035	.03,211,099,925
27 45	.117,287,225,912	.018,935,196,430	.108,398,440,769	.018,714,347,103	.002,274,863,375	.03,220,849,327
28 00	.119,410,028,556	.019,451,582,810	.110,201,774,132	.019,220,627,772	.002,357,621,415	.03,230,955,038
28 15	.121,551,869,789	.019,977,273,200	.112,015,753,672	.019,735,846,503	.002,442,607,994	.03,241,426,697
28 30	.123,712,749,611	.020,512,350,670	.113,840,241,246	.020,260,076,558	.002,529,862,273	.03,252,274,112
28 45	.125,892,668,021	.021,056,898,294	.115,675,097,913	.020,793,391,029	.002,619,423,729	.03,263,507,265
29 00	.128,091,625,020	.021,610,999,141	.117,520,183,942	.021,335,862,832	.002,711,332,159	.03,275,136,309
29 15	.130,309,620,608	.022,174,736,285	.119,375,358,821	.021,887,564,711	.002,805,627,681	.03,287,171,574
29 30	.132,546,654,785	.022,748,192,795	.121,240,481,272	.022,448,569,235	.002,902,350,725	.03,299,623,560
29 45	.134,802,727,550	.023,331,451,744	.123,115,409,260	.023,018,948,793	.003,001,542,039	.03,312,502,951
30 00	.137,077,838,904	.023,924,596,204	.125,000,000,000	.023,598,775,598	.003,103,242,688	.03,325,820,606
30 15	.139,371,988,847	.024,527,709,245	.126,894,109,974	.024,188,121,682	.003,207,494,051	.03,339,587,563
30 30	.141,685,177,378	.025,140,873,940	.128,797,594,939	.024,787,058,897	.003,314,337,820	.03,353,815,043
30 45	.144,017,404,498	.025,764,173,359	.130,710,309,935	.025,395,658,911	.003,423,815,999	.03,368,514,448
31 00	.146,368,670,207	.026,397,690,574	.132,632,109,303	.026,013,993,208	.003,535,970,909	.03,383,697,366
31 15	.148,738,974,505	.027,041,508,658	.134,562,846,691	.026,642,133,087	.003,650,845,178	.03399,375,571
31 30	.151,128,317,392	.027,695,710,681	.136,502,375,065	.027,280,149,662	.003,768,481,746	.03,415,561,019
31 45	.153,536,698,867	.028,360,379,714	.138,450,546,723	.027,928,113,856	.003,888,923,863	.03,432,265,858
32 00	.155,964,118,931	.029,035,598,830	.140,407,213,303	.028,586,096,405	.004,012,215,087	.03,449,502,425
32 15	.158,410,577,583	.029,721,451,100	.142,372,225,798	.029,254,167,852	.004,138,399,287	.03,467,283,248
32 30	.160,876,074,825	.030,418,019,595	.144,345,434,565	.029,932,398,551	.004,267,520,638	.03,485,621,044
32 45	.163,360,610,655	.031,125,387,387	.146,326,689,336	.030,620,858,660	.004,399,623,619	.03,504,528,727
33 00	.165,864,185,074	.031,843,637,547	.148,315,839,231	.031,319,618,143	.004,534,753,019	.03,524,019,404
33 15	.168,386,798,081	.032,572,853,148	.150,312,732,769	.032,028,746,768	.004,672,953,929	.03,544,106,380
33 30	.170,928,449,678	.033,313,117,260	.152,317,217,878	.032,748,314,106	.004,814,271,745	.03,564,803,154
33 45	.173,489,139,863	.034,064,512,954	.154,329,141,909	.033,478,389,528	.004,958,752,166	.03,586,123,426
34 00	.176,068,868,637	.034,827,123,304	.156,348,351,646	.034,219,042,207	.005,106,441,192	.03,608,081,097
34 15	.178,667,635,999	.035,601,031,379	.158,374,693,319	.034,970,341,113	.005,257,385,126	.03,630,690,266
34 30	.181,285,441,951	.036,386,320,252	.160,408,012,614	.035,732,355,013	.005,411,630,573	.03,653,965,239
34 45	.183,922,286,491	.037,183,072,993	.162,448,154,685	.036,505,152,472	.005,569,224,433	.03,677,920,521
35 00	.186,578,169,619	.037,991,372,676	.164,494,964,169	.037,288,801,847	.005,730,213,908	.03,702,570,829
35 15	.189,253,091,337	.038,811,302,370	.166,548,285,191	.038,083,371,291	.005,894,646,499	.03,727,931,079
35 30	.191,947,051,643	.039,642,945,148	.168,607,961,386	.038,888,928,747	.006,062,569,999	.03,754,016,401
35 45	.194,660,050,538	.040,486,384,081	.170,673,835,898	.039,705,541,951	.006,234,032,503	.03,780,842,130
36 00	.197,392,088,022	.041,341,702,240	.172,745,751,406	.040,533,278,426	.006,409,082,397	.03,808,423,814
36 15	.200,143,164,094	.042,208,982,698	.174,823,550,124	.041,372,205,484	.006,587,768,361	.03,836,777,214
36 30	.202,913,278,755	.043,088,308,525	.176,907,073,819	.042,222,390,227	.006,770,139,372	.03,865,918,298
36 45	.205,702,432,005	.043,979,762,794	.178,996,163,824	.043,083,899,537	.006,956,244,696	.03,895,863,257
37 00	.208,510,623,844	.044,883,428,575	.181,090,661,046	.043,956,800,086	.007,146,133,891	.03,926,628,489
37 15	.211,337,854,271	.045,799,388,940	.183,190,405,980	.044,841,158,325	.007,339,856,806	.03,958,230,615
37 30	.214,184,123,288	.046,727,726,961	.185,295,238,725	.045,737,040,489	.007,537,463,579	.03,990,686,472
37 45	.217,049,430,892	.047,668,525,709	.187,404,998,986	.046,644,512,594	.007,739,004,636	.001,024,013,115
38 00	.219,933,777,086	.048,621,868,256	.189,519,526,100	.047,563,640,432	.007,944,530,693	.001,058,227,824
38 15	.222,837,161,868	.049,587,837,673	.191,638,659,036	.048,494,489,578	.008,154,092,749	.001,093,348,095
38 30	.225,759,585,240	.050,566,517,031	.193,762,236,414	.049,437,125,380	.008,367,742,092	.001,129,391,651
38 45	.228,701,047,199	.051,557,989,403	.195,890,096,515	.050,391,612,964	.008,585,530,292	.001,166,376,439
39 00	.231,661,547,748	.052,562,337,860	.198,022,077,296	.051,358,017,228	.008,807,509,205	.001,204,320,632
39 15	.234,641,086,885	.053,579,645,473	.200,158,016,396	.052,336,402,845	.009,033,730,967	.001,243,242,628
39 30	.237,639,664,611	.054,609,995,314	.202,297,751,156	.053,326,834,260	.009,264,247,999	.001,283,161,054
39 45	.240,657,280,926	.055,653,470,454	.204,441,118,627	.054,329,375,687	.009,499,112,999	.001,324,094,767
40 00	.243,693,935,830	.056,710,153,965	.206,587,955,584	.055,344,091,111	.009,738,378,949	.001,366,062,854
40 15	.246,749,629,322	.057,780,128,918	.208,738,098,535	.056,371,044,285	.009,982,099,105	.001,409,084,633
40 30	.249,824,361,403	.058,863,478,385	.210,891,383,740	.057,410,298,728	.010,230,327,003	.001,453,179,657
40 45	.252,918,132,073	.059,960,285,438	.213,047,647,218	.058,461,917,725	.010,483,116,457	.001,498,367,713
41 00	.256,030,941,331	.061,070,633,147	.215,206,724,760	.059,525,964,327	.010,740,521,554	.001,544,668,820
41 15	.259,162,789,178	.062,194,604,585	.217,368,451,945	.060,602,501,348	.011,002,596,657	.001,592,103,237
41 30	.262,313,675,614	.063,332,282,823	.219,532,664,149	.061,691,591,362	.011,269,396,403	.001,640,691,461
41 45	.265,483,600,638	.064,483,750,931	.221,699,196,558	.062,793,296,706	.011,540,975,700	.001,690,454,225
42 00	.268,672,564,251	.065,649,091,983	.223,867,884,183	.063,907,679,478	.011,817,389,728	.001,741,412,505
42 15	.271,880,566,454	.066,828,389,050	.226,038,561,870	.065,034,801,532	.012,098,693,941	.001,793,587,518
42 30	.275,107,607,244	.068,021,725,202	.228,211,064,313	.066,174,724,481	.012,384,944,054	.001,847,000,721
42 45	.278,353,686,624	.069,229,183,512	.230,385,226,068	.067,327,509,694	.012,676,196,060	.001,901,673,818
43 00	.281,618,804,592	.070,450,847,051	.232,560,881,564	.068,493,218,295	.012,972,506,211	.001,957,628,756
43 15	.284,902,961,149	.071,686,798,890	.234,737,865,116	.069,671,911,161	.013,273,931,031	.002,014,887,729
43 30	.288,206,156,295	.072,937,122,102	.236,916,010,939	.070,863,648,923	.013,580,527,307	.002,073,473,179
43 45	.291,528,390,030	.074,201,899,757	.239,095,153,158	.072,068,491,965	.013,892,352,090	.002,133,407,792
44 00	.294,869,662,353	.075,481,214,927	.241,275,125,824	.073,286,500,418	.014,209,462,692	.002,194,714,509
44 15	.298,229,973,265	.076,775,150,684	.243,455,762,923	.074,517,734,165	.014,531,916,690	.002,257,416,519
44 30	.301,609,322,766	.078,083,790,098	.245,636,898,391	.075,762,252,837	.014,859,771,920	.002,321,537,261
44 45	.305,007,710,855	.079,407,216,243	.247,818,366,125	.077,020,115,811	.015,193,086,478	.002,387,100,432
45 00	.308,425,137,534	.080,745,512,188	.250,000,000,000	.078,291,382,210	.015,531,918,721	.002,454,129,978
° ′ ϕ_1	$\int \phi_1 d\phi_1$	$\int \frac{\phi_1^2}{2} d\phi_1$		$\int \text{VERS}\,\phi_1 d\phi_1$	$\int (\phi_1 - \text{SIN}\,\phi_1) d\phi_1$	$\int (\frac{\phi_1^2}{2} - \text{VERS}\,\phi_1) d\phi_1$

TABLE 13—Continued

ϕ_1 ° ′	$\frac{\phi_1^2}{2}$	$\frac{\phi_1^3}{6}$	$\frac{\sin^2\phi_1}{2}$	$\phi_1 - \sin\phi_1$	$\frac{\phi_1^2}{2} - \text{vers}\,\phi_1$	$\sin\phi_1 - \phi_1 + \frac{\phi_1^3}{6}$
45 15	.311,861,602,801	.082,098,761,006	.252,181,633,874	.079,576,110,904	.015,876,327,257	.002,522,650,102
45 30	.315,317,106,656	.083,467,045,768	.254,363,101,609	.080,874,360,503	.016,226,370,956	.002,592,685,265
45 45	.318,791,649,101	.084,850,449,546	.256,544,237,077	.082,186,189,362	.016,582,108,943	.002,664,260,184
46 00	.322,285,230,134	.086,249,055,411	.258,724,874,176	.083,511,655,578	.016,943,600,593	.002,737,399,833
46 15	.325,797,849,756	.087,662,946,435	.260,904,846,842	.084,850,816,987	.017,310,905,538	.002,812,129,448
46 30	.329,329,507,967	.089,092,205,689	.263,083,989,061	.086,203,731,165	.017,684,083,661	.002,888,474,524
46 45	.332,880,204,766	.090,536,916,244	.265,262,134,883	.087,570,455,425	.018,063,195,092	.002,966,460,819
47 00	.336,449,940,154	.091,997,161,173	.267,439,118,436	.088,951,046,818	.018,448,300,216	.003,046,114,355
47 15	.340,038,714,131	.093,473,023,547	.269,614,773,932	.090,345,562,131	.018,839,459,664	.003,127,461,416
47 30	.343,646,526,697	.094,964,586,437	.271,788,935,687	.091,754,057,887	.019,236,734,313	.003,210,528,550
47 45	.347,273,377,851	.096,471,932,914	.273,961,438,130	.093,176,590,340	.019,640,185,286	.003,295,342,574
48 00	.350,919,267,594	.097,995,146,051	.276,132,115,817	.094,613,215,480	.020,049,873,953	.003,381,930,571
48 15	.354,584,195,926	.099,534,308,919	.278,300,803,442	.096,063,989,025	.020,465,861,927	.003,470,319,894
48 30	.358,268,162,846	.101,089,504,589	.280,467,335,851	.097,528,966,428	.020,888,211,062	.003,560,538,161
48 45	.361,971,168,356	.102,660,816,133	.282,631,548,055	.099,008,202,868	.021,316,983,456	.003,652,613,265
49 00	.365,693,212,454	.104,248,326,622	.284,793,275,240	.100,501,753,254	.021,752,241,445	.003,746,573,368
49 15	.369,434,295,141	.105,852,119,128	.286,952,352,782	.102,009,672,223	.022,194,047,604	.003,842,446,905
49 30	.373,194,416,416	.107,472,276,723	.289,108,616,260	.103,532,014,137	.022,642,464,746	.003,940,262,586
49 45	.376,973,576,280	.109,108,882,477	.291,261,901,466	.105,068,833,084	.023,097,555,923	.004,040,049,393
50 00	.380,771,774,733	.110,762,019,463	.293,412,044,417	.106,620,182,878	.023,559,384,420	.004,141,836,585
50 15	.384,589,011,775	.112,431,770,751	.295,558,881,373	.108,186,117,054	.024,028,013,756	.004,245,653,697
50 30	.388,425,287,405	.114,118,219,414	.297,702,248,844	.109,766,688,869	.024,503,507,683	.004,351,530,545
50 45	.392,280,601,625	.115,821,448,524	.299,841,983,604	.111,361,951,305	.024,985,930,188	.004,459,497,219
51 00	.396,154,954,433	.117,541,541,150	.301,977,922,704	.112,971,957,060	.025,475,345,483	.004,569,584,090
51 15	.400,048,345,829	.119,278,580,365	.304,109,903,485	.114,596,758,554	.025,971,818,013	.004,681,821,811
51 30	.403,960,775,815	.121,032,649,241	.306,237,763,586	.116,236,407,925	.026,475,412,453	.004,796,241,316
51 45	.407,892,244,389	.122,803,830,849	.308,361,340,964	.117,890,957,026	.026,986,193,699	.004,912,873,823
52 00	.411,842,751,552	.124,592,208,261	.310,480,473,900	.119,560,457,430	.027,504,226,878	.005,031,750,831
52 15	.415,812,297,303	.126,397,864,547	.312,595,001,014	.121,244,960,423	.028,029,577,337	.005,152,904,124
52 30	.419,800,881,644	.128,220,882,781	.314,704,761,275	.122,944,517,006	.028,562,310,653	.005,276,365,775
52 45	.423,808,504,573	.130,061,346,032	.316,809,594,020	.124,659,177,892	.029,102,492,616	.005,402,168,140
53 00	.427,835,166,090	.131,919,337,372	.318,909,338,954	.126,388,993,510	.029,650,189,242	.005,530,343,862
53 15	.431,880,866,197	.133,794,939,874	.321,003,836,176	.128,134,013,996	.030,205,466,768	.005,660,925,878
53 30	.435,945,604,892	.135,688,236,609	.323,092,926,181	.129,894,289,200	.030,768,391,643	.005,793,947,409
53 45	.440,029,382,176	.137,599,310,647	.325,176,449,876	.131,669,868,680	.031,339,030,540	.005,929,441,967
54 00	.444,132,198,049	.139,528,245,061	.327,254,248,594	.133,460,801,702	.031,917,450,341	.006,067,443,359
54 15	.448,254,052,511	.141,475,122,923	.329,326,164,101	.135,267,137,242	.032,503,718,148	.006,207,985,681
54 30	.452,394,945,561	.143,440,027,303	.331,392,038,614	.137,088,923,981	.033,097,901,272	.006,351,103,322
54 45	.456,554,877,200	.145,423,041,273	.333,451,714,809	.138,926,210,305	.033,700,067,237	.006,496,830,968
55 00	.460,733,847,428	.147,424,247,905	.335,505,035,831	.140,779,044,308	.034,310,283,779	.006,645,203,597
55 15	.464,931,856,244	.149,443,730,270	.337,551,845,315	.142,647,473,785	.034,928,618,840	.006,796,256,485
55 30	.469,148,903,649	.151,481,571,440	.339,591,987,386	.144,531,546,235	.035,555,140,574	.006,950,025,205
55 45	.473,384,989,643	.153,537,854,486	.341,625,306,681	.146,431,308,860	.036,189,917,338	.007,106,545,626
56 00	.477,640,114,226	.155,612,662,480	.343,651,648,354	.148,346,808,562	.036,833,017,697	.007,265,853,918
56 15	.481,914,277,397	.157,706,078,493	.345,670,858,092	.150,278,091,944	.037,484,510,417	.007,427,986,549
56 30	.486,207,479,157	.159,818,185,597	.347,682,782,122	.152,225,205,310	.038,144,464,469	.007,592,980,287
56 45	.490,519,719,506	.161,949,066,863	.349,687,267,232	.154,188,194,659	.038,812,949,026	.007,760,872,204
57 00	.494,850,998,444	.164,098,805,363	.351,684,160,769	.156,167,105,692	.039,490,033,459	.007,931,699,671
57 15	.499,201,315,970	.166,267,484,168	.353,673,310,664	.158,161,983,803	.040,175,787,338	.008,105,500,365
57 30	.503,570,672,085	.168,455,186,350	.355,654,565,435	.160,172,874,084	.040,870,280,432	.008,282,312,266
57 45	.507,959,066,789	.170,661,994,981	.357,627,774,202	.162,199,821,323	.041,573,582,705	.008,462,173,658
58 00	.512,366,500,082	.172,887,993,131	.359,592,786,697	.164,242,870,001	.042,285,764,315	.008,645,123,130
58 15	.516,792,971,963	.175,133,263,873	.361,549,453,278	.166,302,064,291	.043,006,895,615	.008,831,199,582
58 30	.521,238,482,432	.177,397,890,277	.363,497,624,935	.168,377,448,062	.043,737,047,148	.009,020,442,215
58 45	.525,703,031,491	.179,681,955,417	.365,437,153,309	.170,469,064,873	.044,476,289,652	.009,212,890,544
59 00	.530,186,619,138	.181,985,542,361	.367,367,890,696	.172,576,957,974	.045,224,694,048	.009,408,584,387
59 15	.534,689,245,374	.184,308,734,184	.369,289,690,065	.174,701,170,305	.045,982,331,451	.009,607,563,879
59 30	.539,210,910,199	.186,651,613,955	.371,202,405,062	.176,841,744,494	.046,749,273,160	.009,809,869,461
59 45	.543,751,613,613	.189,014,264,747	.373,105,890,026	.178,998,722,862	.047,525,590,659	.010,015,541,885
60 00	.548,311,355,615	.191,396,769,631	.375,000,000,000	.181,172,147,412	.048,311,355,615	.010,224,622,219
60 15	.552,890,136,207	.193,799,211,678	.376,884,590,740	.183,362,059,837	.049,106,639,882	.010,437,151,841
60 30	.557,487,955,387	.196,221,673,961	.378,759,518,728	.185,568,501,516	.049,911,515,490	.010,653,172,445
60 45	.562,104,813,155	.198,664,239,550	.380,624,641,179	.187,791,513,513	.050,726,054,652	.010,872,726,037
61 00	.566,740,709,513	.201,126,991,517	.382,479,816,058	.190,031,136,577	.051,550,329,759	.011,095,854,940
61 15	.571,395,644,459	.203,610,012,934	.384,324,902,087	.192,287,411,138	.052,384,413,378	.011,322,601,796
61 30	.576,069,617,994	.206,113,386,871	.386,159,758,754	.194,560,377,314	.053,228,378,254	.011,553,009,557
61 45	.580,762,630,117	.208,637,196,401	.387,984,246,328	.196,850,074,901	.054,082,297,302	.011,787,121,500
62 00	.585,474,680,830	.211,181,524,596	.389,798,225,868	.199,156,543,377	.054,946,243,616	.012,024,981,219
62 15	.590,205,770,131	.213,746,454,526	.391,601,559,231	.201,479,821,903	.055,820,290,456	.012,266,632,623
62 30	.594,955,898,020	.216,332,069,263	.393,394,109,088	.203,819,949,318	.056,704,511,255	.012,512,119,945
62 45	.599,725,064,499	.218,938,451,878	.395,175,738,928	.206,176,964,140	.057,598,979,616	.012,761,487,738
63 00	.604,513,269,566	.221,565,685,444	.396,946,313,073	.208,550,904,568	.058,503,769,306	.013,014,780,876
63 15	.609,320,513,222	.224,213,853,032	.398,705,696,688	.210,941,808,475	.059,418,954,259	.013,272,044,557
63 30	.614,146,795,467	.226,883,037,713	.400,453,755,788	.213,349,713,414	.060,344,608,577	.013,533,324,299
63 45	.618,992,116,301	.229,573,322,558	.402,190,357,252	.215,774,656,613	.061,280,806,520	.013,798,665,945
64 00	.623,856,475,723	.232,284,790,640	.403,915,368,831	.218,216,674,977	.062,227,622,512	.014,068,115,663
64 15	.628,739,873,734	.235,017,525,029	.405,628,659,160	.220,675,805,083	.063,185,131,138	.014,341,719,946
64 30	.633,642,310,333	.237,771,608,798	.407,330,097,763	.223,152,083,186	.064,153,407,141	.014,619,525,612
64 45	.638,563,785,522	.240,547,125,018	.409,019,555,069	.225,645,545,212	.065,132,525,423	.014,901,579,806
65 00	.643,504,299,299	.243,344,156,759	.410,696,902,422	.228,156,226,759	.066,122,561,040	.015,187,930,000
65 15	.648,463,851,665	.246,162,787,095	.412,362,012,082	.230,684,163,101	.067,123,589,202	.015,478,623,994
65 30	.653,442,442,620	.249,003,099,096	.414,014,757,248	.233,229,389,179	.068,135,685,276	.015,773,709,917
65 45	.658,440,072,163	.251,865,175,833	.415,655,012,054	.235,791,939,609	.069,158,924,776	.016,073,236,224
66 00	.663,456,740,295	.254,749,100,379	.417,282,651,590	.238,371,848,673	.070,193,383,371	.016,377,251,706
66 15	.668,492,447,016	.257,654,955,805	.418,897,551,904	.240,969,150,327	.071,239,136,875	.016,685,805,478
66 30	.673,547,192,326	.260,582,825,183	.420,499,590,016	.243,583,878,191	.072,296,261,251	.016,998,946,992
66 45	.678,620,976,224	.263,532,791,583	.422,088,643,924	.246,216,065,557	.073,364,832,608	.017,316,726,026
67 00	.683,713,798,711	.266,504,938,077	.423,664,592,614	.248,865,745,384	.074,444,927,200	.017,639,192,693
67 15	.688,825,659,787	.269,499,347,738	.425,227,316,075	.251,532,950,296	.075,536,621,424	.017,966,397,442
67 30	.693,956,559,451	.272,516,103,635	.426,776,695,296	.254,217,712,585	.076,639,991,816	.018,298,391,050
° ′ ϕ_1	$\int \phi_1 d\phi_1$	$\int \frac{\phi_1^2}{2} d\phi_1$		$\int \text{VERS}\,\phi_1 d\phi_1$	$\int (\phi_1 - \sin\phi_1) d\phi_1$	$\int (\frac{\phi_1^2}{2} - \text{VERS}\,\phi_1) d\phi_1$

TABLE 13—Continued

ϕ_1 ° ′	$\frac{\phi_1^2}{2}$	$\frac{\phi_1^3}{6}$	$\frac{\sin^2\phi_1}{2}$	$\phi_1 - \sin\phi_1$	$\frac{\phi_1^2}{2} - \text{vers}\,\phi_1$	$\sin\phi_1 - \phi_1 + \frac{\phi_1^3}{6}$
67 45	.699,106,497,705	.275,555,288,842	.428,312,612,289	.256,920,064,208	.077,755,115,057	.018,635,224,634
68 00	.704,275,474,547	.278,616,986,429	.429,834,950,085	.259,640,036,789	.078,882,067,963	.018,976,949,640
68 15	.709,463,489,977	.281,701,279,468	.431,343,592,753	.262,377,661,614	.080,020,927,487	.019,323,617,854
68 30	.714,670,543,997	.284,808,251,031	.432,838,425,405	.265,132,969,634	.081,171,770,721	.019,675,281,397
68 45	.719,896,636,605	.287,937,984,189	.434,319,334,203	.267,905,991,463	.082,334,674,889	.020,031,992,726
69 00	.725,141,767,802	.291,090,562,013	.435,786,206,369	.270,696,757,379	.083,509,717,347	.020,393,804,634
69 15	.730,405,937,588	.294,266,067,576	.437,238,930,197	.273,505,297,320	.084,696,975,586	.020,760,770,256
69 30	.735,689,145,962	.297,464,583,947	.438,677,395,055	.276,331,640,888	.085,896,527,221	.021,132,943,059
69 45	.740,991,392,926	.300,686,194,200	.440,101,491,400	.279,175,817,344	.087,108,450,003	.021,510,376,856
70 00	.746,312,678,477	.303,930,981,405	.441,511,110,780	.282,037,855,610	.088,332,821,803	.021,893,125,795
70 15	.751,653,002,618	.307,199,028,635	.442,906,145,847	.284,917,784,270	.089,569,720,621	.022,281,244,365
70 30	.757,012,365,348	.310,490,418,960	.444,286,490,364	.287,815,631,564	.090,819,224,582	.022,674,787,396
70 45	.762,390,766,666	.313,805,235,452	.445,652,039,213	.290,731,425,393	.092,081,411,929	.023,073,810,059
71 00	.767,788,206,572	.317,143,561,183	.447,002,688,401	.293,665,193,317	.093,356,361,029	.023,478,367,866
71 15	.773,204,685,068	.320,505,479,224	.448,338,335,073	.296,616,962,551	.094,644,150,371	.023,888,516,673
71 30	.778,640,202,153	.323,891,072,647	.449,658,877,512	.299,586,759,970	.095,944,858,558	.024,304,312,677
71 45	.784,094,757,825	.327,300,424,523	.450,964,215,154	.302,574,612,104	.097,258,564,309	.024,725,812,419
72 00	.789,568,352,087	.330,733,617,923	.452,254,248,594	.305,580,545,141	.098,585,346,462	.025,153,072,782
72 15	.795,060,984,938	.334,190,735,920	.453,528,879,589	.308,604,584,923	.099,925,283,966	.025,586,150,997
72 30	.800,572,656,377	.337,671,861,584	.454,788,011,072	.311,646,756,948	.101,278,455,881	.026,025,104,636
72 45	.806,103,366,405	.341,177,077,988	.456,031,547,155	.314,707,086,369	.102,644,941,381	.026,469,991,619
73 00	.811,653,115,022	.344,706,468,203	.457,259,393,139	.317,785,597,993	.104,024,819,745	.026,920,870,210
73 15	.817,221,902,228	.348,260,115,299	.458,471,455,517	.320,882,316,281	.105,418,170,362	.027,377,799,018
73 30	.822,809,728,022	.351,838,102,350	.459,667,641,986	.323,997,265,348	.106,825,072,726	.027,840,837,002
73 45	.828,416,592,405	.355,440,512,425	.460,847,861,453	.327,130,468,960	.108,245,606,436	.028,310,043,465
74 00	.834,042,495,376	.359,067,428,598	.462,012,024,039	.330,281,950,538	.109,679,851,193	.028,785,478,060
74 15	.839,687,436,937	.362,718,933,939	.463,160,041,089	.333,451,733,152	.111,127,886,802	.029,267,200,787
74 30	.845,351,417,086	.366,395,111,520	.464,291,825,176	.336,639,839,527	.112,589,793,164	.029,755,271,993
74 45	.851,034,435,824	.370,096,044,412	.465,407,290,110	.339,846,292,037	.114,065,650,282	.030,249,752,375
75 00	.856,736,493,150	.373,821,815,687	.466,506,350,946	.343,071,112,707	.115,555,538,253	.030,750,702,980
75 15	.862,457,589,065	.377,572,508,415	.467,588,923,985	.346,314,323,211	.117,059,537,271	.031,258,185,204
75 30	.868,197,723,569	.381,348,205,670	.468,654,926,785	.349,575,944,877	.118,577,727,623	.031,772,260,793
75 45	.873,956,896,662	.385,148,990,523	.469,704,278,166	.352,855,998,678	.120,110,189,691	.032,292,991,845
76 00	.879,735,108,343	.388,974,946,044	.470,736,898,215	.356,154,505,239	.121,657,003,943	.032,820,440,805
76 15	.885,532,358,613	.392,826,155,306	.471,752,708,294	.359,471,484,832	.123,218,250,939	.033,354,670,474
76 30	.891,348,647,473	.396,702,701,380	.472,751,631,047	.362,806,957,377	.124,794,011,329	.033,895,744,003
76 45	.897,183,974,920	.400,604,667,338	.473,733,590,400	.366,160,942,445	.126,384,365,842	.034,443,724,893
77 00	.903,038,340,957	.404,532,136,250	.474,698,511,575	.369,533,459,250	.127,989,395,301	.034,998,677,000
77 15	.908,911,745,582	.408,485,191,189	.475,646,321,088	.372,924,526,656	.129,609,180,604	.035,560,664,533
77 30	.914,804,188,796	.412,463,915,226	.476,576,946,759	.376,334,163,175	.131,243,802,734	.036,129,752,051
77 45	.920,715,670,599	.416,468,391,432	.477,490,317,719	.379,762,386,962	.132,893,342,755	.036,706,004,470
78 00	.926,646,190,990	.420,498,702,880	.478,386,364,411	.383,209,215,821	.134,557,881,808	.037,289,487,059
78 15	.932,595,749,971	.424,554,932,640	.479,265,018,597	.386,674,667,200	.136,237,501,111	.037,880,265,440
78 30	.938,564,347,540	.428,637,163,784	.480,126,213,363	.390,158,758,194	.137,932,281,957	.038,478,405,590
78 45	.944,551,983,697	.432,745,479,383	.480,969,883,128	.393,661,505,542	.139,642,305,713	.039,083,973,841
79 00	.950,558,658,444	.436,879,962,510	.481,795,963,642	.397,182,925,627	.141,367,653,821	.039,697,036,883
79 15	.956,584,371,779	.441,040,696,236	.482,604,391,996	.400,723,034,479	.143,108,407,788	.040,317,661,757
79 30	.962,629,123,703	.445,227,763,631	.483,395,106,624	.404,281,847,771	.144,864,649,195	.040,945,915,860
79 45	.968,692,914,215	.449,441,247,768	.484,168,047,312	.407,859,380,819	.146,636,459,689	.041,581,866,949
80 00	.974,775,743,317	.453,681,231,719	.484,923,155,196	.411,455,648,583	.148,423,920,984	.042,225,583,136
80 15	.980,877,611,007	.457,947,798,554	.485,660,372,773	.415,070,665,667	.150,227,114,856	.042,877,132,887
80 30	.986,998,517,286	.462,241,031,345	.486,379,643,900	.418,704,446,318	.152,046,123,147	.043,536,585,027
80 45	.993,138,462,153	.466,561,013,164	.487,080,913,801	.422,357,004,425	.153,881,027,757	.044,204,008,739
81 00	.999,297,445,610	.470,907,827,082	.487,764,129,074	.426,028,353,520	.155,731,910,650	.044,879,473,562
81 15	1.005,475,467,655	.475,281,556,170	.488,429,237,687	.429,718,506,777	.157,598,853,845	.045,563,049,393
81 30	1.011,672,528,289	.479,682,283,502	.489,076,188,991	.433,427,477,013	.159,481,939,419	.046,254,806,489
81 45	1.017,888,627,511	.484,110,092,146	.489,704,933,717	.437,155,276,685	.161,381,249,502	.046,954,815,461
82 00	1.024,123,765,322	.488,565,065,176	.490,315,423,985	.440,901,917,893	.163,296,866,282	.047,663,147,283
82 15	1.030,377,941,722	.493,047,285,663	.490,907,613,302	.444,667,412,378	.165,228,871,996	.048,379,873,285
82 30	1.036,651,156,711	.497,556,836,678	.491,481,456,573	.448,451,771,521	.167,177,348,931	.049,105,065,157
82 45	1.042,943,410,289	.502,093,801,294	.492,036,910,095	.452,255,006,345	.169,142,379,425	.049,838,794,949
83 00	1.049,254,702,455	.506,658,262,580	.492,573,931,569	.456,077,127,514	.171,124,045,860	.050,581,135,066
83 15	1.055,585,033,210	.511,250,303,609	.493,092,480,100	.459,918,145,330	.173,122,430,668	.051,332,158,279
83 30	1.061,934,402,554	.515,870,007,453	.493,592,516,197	.463,778,069,738	.175,137,616,322	.052,091,937,715
83 45	1.068,302,810,486	.520,517,457,182	.494,074,001,780	.467,656,910,323	.177,169,685,338	.052,860,546,859
84 00	1.074,690,257,007	.525,192,735,868	.494,536,900,183	.471,554,676,307	.179,218,720,275	.053,638,059,561
84 15	1.081,096,742,117	.529,895,926,584	.494,981,176,156	.475,471,376,554	.181,284,803,729	.054,424,550,030
84 30	1.087,522,265,816	.534,627,112,400	.495,406,795,862	.479,407,019,568	.183,368,018,336	.055,220,092,832
84 45	1.093,966,828,103	.539,386,376,388	.495,813,726,892	.483,361,613,490	.185,468,446,766	.056,024,762,898
85 00	1.100,430,428,979	.544,173,801,620	.496,201,938,253	.487,335,166,103	.187,586,171,727	.056,838,635,517
85 15	1.106,913,068,444	.548,989,471,166	.496,571,400,385	.491,327,684,827	.189,721,275,956	.057,661,786,339
85 30	1.113,414,746,498	.553,833,468,099	.496,922,085,149	.495,339,176,722	.191,873,842,226	.058,494,291,377
85 45	1.119,935,463,140	.558,705,875,490	.497,253,965,840	.499,369,648,486	.194,043,953,335	.059,336,227,004
86 00	1.126,475,218,371	.563,606,776,411	.497,567,017,186	.503,419,106,455	.196,231,692,115	.060,187,669,956
86 15	1.133,034,012,191	.568,536,253,932	.497,861,215,344	.507,487,556,606	.198,437,141,421	.061,048,697,326
86 30	1.139,611,844,599	.573,494,391,126	.498,136,537,910	.511,575,004,553	.200,660,384,134	.061,919,386,573
86 45	1.146,208,715,597	.578,481,271,064	.498,392,963,919	.515,681,455,548	.202,901,503,160	.062,799,815,516
87 00	1.152,824,625,183	.583,496,976,818	.498,630,473,842	.519,806,914,480	.205,160,581,426	.063,690,062,338
87 15	1.159,459,573,357	.588,541,591,458	.498,849,049,592	.523,951,385,880	.207,437,701,878	.064,590,205,578
87 30	1.166,113,560,121	.593,615,198,058	.499,048,674,523	.528,114,873,913	.209,732,947,486	.065,500,324,145
87 45	1.172,786,585,473	.598,717,879,687	.499,229,333,434	.532,297,382,384	.212,046,401,232	.066,420,497,303
88 00	1.179,478,649,414	.603,849,719,418	.499,391,012,565	.536,498,914,736	.214,378,146,117	.067,350,804,682
88 15	1.186,189,751,944	.609,010,800,322	.499,533,699,606	.540,719,474,048	.216,728,265,154	.068,291,326,274
88 30	1.192,919,893,062	.614,201,205,471	.499,657,383,689	.544,959,063,039	.219,096,841,370	.069,242,142,432
88 45	1.199,669,072,769	.619,421,017,936	.499,762,055,396	.549,217,684,065	.221,483,957,804	.070,203,333,871
89 00	1.206,437,291,065	.624,670,320,788	.499,847,706,754	.553,495,339,119	.223,889,697,502	.071,174,981,669
89 15	1.213,224,547,950	.629,949,197,100	.499,914,331,244	.557,792,029,831	.226,314,143,521	.072,157,167,269
89 30	1.220,030,843,423	.635,257,729,943	.499,961,923,789	.562,107,757,471	.228,757,378,921	.073,149,972,472
89 45	1.226,856,177,485	.640,596,002,388	.499,990,480,766	.566,442,522,944	.231,219,486,770	.074,153,479,444
90 00	1.233,700,550,136	.645,964,097,506	.500,000,000,000	.570,796,326,795	.233,700,550,136	.075,167,770,711
° ϕ_1 ′	$\int \phi_1 d\phi_1$	$\int \frac{\phi_1^2}{2} d\phi_1$		$\int \text{vers}\,\phi_1 d\phi_1$	$\int (\phi_1 - \sin\phi_1) d\phi_1$	$\int (\frac{\phi_1^2}{2} - \text{vers}\,\phi_1) d\phi_1$

TABLE 14—INTEGRALS FOR ARCH LOAD CONSTANTS

ϕ_1	$\int (\phi_1 - \sin\phi_1) \sin\phi\, d\phi_1$		$\int \text{vers}\,\phi_1 \sin\phi\, d\phi_1$		$\int \sin\phi_1 \sin\phi\, d\phi_1$	
° ′	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$
0 15	.0^{10}15,102,722	.0^{13}52,718,658	.0^{7}13,845,160	.0^{10}45,308,339	.0^{5}9,519,233,958	.0^{7}27,690,399
0 30	.0^{9},241,638,493	.0^{11}11,686,982	.0^{6},110,759,067	.0^{9},724,926,521	.0^{4}38,076,210,902	.0^{6},221,520,664
0 45	.0^{8}1,223,252	.0^{10}12,810,332	.0^{6},373,799,397	.0^{8}3,669,882	.0^{4}85,668,756,111	.0^{6},747,618,010
1 00	.0^{8}3,865,892	.0^{9},053,981,520	.06,886,001,687	.0711,598,383	.0^{3},152,293,245	.0^{5}1,772,084
1 15	.0^{8}9,437,620	.09,164,734 157	.0^{5}1,730,368	.0^{7}28,315,555	.0^{3},237,944,605	.053,460,984
1 30	.0^{7}19,568,346	.09,409,898,012	.0^{5}2,989,857	.0^{7}58,713,086	.0^{3},342,616,311	.0^{5}5,980,329
1 45	.0^{7}36,249,487	.09,885,917,181	.0^{5}4,747,371	.06,108,768,754	.0^{3},466,300,395	.0^{5}9,496,071
2 00	.0^{7}61,833,547	.081,727,164	.057,085,747	.06,185,545,860	.0^{3},608,987,435	.0^{4}14,174,084
2 15	.0^{7}99,033,630	.0^{8}3,112,250	.0^{4}10,087,742	.06,297,192,559	.0^{3},770,666,567	.0^{4}20,180,153
2 30	.06,150,922,864	.0^{8}5,270,328	.0^{4}13,836,028	.06,452,941,079	.0^{3},951,325,477	.0^{4}27,679,963
2 45	.06,220,933,762	.0^{8}8,487,391	.0^{4}18,413,176	.06,663,106 844	.021,150,950,408	.0^{4}36,839,085
3 00	.06,312,857,499	.0713,112,563	.0^{4}23,901,646	.06,939,087,494	.021,369,526,158	.0^{4}47,822,963
3 15	.06,430,843,118	.0719,564,382	.0^{4}30,383,776	.0^{5}1,293,362	.021,607,036,081	.0^{4}60,796,903
3 30	.06,579,396,658	.0^{7}28,337,073	.0^{4}37,941,774	.0^{5}1,739,488	.021,863,462,090	.0^{4}75,926,059
3 45	.06,763,380,206	.0^{7}40,006,816	.0^{4}46,657,700	.0^{5}2,292,105	.022,138,784,657	.0^{4}93,375,420
4 00	.06,988,010,881	.0^{7}55,237,999	.0^{4}56,613,464	.0^{5}2,966,925	.022,432,982,815	.03,113,309,800
4 15	.0^{5}1,258,860	.0^{7}74,789,459	.0^{4}67,890,808	.0^{5}3,780,741	.022,746,034,159	.0^{3},135,893,822
4 30	.0^{5}1,581,851	.0^{7}99,520,722	.0^{4}80,571,298	.0^{5}4,751,416	.023,077,914 851	.0^{3},161,291,910
4 45	.0^{5}1,963,259	.06,130,398,214	.0^{4}94,736,312	.0^{5}5,897,886	.023,428,599,616	.0^{3},189,668,270
5 00	.0^{5}2,409,710	.06,168,501,473	.0^{3},110,467,031	.0^{5}7,240,161	.023,798,061,747	.0^{3},221,186,883
5 15	.0^{5}2,928,178	.06,215,029,340	.0^{3},127,844,425	.0^{5}8,799,316	.024,186,273,109	.0^{3},256,011,492
5 30	.0^{5}3,525,984	.06,271,306,136	.0^{3},146,949,246	.0^{4}10,597,495	.024,593,204,138	.0^{3},294,305,586
5 45	.0^{5}4,210,797	.06,338,787,824	.0^{3},167,862,013	.0^{4}12,657,904	.025,018,823,845	.0^{3},336,232,391
6 00	.0^{5}4,990,627	.06,419,068,158	.0^{3},190,663,003	.0^{4}15,004,815	.025,463,099,817	.0^{3},381,954,855
6 15	.0^{5}5,873,830	.06,513,884,806	.0^{3},215,432,242	.0^{4}17,663,558	.025,925,998,220	.0^{3},431,635,640
6 30	.0^{5}6,869,100	.06,625,125,466	.0^{3},242,249,492	.0^{4}20,660,520	.026,407,483,804	.0^{3},485,437,104
6 45	.0^{5}7,985,468	.06,754,833,956	.0^{3},271,194,239	.0^{4}24,023,144	.026,907,519,901	.0^{3},543,521,291
7 00	.059,232,308	.06,905,216,284	.0^{3},302,345,685	.0^{4}27,779,928	.027,426,068,431	.0^{3},606,049,920
7 15	.0^{4}10,619,324	.0^{5}1,078,647	.0^{3},335,782,737	.0^{4}31,960,415	.0^{2}7,963,089,905	.0^{3},673,184,371
7 30	.0^{4}12,156,554	.0^{5}1,277,674	.0^{3},371,583,994	.0^{4}36,595,199	.028,518,543,428	.0^{3},745,085,674
7 45	.0^{4}13,854,367	.0^{5}1,505,026	.0^{3},409,827,739	.0^{4}41,715,915	.029,092,386,698	.0^{3},821,914,495
8 00	.0^{4}15,723,463	.0^{5}1,763,619	.0^{3},450,591,926	.0^{4}47,355,243	.029,684,576,015	.0^{3},903,831,126
8 15	.0^{4}17,774,862	.0^{5}2,056,560	.0^{3},493,954,170	.0^{4}53,546,898	.010,295,066,283	.0^{3},990,995,469
8 30	.0^{4}20,019,915	.0^{5}2,387,155	.0^{3},539,991,740	.0^{4}60,325,629	.010,923,811,009	.0^{2}1,083,567,029
8 45	.0^{4}22,470,290	.0^{5}2,758,912	.0^{3},588,781,539	.0^{4}67,727,219	.011,570,762,313	.0^{2}1,181,704,898
9 00	.0^{4}25,137,976	.0^{5}3,175,552	.0^{3},640,400,107	.0^{4}75,788,479	.012,235,870,926	.0^{2}1,285,567,746
9 15	.0^{4}28,035,277	.0^{5}3,641,010	.0^{3},694,923,598	.0^{4}84,547,242	.012,919,086,198	.0^{2}1,395,313,803
9 30	.0^{4}31,174,813	.0^{5}4,159,443	.0^{3},752,427,777	.0^{4}94,042,363	.013,620,356,100	.0^{2}1,511,100,855
9 45	.0^{4}34,569,512	.0^{5}4,735,236	.0^{3},812,988,006	.0^{3},104,313,715	.014,339,627,227	.0^{2}1,633,086,226
10 00	.0^{4}38,232,612	.0^{5}5,373,006	.0^{3},876,679,236	.0^{3},115,402,184	.015,076,844,804	.0^{2}1,761,426,768
10 15	.0^{4}42,177,658	.0^{5}6,077,612	.0^{3},943,575,995	.0^{3},127,349,666	.015,831,952,688	.0^{2}1,896,278,850
10 30	.0^{4}46,418,494	.0^{5}6,854,154	.0^{2}1,013,752,376	.0^{3},140,199,060	.016,604,893,376	.0^{2}2,037,798,343
10 45	.0^{4}50,969,267	.0^{5}7,707,985	.0^{2}1,087,282,033	.0^{3},153,994,269	.017,395,608,005	.0^{2}2,186,140,613
11 00	.0^{4}55,844,419	.0^{5}8,644,712	.0^{2}1,164,238,163	.0^{3},168,780,194	.018,204,036,358	.0^{2}2,341,460,505
11 15	.0^{4}61,058,683	.0^{5}9,670,204	.0^{2}1,244,693,500	.0^{3},184,602,725	.019,030,116,872	.0^{2}2,503,912,333
11 30	.0^{4}66,627,086	.0^{4}10,790,602	.0^{2}1,328,720,305	.0^{3},201,508,742	.019,873,786,637	.0^{2}2,673,649,867
11 45	.0^{4}72,564,941	.0^{4}12,012,313	.0^{2}1,416,390,354	.0^{3},219,546,111	.020,734,981,404	.0^{2}2,850,826,323
12 00	.0^{4}78,887,843	.0^{4}13,342,029	.0^{2}1,507,774,929	.0^{3},238,763,677	.021,613,635,589	.0^{2}3,035,594,350
12 15	.0^{4}85,611,667	.0^{4}14,786,717	.0^{2}1,602,944,808	.0^{3},259,211,256	.022,509,682,281	.0^{2}3,228,106,021
12 30	.0^{4}92,752,566	.0^{4}16,353,645	.0^{2}1,701,970,253	.0^{3},280,939,639	.023,423,053,241	.0^{2}3,428,512,814
12 45	.0^{3},100,326,967	.0^{4}18,050,367	.0^{2}1,804,921,005	.0^{3},304,000,578	.024,353,678,913	.0^{2}3,636,965,612
13 00	.0^{3},108,351,564	.0^{4}19,884,738	.0^{2}1,911,866,267	.0^{3},328,446,790	.025,301,488,425	.0^{2}3,853,614,682
13 15	.0^{3},116,843,318	.0^{4}21,864,923	.0^{2}2,022,874,700	.0^{3},354,331,941	.026,266,409,599	.0^{2}4,078,609,667
13 30	.0^{3},125,819,453	.0^{4}23,999,394	.0^{2}2,138,014,412	.0^{3},381,710,649	.027,248,368,953	.0^{2}4,312,099,575
13 45	.0^{3},135,297,49	.0^{4}26,296,941	.0^{2}2,257,352,943	.0^{3},410,638,482	.028,247,291,705	.0^{2}4,554,232,766
14 00	.0^{3},145,295,043	.0^{4}28,766,676	.0^{2}2,380,957,264	.0^{3},441,171,939	.029,263,101,785	.0^{2}4,805,156,943
14 15	.0^{3},155,830,221	.0^{4}31,418,032	.0^{2}2,508,893,760	.0^{3},473,368,458	.030,295,721,835	.0^{2}5,065,019,140
14 30	.0^{3},166,921,215	.0^{4}34,260,781	.0^{2}2,641,228,223	.0^{3},507,286,407	.031,345,073,215	.0^{2}5,333,965,708
14 45	.0^{3},178,586,502	.0^{4}37,305,030	.0^{2}2,778,025,845	.0^{3},542,985,071	.032,411,076,015	.0^{2}5,612,142,308
15 00	.0^{3},190,844,794	.0^{4}40,561,221	.0^{2}2,919,351,203	.0^{3},580,524,657	.033,493,649,054	.0^{2}5,899,693,900
15 15	.0^{3},203,715,038	.0^{4}44,040,154	.0^{2}3,065,268,253	.0^{3},619,966,281	.034,592,709,890	.0^{2}6,196,764,724
15 30	.0^{3},217,216,414	.0^{4}47,752,971	.0^{2}3,215,840,321	.0^{3},661,371,967	.035,708,174,824	.0^{2}6,503,498,302
15 45	.0^{3},231,368,321	.0^{4}51,711,177	.0^{2}3,371,130,092	.0^{3},704,804,635	.036,839,958,911	.0^{2}6,820,037,416
16 00	.0^{3},246,190,385	.0^{4}55,926,637	.0^{2}3,531,199,599	.0^{3},750,328,101	.037,987,975,961	.0^{2}7,146,524,101
16 15	.0^{3},261,702,448	.0^{4}60,411,580	.0^{2}3,696,110,220	.0^{3},798,007,067	.039,152,138,547	.0^{2}7,483,099,638
16 30	.0^{3},277,924,560	.0^{4}65,178,610	.0^{2}3,865,922,661	.0^{3},847,907,118	.040,332,358,014	.0^{2}7,829,904,536
16 45	.0^{3},294,876,985	.0^{4}70,240,704	.0^{2}4,040,696,952	.0^{3},900,094,712	.041,528,544,483	.0^{2}8,187,078,527
17 00	.0^{3},312,580,182	.0^{4}75,611,222	.0^{2}4,220,492,436	.0^{3},954,637,176	.042,740,606,861	.0^{2}8,554,760,552
17 15	.0^{3},331,054,816	.0^{4}81,303,909	.0^{2}4,405,367,760	.0^{2}1,011,602,698	.043,968,452,845	.0^{2}8,933,088,753
17 30	.0^{3},350,321,741	.0^{4}87,332,896	.0^{2}4,595,380,867	.0^{2}1,071,060,324	.045,211,988,928	.0^{2}9,322,200,462
17 45	.0^{3},370,402,001	.0^{4}93,712,716	.0^{2}4,790,588,986	.0^{2}1,133,079,946	.046,471,120,411	.0^{2}9,722,232,187
18 00	.0^{3},391,316,825	.0^{3},100,458,295	.0^{2}4,991,048,622	.0^{2}1,197,732,299	.047,745,751,406	.010,133,319,606
18 15	.0^{3},413,087,618	.0^{3},107,584,963	.0^{2}5,196,815,552	.0^{2}1,265,088,952	.049,035,784,846	.010,555,597,557
18 30	.0^{3},435,735,962	.0^{3},115,108,462	.0^{2}5,407,944,808	.0^{2}1,335,222,306	.050,341,122,488	.010,989,200,022
18 45	.0^{3},459,283,607	.0^{3},123,044,943	.0^{2}5,624,490,676	.0^{2}1,408,205,578	.051,661,664,927	.011,434,260,122
19 00	.0^{3},483,752,464	.0^{3},131,410,973	.0^{2}5,846,506,686	.0^{2}1,484,112,803	.052,997,311,598	.011,890,910,108
19 15	.0^{3},509,164,609	.0^{3},140,223,541	.0^{2}6,074,045,599	.0^{2}1,563,018,820	.054,347,960,787	.012,359,281,345
19 30	.0^{3},535,542,264	.0^{3},149,500,060	.0^{2}6,307,159,402	.0^{2}1,644,999,272	.055,713,509,636	.012,839,504,307
19 45	.0^{3},562,907,805	.0^{3},159,258,374	.0^{2}6,545,899,299	.0^{2}1,730,130,591	.057,093,854,153	.013,331,708,565
20 00	.0^{3},591,283,750	.0^{3},169,516,760	.0^{2}6,790,315,705	.0^{2}1,818,489,994	.058,488,889,220	.013,836,022,778
20 15	.0^{3},620,692,749	.0^{3},180,293,928	.0^{2}7,040,458,230	.0^{2}1,910,155,478	.059,898,508,600	.014,352,574,682
20 30	.0^{3},651,157,593	.0^{3},191,609,035	.0^{2}7,296,375,682	.0^{2}2,005,205,808	.061,322,604,944	.014,881,491,082
20 45	.0^{3},682,701,193	.0^{3},203,481,681	.0^{2}7,558,116,049	.0^{2}2,103,720,511	.062,761,069,803	.015,422,897,840
21 00	.0^{3},715,346,583	.0^{3},215,931,909	.0^{2}7,825,726,496	.0^{2}2,205,779,872	.064,213,793,631	.015,976,919,870
21 15	.0^{3},749,116,914	.0^{3},228,980,227	.0^{2}8,099,253,355	.0^{2}2,311,464,919	.065,680,665,798	.016,543,681,120
21 30	.0^{3},784,035,444	.0^{3},242,647,587	.0^{2}8,378,742,119	.0^{2}2,420,857,423	.067,161,574,595	.017,123,304,574
21 45	.0^{3},820,125,538	.0^{3},256,955,409	.0^{2}8,664,237,432	.0^{2}2,534,039,881	.068,656,407,247	.017,715,912,231
22 00	.0^{3},857,410,657	.0^{3},271,925,573	.0^{2}8,955,783,082	.0^{2}2,651,095,518	.070,165,049,915	.018,321,625,105
22 15	.0^{3},895,914,357	.0^{3},287,580,428	.0^{2}9,253,421,993	.0^{2}2,772,108,271	.071,687,387,711	.018,940,563,210
22 30	.0^{3},935,660,279	.0^{3},303,942,794	.0^{2}9,557,196,219	.0^{2}2,897,162,786	.073,223,304,703	.019,572,845,553
° ′ ϕ_1	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$
	$\int (\phi_1 - \sin\phi_1) \cos\phi\, d\phi_1$		$\int \text{vers}\,\phi_1 \cos\phi\, d\phi_1$		$\int \sin\phi_1 \cos\phi\, d\phi_1$	

TABLE 14—Continued

ϕ_1	$\int (\phi_1 - \sin\phi_1) \sin\phi\, d\phi_1$		$\int \text{vers}\ \phi_1 \sin\phi\, d\phi_1$		$\int \sin\phi_1 \sin\phi\, d\phi_1$	
° ′	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$
22 45	.0³, 976,672,146	.0³, 321,035,964	.009,867,146,934	.003,026,344,405	.074,772,683,925	.020,218,590,126
23 00	.001,018,973,757	.0³, 338,883,707	.010,183,314,425	.003,159,739,163	.076,335,407,385	.020,877,913,895
23 15	.001,062,588,978	.0³, 357,510,277	.010,505,738,087	.003,297,433,775	.077,911,356,076	.021,550,932,792
23 30	.001,107,541,738	.0³, 376,940,410	.010,834,456,411	.003,439,515,631	.079,500,409,984	.022,237,761,705
23 45	.001,153,856,025	.0³, 397,199,328	.011,169,506,982	.003,586,072,785	.081,102,448,096	.022,938,514,472
24 00	.001,201,555,878	.0³, 418,312,745	.011,510,926,467	.003,737,193,947	.082,717,348,410	.023,653,303,870
24 15	.001,250,665,375	.0³, 440,306,869	.011,858,850,611	.003,892,968,477	.084,344,987,946	.024,382,241,607
24 30	.001,301,208,643	.0³, 463,208,402	.012,213,014,231	.004,053,486,371	.085,985,242,752	.025,125,438,314
24 45	.001,353,209,831	.0³, 487,044,548	.012,573,751,203	.004,218,838,258	.087,637,987,917	.025,883,003,535
25 00	.001,406,693,120	.0³, 511,843,015	.012,940,994,461	.004,389,115,385	.089,303,097,578	.026,655,045,719
25 15	.001,461,682,707	.0³, 537,632,009	.013,314,775,990	.004,564,409,616	.090,980,444,930	.027,441,672,217
25 30	.001,518,202,807	.0³, 564,440,249	.013,695,126,815	.004,744,813,413	.092,669,902,237	.028,242,989,265
25 45	.001,576,277,638	.0³, 592,296,964	.014,082,076,997	.004,930,419,837	.094,371,340,840	.029,059,101,981
26 00	.001,635,931,417	.0³, 621,231,896	.014,475,655,628	.005,121,322,532	.096,084,631,169	.029,890,114,357
26 15	.001,697,188,362	.0³, 651,275,301	.014,875,890,822	.005,317,615,719	.097,809,642,748	.030,736,129,251
26 30	.001,760,072,670	.0³, 682,457,955	.015,282,809,709	.005,519,394,186	.099,546,244,212	.031,597,248,377
26 45	.001,824,608,523	.0³, 714,811,151	.015,696,438,429	.005,726,753,277	.101,294,303,312	.032,473,572,300
27 00	.001,890,820,078	.0³, 748,366,712	.016,116,802,127	.005,939,788,885	.103,053,686,927	.033,365,200,425
27 15	.001,958,731,459	.0³, 783,156,975	.016,543,924,944	.006,158,597,442	.104,824,261,072	.034,272,230,995
27 30	.002,028,366,746	.0³, 819,214,813	.016,977,830,014	.006,383,275,910	.106,605,890,912	.035,194,761,077
27 45	.002,099,749,981	.0³, 856,573,626	.017,418,539,456	.006,613,921,768	.108,398,440,769	.036,132,886,559
28 00	.002,172,905,150	.0³, 895,267,344	.017,866,074,368	.006,850,633,009	.110,201,774,132	.037,086,702,140
28 15	.002,247,856,176	.0³, 935,330,429	.018,320,454,824	.007,093,508,123	.112,015,753,672	.038,056,301,327
28 30	.002,324,626,922	.0³, 976,797,882	.018,781,699,864	.007,342,646,092	.113,840,241,246	.039,041,776,422
28 45	.002,403,241,174	.001,019,705,235	.019,249,827,492	.007,598,146,379	.115,675,097,913	.040,043,218,521
29 00	.002,483,722,637	.001,064,088,567	.019,724,854,668	.007,860,108,919	.117,520,183,942	.041,060,717,500
29 15	.002,566,094,937	.001,109,984,490	.020,206,797,304	.008,128,634,106	.119,375,358,821	.042,094,362,015
29 30	.002,650,381,594	.001,157,430,159	.020,695,670,259	.008,403,822,788	.121,240,481,272	.043,144,239,494
29 45	.002,736,606,037	.001,206,463,278	.021,191,487,331	.008,685,776,251	.123,115,409,260	.044,210,436,124
30 00	.002,824,791,583	.001,257,122,089	.021,694,261,255	.008,974,596,216	.125,000,000,000	.045,293,036,853
30 15	.002,914,961,436	.001,309,445,384	.022,204,003,697	.009,270,384,822	.126,894,109,974	.046,392,125,379
30 30	.003,007,138,675	.001,363,472,500	.022,720,725,247	.009,573,244,619	.128,797,594,939	.047,507,784,144
30 45	.003,101,346,254	.001,419,243,325	.023,244,435,418	.009,883,278,564	.130,710,309,935	.048,640,094,329
31 00	.003,197,606,989	.001,476,798,298	.023,775,142,636	.010,200,589,995	.132,632,109,303	.049,789,135,844
31 15	.003,295,943,551	.001,536,178,406	.024,312,854,242	.010,525,282,636	.134,562,846,691	.050,954,987,329
31 30	.003,396,378,463	.001,597,425,188	.024,857,576,480	.010,857,460,581	.136,502,375,065	.052,137,726,142
31 45	.003,498,934,091	.001,660,580,740	.025,409,314,497	.011,197,228,281	.138,450,546,723	.053,337,428,353
32 00	.003,603,632,632	.001,725,687,707	.025,968,072,339	.011,544,690,541	.140,407,213,303	.054,554,168,744
32 15	.003,710,496,119	.001,792,789,291	.026,533,852,944	.011,899,952,498	.142,372,225,798	.055,788,020,796
32 30	.003,819,546,396	.001,861,929,248	.027,106,658,139	.012,263,119,622	.144,345,434,565	.057,039,056,690
32 45	.003,930,805,126	.001,933,151,891	.027,686,488,635	.012,634,297,700	.146,326,689,336	.058,307,347,295
33 00	.004,044,293,779	.002,006,502,090	.028,273,344,025	.013,013,592,824	.148,315,939,231	.059,592,962,168
33 15	.004,160,033,619	.002,082,025,268	.028,867,222,780	.013,401,111,383	.150,312,732,769	.060,895,969,548
33 30	.004,278,045,706	.002,159,767,410	.029,468,122,240	.013,796,960,055	.152,317,217,878	.062,216,436,346
33 45	.004,398,350,883	.002,239,775,056	.030,076,038,618	.014,201,245,788	.154,329,141,909	.063,554,428,146
34 00	.004,520,969,767	.002,322,095,304	.030,690,966,990	.014,614,075,799	.156,348,351,646	.064,910,009,197
34 15	.004,645,922,747	.002,406,775,808	.031,312,901,296	.015,035,557,554	.158,374,693,319	.066,283,242,409
34 30	.004,773,229,975	.002,493,864,784	.031,941,834,332	.015,465,798,764	.160,408,012,614	.067,674,189,345
34 45	.004,902,911,354	.002,583,411,002	.032,577,757,750	.015,904,907,373	.162,448,154,685	.069,082,910,222
35 00	.005,034,986,536	.002,675,463,793	.033,220,662,056	.016,352,991,542	.164,494,964,169	.070,509,463,903
35 15	.005,169,474,915	.002,770,073,043	.033,870,536,600	.016,810,159,647	.166,548,285,191	.071,953,907,891
35 30	.005,306,395,609	.002,867,289,197	.034,527,369,582	.017,276,520,258	.168,607,961,386	.073,416,298,329
35 45	.005,445,767,473	.002,967,163,254	.035,191,148 042	.017,752,182,137	.170,673,835,898	.074,896,689,993
36 00	.005,587,609,067	.003,069,746,775	.035,861,857,859	.018,237,254,219	.172,745,751,406	.076,395,136,285
36 15	.005,731,938,668	.003,175,091,874	.036,539,483,753	.018,731,845,609	.174,823,550,124	.077,911,689,237
36 30	.005,878,774,254	.003,283,251,216	.037,224,009,271	.019,236,065,564	.176,907,073,819	.079,446,399,498
36 45	.006,028,133,497	.003,394,278,030	.037,915,416,800	.019,750,023,485	.178,996,163,824	.080,999,316,337
37 00	.006,180,033,754	.003,508,226,091	.038,613,687,549	.020,273,828,907	.181,090,661,046	.082,570,487,635
37 15	.006,334,492,067	.003,625,149,734	.039,318,801,557	.020,807,591,485	.183,190,405,980	.084,159,959,882
37 30	.006,491,525,142	.003,745,103,840	.040,030,737,688	.021,351,420,984	.185,295,238,725	.085,767,778,177
37 45	.006,651,149,359	.003,868,143,845	.040,749,473,626	.021,905,427,270	.187,404,998,986	.087,393,986,220
38 00	.006,813,380,748	.003,994,325,736	.041,474,985,878	.022,469,720,293	.189,519,526,100	.089,038,626,310
38 15	.006,978,234,990	.004,123,706,047	.042,207,249,766	.023,044,410,083	.191,638,659,036	.090,701,739,344
38 30	.007,145,727,409	.004,256,341,859	.042,946,239,433	.023,629,606,734	.193,762,236,414	.092,383,364,813
38 45	.007,315,872,964	.004,392,290,801	.043,691,927,830	.024,225,420,392	.195,890,096,515	.094,083,540,794
39 00	.007,488,686,238	.004,531,611,051	.044,444,286,727	.024,831,961,247	.198,022,077,296	.095,802,303,955
39 15	.007,664,181,436	.004,674,361,322	.045,203,286,704	.025,449,339,522	.200,158,016,396	.097,539,689,549
39 30	.007,842,372,373	.004,820,600,878	.045,968,897,147	.026,077,665,456	.202,297,751,156	.099,295,731,407
39 45	.008,023,272,468	.004,970,389,517	.046,741,086,256	.026,717,049,300	.204,441,118,627	.101,070,461,943
40 00	.008,206,894,738	.005,123,787,579	.047,519,821,035	.027,367,601,297	.206,587,955,584	.102,863,912,146
40 15	.008,393,251,788	.005,280,855,940	.048,305,067,295	.028,029,431,682	.208,738,098,535	.104,676,111,580
40 30	.008,582,355,803	.005,441,656,013	.049,096,789,652	.028,702,650,660	.210,891,383,740	.106,507,088,380
40 45	.008,774,218,544	.005,606,249,740	.049,894,951,528	.029,387,368,398	.213,047,647,218	.108,356,869,253
41 00	.008,968,851,340	.005,774,699,597	.050,699,515,146	.030,083,695,017	.215,206,724,760	.110,225,479,473
41 15	.009,166,265,072	.005,947,068,586	.051,510,441,533	.030,791,740,576	.217,368,451,945	.112,112,942,881
41 30	.009,366,470,181	.006,123,420,241	.052,327,690,517	.031,511,615,062	.219,532,664,149	.114,019,281,879
41 45	.009,569,476,645	.006,303,818,616	.053,151,220,728	.032,243,428,380	.221,699,196,558	.115,944,517,434
42 00	.009,775,293,980	.006,488,328,286	.053,980,989,599	.032,987,290,340	.223,867,884,183	.187,888,669,077
42 15	.009,983,931,236	.006,677,014,348	.054,816,953,360	.033,743,310,643	.226,038,561,870	.119,851,754,892
42 30	.010,195,396,975	.006,869,942,415	.055,659,067,044	.034,511,598,877	.228,211,064,313	.121,833,791,525
42 45	.010,409,699,279	.007,067,178,614	.056,507,284,486	.035,292,264,496	.230,385,226,068	.123,834,794,180
43 00	.010,626,845,735	.007,268,789,583	.057,361,558,319	.036,085,416,817	.232,560,881,564	.125,854,776,614
43 15	.010,846,843,429	.007,474,842,472	.058,221,839,977	.036,891,165,002	.234,737,865,116	.127,893,751,138
43 30	.011,069,698,938	.007,685,404,932	.059,088,079,697	.037,709,618,049	.236,916,010,939	.129,951,728,620
43 45	.011,295,418,323	.007,900,545,118	.059,960,226,513	.038,540,884,782	.239,095,153,158	.132,028,718,478
44 00	.011,524,007,121	.008,120,331,687	.060,838,228,266	.039,385,073,837	.241,275,125,824	.134,124,728,684
44 15	.011,755,470,338	.008,344,833,793	.061,722,031,594	.040,242,293,652	.243,455,762,923	.136,239,765,759
44 30	.011,989,812,442	.008,574,121,078	.062,611,581,942	.041,112,652,455	.245,636,898,391	.138,373,834,779
44 45	.012,227,037,359	.008,808,263,678	.063,506,823,557	.041,996,258,252	.247,818,366,125	.140,526,939,368
45 00	.012,467,148,457	.009,047,332,218	.064,407,699,488	.042,893,218,813	.250,000,000,000	.142,699,081,699
° ′	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$
ϕ_1	$\int (\phi_1 - \sin\phi_1) \cos\phi\, d\phi_1$		$\int \text{vers}\ \phi_1 \cos\phi\, d\phi_1$		$\int \sin\phi_1 \cos\phi\, d\phi_1$	

TABLE 14—Continued

ϕ_1	$\int(\phi_1-\sin\phi_1)\sin\phi\,d\phi_1$		$\int \text{vers}\,\phi_1 \sin\phi\,d\phi_1$		$\int \sin\phi_1 \sin\phi\,d\phi_1$	
° ′	+SIN Φ_o	+COS Φ_o	+SIN Φ_o	+COS Φ_o	+SIN Φ_o	+COS Φ_o
45 15	.012,710,148,544	.009,291,397,802	.065,314,151,594	.043,803,641,670	.252,181,633,874	.144,890,262,498
45 30	.012,956,039,863	.009,540,532,016	.066,226,120,536	.044,727,634,091	.254,363,101,609	.147,100,481,039
45 45	.013,204,824,082	.009,794,806,920	.067,143,545,787	.045,665,303,081	.256,544,237,077	.149,329,735,149
46 00	.013,456,502,284	.010,054,295,047	.068,066,365,626	.046,616,755,365	.258,724,874,176	.151,578,021,204
46 15	.013,711,074,966	.010,319,069,401	.068,994,517,141	.047,582,097,376	.260,904,846,842	.153,845,334,128
46 30	.013,968,542,027	.010,589,203,445	.069,927,936,235	.048,561,435,245	.263,083,989,061	.156,131,667,400
46 45	.014,228,902,761	.010,864,771,111	.070,866,557,623	.049,554,874,791	.265,262,134,883	.158,437,013,048
47 00	.014,492,155,851	.011,145,846,780	.071,810,314,836	.050,562,521,502	.267,439,118,436	.160,761,361,654
47 15	.014,758,299,366	.011,432,505,289	.072,759,140,219	.051,584,480,535	.269,614,773,932	.163,104,702,350
47 30	.015,027,330,743	.011,724,821,921	.073,712,964,938	.052,620,856,697	.271,788,935,687	.165,467,022,825
47 45	.015,299,246,792	.012,022,872,406	.074,671,718,982	.053,671,754,435	.273,961,438,130	.167,848,309,322
48 00	.015,574,043,681	.012,326,732,912	.075,635,331,157	.054,737,277,824	.276,132,115,817	.170,248,546,637
48 15	.015,851,716,933	.012,636,480,045	.076,603,729,099	.055,817,530,557	.278,300,803,442	.172,667,718,124
48 30	.016,132,261,416	.012,952,190,833	.077,576,839,270	.056,912,615,933	.280,467,335,851	.175,105,805,698
48 45	.016,415,671,338	.013,273,942,741	.078,554,586,962	.058,022,636,845	.282,631,548,055	.177,562,789,830
49 00	.016,701,940,241	.013,601,813,647	.079,536,896,299	.059,147,695,769	.284,793,275,240	.180,038,649,553
49 15	.016,991,060,990	.013,935,881,851	.080,523,690,240	.060,287,894,755	.286,952,352,782	.182,533,362,463
49 30	.017,283,025,770	.014,276,226,062	.081,514,890,583	.061,443,335,410	.289,108,616,260	.185,046,904,720
49 45	.017,577,826,080	.014,622,925,396	.082,510,417,965	.062,614,118,891	.291,261,901,466	.187,579,251,049
50 00	.017,875,452,722	.014,976,059,371	.083,510,191,867	.063,800,345,896	.293,412,044,417	.190,130,374,745
50 15	.018,175,895,794	.015,335,707,902	.084,514,130,618	.065,002,116,646	.295,558,881,373	.192,700,247,672
50 30	.018,479,144,691	.015,701,951,296	.085,504,151,397	.066,219,530,878	.297,702,248,844	.195,288,840,266
50 45	.018,785,188,089	.016,074,870,243	.086,534,170,233	.067,452,687,833	.299,841,983,604	.197,896,121,538
51 00	.019,094,013,942	.016,454,545,820	.087,550,102,015	.068,701,686,246	.301,977,922,704	.200,522,059,075
51 15	.019,405,609,477	.016,841,059,474	.088,569,860,490	.069,966,624,331	.304,109,903,485	.203,166,619,044
51 30	.019,719,961,189	.017,234,493,024	.089,593,358,267	.071,247,599,776	.306,237,763,586	.205,829,766,192
51 45	.020,037,054,825	.017,634,928,655	.090,620,506,828	.072,544,709,726	.308,361,340,964	.208,511,463,854
52 00	.020,356,875,387	.018,042,448,908	.091,651,216,519	.073,858,050,774	.310,480,473,900	.211,211,673,949
52 15	.020,679,407,123	.018,457,136,678	.092,685,396,566	.075,187,718,952	.312,595,001,014	.213,930,356,989
52 30	.021,004,633,522	.018,879,075,209	.093,722,955,070	.076,533,809,716	.314,704,761,275	.216,667,472,076
52 45	.021,332,537,299	.019,308,348,087	.094,763,799,019	.077,896,417,937	.316,809,594,020	.219,422,976,911
53 00	.021,663,100,402	.019,745,039,229	.095,807,834,284	.079,275,637,894	.318,909,338,954	.222,196,827,794
53 15	.021,996,303,996	.020,189,232,887	.096,854,965,630	.080,671,563,253	.321,003,836,176	.224,988,979,626
53 30	.022,332,128,459	.020,641,013,633	.097,905,086,718	.082,084,287,068	.323,092,926,181	.227,799,385,918
53 45	.022,670,553,381	.021,100,466,358	.098,958,130,106	.083,513,901,760	.325,176,449,876	.230,627,998,786
54 00	.023,011,557,545	.021,567,676,263	.100,013,967,263	.084,960,499,114	.327,254,248,594	.233,474,768,965
54 15	.023,355,118,939	.022,042,728,855	.101,036,508,560	.086,424,170,262	.329,326,164,101	.236,339,645,802
54 30	.023,701,214,733	.022,525,709,936	.102,133,653,288	.087,905,005,675	.331,392,038,614	.239,222,577,269
54 45	.024,049,821,282	.023,016,705,608	.103,197,299,655	.089,403,095,154	.333,451,714,809	.242,123,509,960
55 00	.024,400,914,121	.023,515,802,247	.104,263,344,794	.090,918,527,818	.335,505,035,831	.245,042,389,102
55 15	.024,754,467,951	.024,023,086,516	.105,331,684,767	.092,451,392,089	.337,551,845,315	.247,979,158,552
55 30	.025,110,456,646	.024,538,645,349	.106,402,214,569	.094,001,775,689	.339,591,987,386	.250,933,760,804
55 45	.025,468,853,231	.025,062,565,943	.107,474,828,138	.095,569,765,624	.341,625,306,681	.253,906,136,998
56 00	.025,829,629,891	.025,594,935,754	.108,549,418,355	.097,155,448,175	.343,651,648,354	.256,896,226,917
56 15	.026,192,757,958	.026,135,842,493	.109,625,877,051	.098,758,908,888	.345,670,858,092	.259,903,968,995
56 30	.026,558,207,904	.026,685,374,109	.110,704,095,015	.100,380,232,566	.347,682,782,122	.262,929,300,325
56 45	.026,925,949,340	.027,243,618,795	.111,783,961,998	.102,019,503,248	.349,687,267,232	.265,972,156,657
57 00	.027,295,951,008	.027,810,664,971	.112,865,366,716	.103,676,804,216	.351,684,160,769	.269,032,472,408
57 15	.027,668,180,778	.028,386,601,284	.113,948,196,861	.105,352,217,968	.353,673,310,664	.272,110,180,664
57 30	.028,042,605,636	.028,971,516,592	.115,034,339,105	.107,045,826,218	.355,654,565,435	.275,205,213,189
57 45	.028,419,191,686	.029,565,499,966	.116,117,679,103	.108,757,709,882	.357,627,774,202	.278,317,500,426
58 00	.028,797,904,141	.030,168,640,676	.117,204,101,503	.110,487,949,070	.359,592,786,697	.281,446,971,504
58 15	.029,178,707,321	.030,781,028,190	.118,291,489,952	.112,236,623,070	.361,549,453,278	.284,593,554,243
58 30	.029,561,564,640	.031,402,752,156	.119,379,727,099	.114,003,810,349	.363,497,624,935	.287,757,175,161
58 45	.029,946,438,612	.032,033,902,406	.120,468,694,605	.115,789,588,530	.365,437,153,309	.290,937,759,478
59 00	.030,333,290,837	.032,674,568,941	.121,558,273,149	.117,594,034,394	.367,367,890,696	.294,135,231,123
59 15	.030,722,081,998	.033,324,841,926	.122,648,342,433	.119,417,223,858	.369,289,690,065	.297,349,512,738
59 30	.031,112,771,862	.033,984,811,685	.123,801,781,189	.121,259,231,977	.371,202,405,062	.300,580,525,683
59 45	.031,505,319,265	.034,654,568,683	.124,829,467,186	.123,120,132,928	.373,105,890,026	.303,828,190,048
60 00	.031,899,682,116	.035,334,203,534	.125,920,277,240	.125,000,000,000	.375,000,000,000	.307,092,424,652
60 15	.032,295,817,388	.036,023,806,977	.127,011,087,216	.126,898,905,585	.376,884,590,740	.310,373,147,053
60 30	.032,693,681,112	.036,723,469,877	.128,101,772,037	.128,816,921,169	.378,759,518,728	.313,670,273,553
60 45	.033,093,228,379	.037,433,283,216	.129,192,205,691	.130,754,117,324	.380,624,641,179	.316,983,719,204
61 00	.033,494,413,324	.038,153,338,083	.130,282,261,242	.132,710,563,696	.382,479,816,058	.320,313,397,819
61 15	.033,897,189,137	.038,883,725,670	.131,371,810,832	.134,686,328,994	.384,324,902,087	.323,659,221,970
61 30	.034,301,508,043	.039,624,537,252	.132,460,725,687	.136,681,480,986	.386,159,758,754	.327,021,103,002
61 45	.034,707,321,308	.040,375,864,198	.133,548,876,135	.138,696,086,487	.387,984,246,328	.330,398,951,036
62 00	.035,114,579,230	.041,137,797,944	.134,636,131,602	.140,730,211,346	.389,798,225,868	.333,792,674,979
62 15	.035,523,231,139	.041,910,429,994	.135,722,360,625	.142,783,920,444	.391,601,559,231	.337,202,182,528
62 30	.035,933,225,385	.042,693,851,910	.136,807,430,858	.144,857,277,677	.393,394,109,088	.340,627,380,176
62 45	.036,344,509,345	.043,388,155,306	.137,891,209,084	.146,950,345,955	.395,175,738,928	.344,068,173,224
63 00	.036,757,029,408	.044,293,431,830	.138,973,561,216	.149,063,187,187	.396,946,313,073	.347,524,465,784
63 15	.037,170,730,978	.045,109,773,170	.140,054,352,314	.151,195,862,275	.398,705,696,688	.350,996,160,789
63 30	.037,585,558,470	.045,937,271,029	.141,133,446,582	.153,348,431,102	.400,453,755,788	.354,483,159,996
63 45	.038,001,455,302	.046,776,017,131	.142,210,707,387	.155,520,952,529	.402,190,357,252	.357,985,364,000
64 00	.038,418,363,893	.047,626,103,202	.143,285,997,259	.157,713,484,380	.403,915,368,831	.361,502,672,236
64 15	.038,836,225,663	.048,487,620,965	.144,359,177,907	.159,926,083,436	.405,628,659,160	.365,034,982,990
64 30	.039,254,981,026	.049,360,662,130	.145,430,110,218	.162,158,805,429	.407,330,097,763	.368,582,193,404
64 45	.039,674,569,386	.050,245,318,389	.146,498,654,274	.164,411,705,030	.409,019,555,069	.372,144,199,486
65 00	.040,094,929,136	.051,141,681,401	.147,564,669,359	.166,684,835,837	.410,696,902,422	.375,720,896,118
65 15	.040,515,997,650	.052,049,842,785	.148,628,013,962	.168,978,250,381	.412,362,012,082	.379,312,177,063
65 30	.040,937,711,287	.052,969,894,112	.149,688,545,793	.171,292,000,096	.414,014,757,248	.382,917,934,972
65 45	.041,360,005,384	.053,901,926,895	.150,746,121,787	.173,626,135,333	.415,655,012,054	.386,538,061,396
66 00	.041,782,814,251	.054,846,032,582	.151,800,598,116	.175,980,705,334	.417,282,651,590	.390,172,446,789
66 15	.042,206,071,170	.055,802,302,542	.152,851,830,193	.178,355,758,237	.418,897,551,904	.393,820,980,520
66 30	.042,629,708,393	.056,770,828,059	.153,899,672,692	.180,751,341,059	.420,499,590,016	.397,483,550,883
66 45	.043,053,657,138	.057,751,700,321	.154,943,979,543	.183,167,499,692	.422,088,643,924	.401,160,045,100
67 00	.043,477,847,588	.058,745,010,413	.155,984,603,950	.185,604,278,897	.423,664,592,614	.404,850,349,334
67 15	.043,902,208,881	.059,750,849,307	.157,021,398,398	.188,061,722,288	.425,227,316,075	.408,554,348,694
67 30	.044,326,669,121	.060,769,307,847	.158,054,214,666	.190,539,872,339	.426,776,695,296	.412,271,927,251
° ′	+COS Φ_o	−SIN Φ_o	+COS Φ_o	−SIN Φ_o	+COS Φ_o	−SIN Φ_o
ϕ_1	$\int(\phi_1-\sin\phi_1)\cos\phi\,d\phi_1$		$\int \text{vers}\,\phi_1 \cos\phi\,d\phi_1$		$\int \sin\phi_1 \cos\phi\,d\phi_1$	

TABLE 14—Continued

ϕ_1	$\int(\phi_1-\sin\phi_1)\sin\phi\,d\phi_1$		$\int \text{vers}\,\phi_1 \sin\phi\,d\phi_1$		$\int \sin\phi_1 \sin\phi\,d\phi_1$	
° ′	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$	$+\sin\phi_0$	$+\cos\phi_0$
67 45	.044,751,155,360	.061,800,476,748	.159,082,903,830	.193,038,770,359	.428,312,612,289	.416,002,968,038
68 00	.045,175,593,611	.062,844,446,578	.160,107,316,274	.195,558,456,499	.429,834,950,085	.419,747,353,063
68 15	.045,599,908,831	.063,901,307,758	.161,127,301,705	.198,098,969,737	.431,343,592,753	.423,504,963,319
68 30	.046,024,024,927	.064,971,150,542	.162,142,709,159	.200,660,347,871	.432,838,425,405	.427,275,678,793
68 45	.046,447,864,758	.066,054,065,015	.163,153,387,006	.203,242,627,513	.434,319,334,203	.431,059,378,469
69 00	.046,871,350,120	.067,150,141,080	.164,159,182,969	.205,845,844,086	.435,786,206,369	.434,855,940,348
69 15	.047,294,401,755	.068,259,468,443	.165,159,944,129	.208,470,031,805	.437,238,930,197	.438,665,241,449
69 30	.047,716,939,344	.069,382,136,618	.166,155,516,933	.211,115,223,686	.438,677,395,055	.442,487,157,821
69 45	.048,138,881,507	.070,518,234,900	.167,145,747,207	.213,781,451,523	.440,101,491,400	.446,321,564,551
70 00	.048,560,145,805	.071,667,852,367	.168,130,480,166	.216,468,745,894	.441,511,110,780	.450,168,335,776
70 15	.048,980,648,724	.072,831,077,862	.169,109,560,424	.219,177,136,150	.442,906,145,847	.454,027,344,694
70 30	.049,400,305,695	.074,007,999,991	.170,082,832,001	.221,906,650,402	.444,286,490,364	.457,898,463,565
70 45	.049,819,031,073	.075,198,707,105	.171,050,138,340	.224,657,315,524	.445,652,039,213	.461,781,563,733
71 00	.050,236,738,147	.076,403,287,294	.172,011,322,309	.227,429,157,142	.447,002,688,401	.465,676,515,626
71 15	.050,653,339,134	.077,621,828,378	.172,966,226,220	.230,222,199,624	.448,338,335,073	.469,583,188,771
71 30	.051,068,745,182	.078,854,417,894	.173,914,691,830	.233,036,466,083	.449,658,877,512	.473,501,451,800
71 45	.051,482,866,365	.080,101,143,089	.174,856,560,361	.235,871,978,362	.450,964,215,154	.477,431,172,465
72 00	.051,895,611,678	.081,362,090,905	.175,791,672,504	.238,728,757,031	.452,254,248,594	.481,372,217,645
72 15	.052,306,889,048	.082,637,347,973	.176,719,868,432	.241,606,821,383	.453,528,879,589	.485,324,453,355
72 30	.052,716,605,323	.083,927,000,602	.177,640,987,812	.244,506,189,424	.454,788,011,072	.489,287,744,760
72 45	.053,124,666,276	.085,231,134,764	.178,554,869,813	.247,426,877,869	.456,031,547,155	.493,261,956,182
73 00	.053,530,976,599	.086,549,836,096	.179,461,353,117	.250,368,902,138	.457,259,393,139	.497,246,951,110
73 15	.053,935,439,910	.087,883,189,872	.180,360,275,934	.253,332,276,349	.458,471,455,517	.501,242,592,215
73 30	.054,337,958,749	.089,231,281,006	.181,251,476,006	.256,317,013,310	.459,667,641,986	.505,248,741,354
73 45	.054,738,434,575	.090,594,194,038	.182,134,790,626	.259,323,124,516	.460,847,861,453	.509,265,259,586
74 00	.055,136,767,767	.091,972,013,122	.183,010,056,641	.262,350,620,144	.462,012,024,039	.513,292,007,179
74 15	.055,532,857,631	.093,364,822,017	.183,877,110,472	.265,399,509,046	.463,160,041,089	.517,328,843,624
74 30	.055,926,602,385	.094,772,704,075	.184,735,788,114	.268,469,798,746	.464,291,825,176	.521,375,627,641
74 45	.056,317,899,177	.096,195,742,232	.185,585,925,156	.271,561,495,432	.465,407,290,110	.525,432,217,193
75 00	.056,706,644,066	.097,634,018,997	.186,427,356,791	.274,674,603,951	.466,506,350,946	.529,498,469,498
75 15	.057,092,732,040	.099,087,616,445	.187,259,917,825	.277,809,127,809	.467,588,923,985	.533,574,241,036
75 30	.057,476,057,004	.100,556,616,194	.188,083,442,689	.280,965,069,161	.468,654,926,785	.537,659,387,566
75 45	.057,856,511,789	.102,041,099,410	.188,897,765,450	.284,142,428,805	.469,704,278,166	.541,753,764,128
76 00	.058,233,988,142	.103,541,146,788	.189,702,719,822	.287,341,206,185	.470,736,898,215	.545,857,225,061
76 15	.058,608,376,736	.105,056,838,544	.190,498,139,182	.290,561,399,380	.471,752,708,294	.549,969,624,014
76 30	.058,979,567,168	.106,588,254,401	.191,283,856,576	.293,803,005,097	.472,751,631,047	.554,090,813,953
76 45	.059,347,447,956	.108,135,473,582	.192,059,704,730	.297,066,018,678	.473,733,590,400	.558,220,647,175
77 00	.059,711,906,548	.109,698,574,797	.192,825,516,070	.300,350,434,081	.474,698,511,575	.562,358,975,320
77 15	.060,072,829,313	.111,277,636,234	.193,581,122,724	.303,656,243,890	.475,646,321,088	.566,505,649,380
77 30	.060,430,101,548	.112,872,735,548	.194,326,356,537	.306,983,439,303	.476,576,946,759	.570,660,519,712
77 45	.060,783,607,480	.114,483,949,848	.195,061,049,087	.310,332,010,125	.477,490,317,719	.574,823,436,049
78 00	.061,133,230,263	.116,111,355,688	.195,785,031,688	.313,701,944,771	.478,386,364,411	.578,994,247,509
78 15	.061,478,851,983	.117,755,029,061	.196,498,135,411	.317,093,230,263	.479,265,018,597	.583,172,802,611
78 30	.061,820,353,659	.119,415,045,376	.197,200,191,091	.320,505,852,220	.480,126,213,363	.587,358,949,285
78 45	.062,157,615,240	.121,091,479,458	.197,891,029,339	.323,939,794,856	.480,969,883,128	.591,552,534,881
79 00	.062,490,515,616	.122,784,405,537	.198,570,480,556	.327,395,040,981	.481,795,963,642	.595,753,406,183
79 15	.062,818,932,607	.124,493,897,230	.199,238,374,942	.330,871,571,995	.482,604,391,996	.599,961,409,421
79 30	.063,142,742,977	.126,220,027,535	.199,894,542,510	.334,369,367,884	.483,395,106,624	.604,176,390,281
79 45	.063,461,822,428	.127,962,868,820	.200,538,813,099	.337,888,407,214	.484,168,047,312	.608,398,193,918
80 00	.063,776,045,609	.129,722,492,816	.201,171,016,383	.341,428,667,137	.484,923,155,196	.612,626,664,966
80 15	.064,085,286,107	.131,498,970,594	.201,790,981,887	.344,990,123,378	.485,660,372,773	.616,861,647,554
80 30	.064,389,416,463	.133,292,372,567	.202,398,538,995	.348,572,750,239	.486,379,643,900	.621,102,985,313
80 45	.064,688,308,164	.135,102,768,474	.202,993,516,966	.352,176,520,595	.487,080,913,801	.625,350,521,391
81 00	.064,981,831,648	.136,930,227,369	.203,575,744,944	.355,801,405,886	.487,764,129,074	.629,604,098,464
81 15	.065,269,856,312	.138,774,817,610	.204,145,051,969	.359,447,376,123	.488,429,237,687	.633,863,558,746
81 30	.065,552,250,504	.140,636,606,848	.204,701,266,994	.363,114,399,879	.489,076,188,991	.638,128,744,007
81 45	.065,828,881,536	.142,515,662,022	.205,244,218,892	.366,802,444,292	.489,704,933,717	.642,399,495,577
82 00	.066,099,615,682	.144,412,049,337	.205,773,736,471	.370,511,475,055	.490,315,423,985	.646,675,654,364
82 15	.066,364,318,181	.146,325,834,263	.206,289,648,485	.374,241,456,424	.490,907,613,302	.650,957,060,863
82 30	.066,622,853,240	.148,257,081,520	.206,791,783,651	.377,992,351,207	.491,481,456,573	.655,243,555,172
82 45	.066,875,084,042	.150,205,855,066	.207,279,970,654	.381,764,120,769	.492,036,910,095	.659,534,976,999
83 00	.067,120,872,739	.152,172,218,091	.207,754,038,164	.385,556,725,026	.492,573,931,569	.663,831,165,678
83 15	.067,360,080,465	.154,156,233,003	.208,213,814,848	.389,370,122,442	.493,092,480,100	.668,131,960,178
83 30	.067,592,567,336	.156,157,961,414	.208,659,129,383	.393,204,270,035	.493,592,516,197	.672,437,199,121
83 45	.067,818,192,451	.158,177,464,135	.209,089,810,465	.397,059,123,368	.494,074,001,780	.676,746,720,788
84 00	.068,036,813,902	.160,214,801,165	.209,505,686,826	.400,934,636,549	.494,536,900,183	.681,060,363,133
84 15	.068,248,288,767	.162,270,031,676	.209,906,587,244	.404,830,762,232	.494,981,176,156	.685,377,963,798
84 30	.068,452,473,130	.164,343,214,002	.210,292,340,555	.408,747,451,618	.495,406,795,862	.689,699,360,123
84 45	.068,649,222,068	.166,434,405,635	.210,662,775,670	.412,684,654,445	.495,813,726,892	.694,024,389,160
85 00	.068,838,389,667	.168,543,663,209	.211,017,721,577	.416,642,318,999	.496,201,938,253	.698,352,887,680
85 15	.069,019,829,017	.170,671,042,489	.211,357,007,370	.420,620,392,103	.496,571,400,385	.702,684,692,197
85 30	.069,193,392,228	.172,816,598,361	.211,680,462,245	.424,618,819,123	.496,922,085,149	.707,019,638,967
85 45	.069,358,930,420	.174,980,384,826	.211,987,915,524	.428,637,543,965	.497,253,965,840	.711,357,564,010
86 00	.069,516,293,743	.177,162,454,980	.212,279,196,663	.432,676,509,070	.497,567,017,186	.715,698,303,118
86 15	.069,665,331,366	.179,362,861,014	.212,554,135,262	.436,735,655,426	.497,861,215,344	.720,041,691,868
86 30	.069,805,891,493	.181,581,654,193	.212,812,561,083	.440,814,922,555	.498,136,537,910	.724,387,565,636
86 45	.069,937,821,361	.183,818,884,853	.213,054,304,063	.444,914,248,518	.498,392,963,919	.728,735,759,611
87 00	.070,060,967,256	.186,074,602,390	.213,279,194,320	.449,033,569,915	.498,630,473,842	.733,086,108,800
87 15	.070,175,174,496	.188,348,855,242	.213,487,062,173	.453,172,821,887	.498,849,049,592	.737,438,448,053
87 30	.070,280,287,462	.190,641,690,889	.213,677,738,148	.457,331,938,112	.499,048,674,523	.741,792,612,061
87 45	.070,376,149,585	.192,953,155,835	.213,851,052,997	.461,510,850,807	.499,229,333,434	.746,148,435,381
88 00	.070,462,603,361	.195,283,295,599	.214,006,837,705	.465,709,490,732	.499,391,012,565	.750,505,752,441
88 15	.070,539,490,346	.197,632,154,709	.214,144,923,511	.469,927,787,184	.499,533,699,606	.754,864,397,559
88 30	.070,606,651,176	.199,999,776,684	.214,265,141,908	.474,165,668,003	.499,657,383,689	.759,224,204,947
88 45	.070,663,925,559	.202,386,204,029	.214,367,324,666	.478,423,059,569	.499,762,055,396	.763,585,008,731
89 00	.070,711,152,289	.204,791,478,224	.214,451,303,843	.482,699,886,809	.499,847,706,754	.767,946,642,962
89 15	.070,748,169,249	.207,215,639,711	.214,516,911,795	.486,996,073,185	.499,914,331,244	.772,308,941,626
89 30	.070,774,813,417	.209,658,727,888	.214,563,981,187	.491,311,540,713	.499,961,923,789	.776,671,738,658
89 45	.070,790,920,871	.212,120,781,094	.214,592,345,014	.495,646,209,949	.499,990,480,766	.781,034,867,958
90 00	.070,796,326,795	.214,601,836,603	.214,601,836,603	.500,000,000,000	.500,000,000,000	.785,398,163,397
° ′	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$	$+\cos\phi_0$	$-\sin\phi_0$
ϕ_1	$\int(\phi_1-\sin\phi_1)\cos\phi\,d\phi_1$		$\int \text{vers}\,\phi_1 \cos\phi\,d\phi_1$		$\int \sin\phi_1 \cos\phi\,d\phi_1$	

TABLE 15—INTEGRALS FOR ARCH LOAD CONSTANTS—Continued

ϕ_1	$\int \frac{\Phi_1^2}{2} \sin\phi\, d\phi_1$		$\int \phi_1 \sin\phi\, d\phi_1$		$\int \left(\frac{\Phi_1^2}{2} - \text{VERS } \phi_1\right) \sin\phi\, d\phi_1$	
° ′	+ SIN Φ_0	+ COS Φ_0	+ SIN Φ_0	+ COS Φ_0	+ SIN Φ_0	+ COS Φ_0
0 15	.0^{7}13,845,173	.0^{10}45,308,387	.0^{5}9,519,249	.0^{7}27,690,452	.0^{13}13,179,608	.0^{16}47,922,617
0 30	.0^{6},110,759,489	.0^{9},724,929,588	.0^{4}38,076,452	.0^{6},221,522,352	.0^{12},421,738,272	.0^{14}3,067,021
0 45	.0^{6},373,802,600	.0^{8}3,669,917	.0485,669,979	.0^{6},747,630,820	.0^{11}3,202,459	.0^{13}34,934,789
1 00	.0^{6},886,015,181	.0^{7}11,598,579	.0^{3},152,297,111	.0^{5}1,772,138	.0^{10}13,494,450	.0^{12},196,282,630
1 15	.0^{5}1,730,409,4	.0^{7}28,316,304	.0^{3},237,954,042	.0^{5}3,461,148	.0^{10}41,179,105	.0^{12},748,739,569
1 30	.0^{5}2,989,959,6	.0^{7}58,715,322	.0^{3},342,635,880	.0^{5}5,980,739	.0^{9},102,458,616	.0^{11}2,235,654
1 45	.0^{5}4,747,592,6	.0^{6},108,774,391	.0^{3},466,336,644	.0^{5}9,496,957	.0^{9},221,432,558	.0^{11}5,637,275
2 00	.0^{5}7,086,178	.0^{6},185,558,421	.0^{3},609,049,269	.0^{4}14,175,811	.0^{9},431,672,074	.0^{10}12,560,366
2 15	.0^{4}10,088,520	.0^{6},297,218,021	.0^{3},770,765,601	.0^{4}20,183,265	.0^{9},777,791,161	.0^{10}25,462,225
2 30	.0^{4}13,837,346	.0^{6},452,988,988	.0^{3},951,476,400	.0^{4}27,685,233	.0^{8}1,317,014,595	.0^{10}47,909,069
2 45	.0^{4}18,415,297	.0^{6},663,191,713	.0^{2}1,151,171,342	.0^{4}36,847,572	.0^{8}2,120,742,000	.0^{10}84,868,749
3 00	.0^{4}23,904,922	.0^{6},939,230,532	.0^{2}1,369,839,016	.0^{4}47,836,076	.0^{8}3,276,107,600	.0^{9},143,037,739
3 15	.0^{4}30,388,664	.0^{5}1,293,593	.0^{2}1,607,466,924	.0^{4}60,816,467	.0^{8}4,887,535,143	.0^{9},231,202,361
3 30	.0^{4}37,948,852	.0^{5}1,739,849	.0^{2}1,864,041,486	.0^{4}75,954,396	.0^{8}7,078,287,561	.0^{9},360,634,170
3 45	.0^{4}46,667,692	.0^{5}2,292,650	.0^{2}2,139,548,037	.0^{4}93,415,427	.0^{8}9,992,010,849	.0^{9},545,519,454
4 00	.0^{4}56,627,258	.0^{5}2,967,729	.0^{2}2,433,970,826	.0^{3}1,113,365,038	.0^{7}13,794,272	.0^{9},803,422,780
4 15	.0^{4}67,909,482	.0^{5}3,781,897	.0^{2}2,747,293,019	.0^{3}1,135,968,612	.0^{7}18,674,088	.0^{8}1,155,788
4 30	.0^{4}80,596,143	.0^{5}4,753,044	.0^{2}3,079,496,702	.0^{3}1,161,391,431	.0^{7}24,845,455	.0^{8}1,628,452
4 45	.0^{4}94,768,861	.0^{5}5,900,139	.0^{2}3,430,562,875	.0^{3}1,189,798,668	.0^{7}32,548,856	.0^{8}2,252,245
5 00	.0^{3}1,110,509,083	.0^{5}7,243,225	.0^{2}3,800,471,457	.0^{3}1,221,355,385	.0^{7}42,052,776	.0^{8}3,063,554
5 15	.0^{3}1,127,898,081	.0^{5}8,803,421	.0^{2}4,189,201,287	.0^{3}1,256,226,521	.0^{7}53,655,198	.0^{8}4,104,975
5 30	.0^{3}1,147,016,931	.0^{4}10,602,921	.0^{2}4,596,730,122	.0^{3}1,294,576,892	.0^{7}67,685,095	.0^{8}5,425,971
5 45	.0^{3}1,167,946,517	.0^{4}12,664,988	.0^{2}5,023,034,642	.0^{3}1,336,571,178	.0^{7}84,503,907	.0^{8}7,083,576
6 00	.0^{3}1,190,767,510	.0^{4}15,013,958	.0^{2}5,468,090,444	.0^{3}1,382,373,924	.0^{6}1,104,507,016	.0^{8}9,143,126
6 15	.0^{3}1,215,560,368	.0^{4}17,675,237	.0^{2}5,931,872,050	.0^{3}1,432,149,525	.0^{6}1,128,125,204	.0^{7}11,679,023
6 30	.0^{3}1,242,405,319	.0^{4}20,675,295	.0^{2}6,414,352,904	.0^{3}1,486,062,229	.0^{6}1,155,826,102	.0^{7}14,775,536
6 45	.0^{3}1,271,382,355	.0^{4}24,041,672	.0^{2}6,915,505,369	.0^{3}1,544,276,125	.0^{6}1,188,115,628	.0^{7}18,527,635
7 00	.0^{3}1,302,571,225	.0^{4}27,802,970	.0^{2}7,435,300,739	.0^{3}1,606,955,136	.0^{6}1,225,539,414	.0^{7}23,041,850
7 15	.0^{3}1,336,051,422	.0^{4}31,988,852	.0^{2}7,973,709,229	.0^{3}1,674,263,018	.0^{6}1,268,684,218	.0^{7}28,437,173
7 30	.0^{3}1,371,902,175	.0^{4}36,630,045	.0^{2}8,530,699,982	.0^{3}1,746,363,347	.0^{6}1,318,179,323	.0^{7}34,845,985
7 45	.0^{3}1,410,202,438	.0^{4}41,758,330	.0^{2}9,106,241,065	.0^{3}1,823,419,521	.0^{6}1,374,697,930	.0^{7}42,415,020
8 00	.0^{3}1,451,030,885	.0^{4}47,406,549	.0^{2}9,700,299,478	.0^{3}1,905,594,744	.0^{6}1,438,958,526	.0^{7}51,306,359
8 15	.0^{3}1,494,465,898	.0^{4}53,608,596	.0^{1}0,312,841,145	.0^{3}1,993,052,028	.0^{6}1,511,726,245	.0^{7}61,698,456,5
8 30	.0^{3}1,540,585,553	.0^{4}60,399,416	.0^{1}0,943,830,924	.0^{2}1,085,954,184	.0^{6}1,593,814,215	.0^{7}73,787,194
8 45	.0^{3}1,589,467,623	.0^{4}67,815,006	.0^{1}1,593,232,603	.0^{2}1,184,463,811	.0^{6}1,686,084,885	.0^{7}87,786,982
9 00	.0^{3}1,641,189,558	.0^{4}75,892,411	.0^{1}2,261,008,902	.0^{2}1,288,743,298	.0^{6}1,789,451,340	.0^{6}1,103,931,866
9 15	.0^{3}1,695,828,475	.0^{4}84,669,717	.0^{1}2,947,121,475	.0^{2}1,398,954,814	.0^{6}1,904,878,599	.0^{6}1,122,476,692
9 30	.0^{3}1,753,461,161	.0^{4}94,186,061	.0^{1}3,651,530,913	.0^{2}1,515,260,299	.0^{5}1,033,385	.0^{6}1,143,698,283
9 45	.0^{3}1,814,164,049	.0^{3}1,104,481,612	.0^{1}4,374,196,739	.0^{2}1,637,821,462	.0^{5}1,176,043	.0^{6}1,167,896,657
10 00	.0^{3}1,878,013,217	.0^{3}1,115,597,581	.0^{1}5,115,077,416	.0^{2}1,766,799,775	.0^{5}1,333,981	.0^{6}1,195,396 273
10 15	.0^{3}1,945,084,379	.0^{3}1,127,576,213	.0^{1}5,874,130,346	.0^{2}1,902,356,462	.0^{5}1,508,384	.0^{6}1,226,547,305
10 30	.0^{2}1,015,452,874	.0^{3}1,140,460,787	.0^{1}6,651,311,870	.0^{2}2,044,652,497	.0^{5}1,700,498	.0^{6}1,261,726,951
10 45	.0^{2}1,089,193,655	.0^{3}1,154,295,611	.0^{1}7,446,577,272	.0^{2}2,193,848,598	.0^{5}1,911,623	.0^{6}1,301,340,768
11 00	.0^{2}1,166,381,286	.0^{3}1,169,126,018	.0^{1}8,259,880,777	.0^{2}2,350,105,217	.0^{5}2,143,124	.0^{6}1,345,824,041
11 15	.0^{2}1,247,089,925	.0^{3}1,184,998,367	.0^{1}9,091,175,555	.0^{2}2,513,582,537	.0^{5}2,396,425	.0^{6}1,395,643,177
11 30	.0^{2}1,331,393,320	.0^{3}1,201,960,039	.0^{1}9,940,413,723	.0^{2}2,684,440,469	.0^{5}2,673,015	.0^{6}1,451,297,131
11 45	.0^{2}1,419,364,801	.0^{3}1,220,059,431	.0^{2}0,807,546,345	.0^{2}2,862,838,636	.0^{5}2,974,447	.0^{6}1,513,318,864
12 00	.0^{2}1,511,077,263	.0^{3}1,239,345,954	.0^{2}1,692,523,432	.0^{3}1,048,936,379	.0^{5}3,302,334	.0^{6}1,582,276,823
12 15	.0^{2}1,606,603,171	.0^{3}1,259,870,034	.0^{2}2,595,293,948	.0^{3}1,242,892,738	.0^{5}3,658,363	.0^{6}1,658,776,457
12 30	.0^{2}1,706,014,536	.0^{3}1,281,683,101	.0^{2}3,515,805,807	.0^{3}1,444,866,459	.0^{5}4,044,283	.0^{6}1,743,461,760
12 45	.0^{2}1,809,382,916	.0^{3}1,304,837,596	.0^{2}4,454,005,880	.0^{3}1,655,015,979	.0^{5}4,461,911	.0^{6}1,837,016,833
13 00	.0^{2}1,916,779,404	.0^{3}1,329,386,957	.0^{2}5,409,839,989	.0^{3}1,873,499,420	.0^{5}4,913,137	.0^{6}1,940,167,491
13 15	.0^{2}2,028,274,617	.0^{3}1,355,385,622	.0^{2}6,383,252,917	.0^{4}1,100,474,590	.0^{5}5,399,917	.0^{5}1,053,683
13 30	.0^{2}2,143,938,692	.0^{3}1,382,889,027	.0^{2}7,374,188,406	.0^{4}1,336,098,969	.0^{5}5,924,280	.0^{5}1,178,377
13 45	.0^{2}2,263,841,271	.0^{3}1,411,953,592	.0^{2}8,382,589,154	.0^{4}1,580,529,707	.0^{5}6,488,328	.0^{5}1,315,111
14 00	.0^{2}2,388,051,496	.0^{3}1,442,636,732	.0^{2}9,408,396,828	.0^{4}1,833,923,619	.0^{5}7,094,232	.0^{5}1,464,793
14 15	.0^{2}2,516,638,005	.0^{3}1,474,996,844	.0^{3}0,451,552,056	.0^{4}2,096,437,172	.0^{5}7,744,245	.0^{5}1,628,386
14 30	.0^{2}2,649,668,909	.0^{3}1,509,093,301	.0^{3}1,511,994,430	.0^{4}2,368,226,489	.0^{5}8,440,686	.0^{5}1,806,894
14 45	.0^{2}2,787,211,796	.0^{3}1,544,986,461	.0^{3}2,589,662,517	.0^{4}2,649,447,338	.0^{5}9,185,951	.0^{5}2,001,390
15 00	.0^{2}2,929,333,721	.0^{3}1,582,737,646	.0^{3}3,684,493,848	.0^{4}2,940,255,121	.0^{5}9,982,518	.0^{5}2,212,989
15 15	.0^{2}3,076,101,191	.0^{3}1,622,409,150	.0^{3}4,796,424,928	.0^{4}3,240,804,878	.0^{4}10,832,938	.0^{5}2,442,869
15 30	.0^{2}3,227,580,161	.0^{3}1,664,064,236	.0^{3}5,925,391,238	.0^{4}3,551,251,273	.0^{4}11,739,840	.0^{5}2,692,269
15 45	.0^{2}3,383,836,022	.0^{3}1,707,767,119	.0^{3}7,071,327,232	.0^{4}3,871,748,593	.0^{4}12,705,930	.0^{5}2,962,484
16 00	.0^{2}3,544,933,598	.0^{3}1,753,582,978	.0^{3}8,234,166,346	.0^{4}4,202,450,738	.0^{4}13,733,999	.0^{5}3,254,877
16 15	.0^{2}3,710,937,132	.0^{3}1,801,577,943	.0^{3}9,413,840,995	.0^{4}4,543,511,218	.0^{4}14,826,912	.0^{5}3,570,876
16 30	.0^{2}3,881,910,278	.0^{3}1,851,819,085	.0^{4}0,610,282,574	.0^{4}4,895,083,146	.0^{4}15,987,617	.0^{5}3,911,967
16 45	.0^{2}4,057,916,098	.0^{3}1,904,374,429	.0^{4}1,823,421,468	.0^{4}5,257,319,231	.0^{4}17,219,146	.0^{5}4,279,717
17 00	.0^{2}4,239,017,044	.0^{3}1,959,312,930	.0^{4}3,053,187,043	.0^{4}5,630,371,774	.0^{4}18,524,608	.0^{5}4,675,754
17 15	.0^{2}4,425,274,957	.0^{2}1,016,704,485	.0^{4}4,299,507,661	.0^{4}6,014,392,662	.0^{4}19,907,197	.0^{5}5,101,787
17 30	.0^{2}4,616,751,058	.0^{2}1,076,619,917	.0^{4}5,562,310,669	.0^{4}6,409,533,358	.0^{4}21,370,191	.0^{5}5,559,593
17 45	.0^{2}4,813,505,934	.0^{2}1,139,130,977	.0^{4}6,841,522,412	.0^{4}6,815,944,903	.0^{4}22,916,948	.0^{5}6,051,031
18 00	.0^{2}5,015,599,537	.0^{2}1,204,310,337	.0^{4}8,137,068,231	.0^{4}7,233,777,901	.0^{4}24,550,915	.0^{5}6,578,038
18 15	.0^{2}5,223,091,170	.0^{2}1,272,231,582	.0^{4}9,448,872,464	.0^{4}7,663,182,520	.0^{4}26,275,618	.0^{5}7,142,630
18 30	.0^{2}5,436,039,480	.0^{2}1,342,969,212	.0^{5}0,776,858,450	.0^{4}8,104,308,484	.0^{4}28,094,672	.0^{5}7,746,906
18 45	.0^{2}5,654,502,452	.0^{2}1,416,598,634	.0^{5}2,120,948,534	.0^{4}8,557,305,065	.0^{4}30,011,776	.0^{5}8,393,056
19 00	.0^{2}5,878,537,396	.0^{2}1,493,196,152	.0^{5}3,481,064,062	.0^{4}9,022,321,081	.0^{4}32,030,710	.0^{5}9,083,349
19 15	.0^{2}6,108,200,947	.0^{2}1,572,838,978	.0^{5}4,857,125,396	.0^{4}9,499,504,886	.0^{4}34,155,348	.0^{5}9,820,158
19 30	.0^{2}6,343,549,048	.0^{2}1,655,605,201	.0^{5}6,249,051,900	.0^{4}9,989,004,367	.0^{4}36,389,646	.0^{6}1,060,592,9
19 45	.0^{2}6,584,636,944	.0^{2}1,741,573,806	.0^{5}7,656,761,958	.0^{4}1,049,096,939	.0^{4}38,737,645	.0^{6}1,144,321,5
20 00	.0^{2}6,831,519,177	.0^{2}1,830,824,663	.0^{5}9,080,172,970	.0^{4}1,100,538,538	.0^{4}41,203,472	.0^{6}1,233,466,9
20 15	.0^{2}7,084,249,579	.0^{2}1,923,438,506	.0^{6}0,519,201 349	.0^{4}1,153,868,610	.0^{4}43,791,349	.0^{6}1,328,302,8
20 30	.0^{2}7,342,881,254	.0^{2}2,019,496,954	.0^{6}1,973,762,537	.0^{4}1,209,117,100	.0^{4}46,505,572	.0^{6}1,429,146,0
20 45	.0^{2}7,607,466,583	.0^{2}2,119,082,483	.0^{6}3,443,770,996	.0^{4}1,266,379,521	.0^{4}49,350,534	.0^{6}1,536,197,2
21 00	.0^{2}7,878,057,208	.0^{2}2,222,278,433	.0^{6}4,929,140,214	.0^{4}1,325,851,779	.0^{4}52,330,712	.0^{6}1,649,856,1
21 15	.0^{2}8,154,704,025	.0^{2}2,329,168,999	.0^{6}6,429,782,712	.0^{4}1,387,772,661	.0^{4}55,450,670	.0^{6}1,770,480,8
21 30	.0^{2}8,437,457,177	.0^{2}2,439,839,222	.0^{6}7,945,610,039	.0^{4}1,452,365,952	.0^{4}58,715,058	.0^{6}1,898,399,7
21 45	.0^{2}8,726,366,048	.0^{2}2,554,374,994	.0^{6}9,476,532,785	.0^{4}1,519,872,867	.0^{4}62,128,616	.0^{6}2,033,911,3
22 00	.0^{2}9,021,479,251	.0^{2}2,672,863,038	.0^{7}1,022,460,572	.0^{4}1,590,550,678	.0^{4}65,696,169	.0^{6}2,177,385,2
22 15	.0^{2}9,322,844,624	.0^{2}2,795,390,915	.0^{7}2,583,302,068	.0^{4}1,664,638,610	.0^{4}69,422,631	.0^{6}2,329,164,4
22 30	.0^{2}9,630,509,218	.0^{2}2,922,047,012	.0^{7}4,158,964,982	.0^{4}1,742,379,788	.0^{4}73,312,999	.0^{6}2,489,615,2
° ′	+ COS Φ_0	− SIN Φ_0	+ COS Φ_0	− SIN Φ_0	+ COS Φ_0	− SIN Φ_0
ϕ_1	$\int \frac{\Phi_1^2}{2} \cos\phi\, d\phi_1$		$\int \phi_1 \cos\phi\, d\phi_1$		$\int \left(\frac{\Phi_1^2}{2} - \text{VERS } \phi_1\right) \cos\phi\, d\phi_1$	

TABLE 15—Continued

ϕ_1	$\int \frac{\phi_1^2}{2}$ SIN $\phi\, d\phi_1$		$\int \phi_1$ SIN $\phi\, d\phi_1$		$\int \left(\frac{\phi_1^2}{2}-\text{VERS } \phi_1\right)$ SIN $\phi\, d\phi_1$	
° ′	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0
22 45	.009,944,519,294	.003,052,920,535	.075,749,356,071	.020,539,626,090	.$0^4$77,372,360	.$0^4$26,576,130
23 00	.010,264,920,315	.003,188,101,508	.077,354,381,142	.021,216,797,602	.$0^4$81,605,890	.$0^4$28,362,345
23 15	.010,591,756,930	.003,327,680,763	.078,973,945,054	.021,908,443,069	.$0^4$86,018,843	.$0^4$30,246,988
23 30	.010,925,072,979	.003,471,749,935	.080,607,951,722	.022,614,702,115	.$0^4$90,616,568	.$0^4$32,234,304
23 45	.011,264,911,477	.003,620,401,460	.082,256,304,121	.023,335,713,800	.$0^4$95,404,495	.$0^4$34,328,675
24 00	.011,611,314,608	.003,773,728,560	.083,918,904,288	.024,071,616,615	.0^3,100,388,141	.$0^4$36,534,613
24 15	.011,964,323,717	.003,931,825,244	.085,595,653,321	.024,822,548,476	.0^3,105,573,106	.$0^4$38,856,767
24 30	.012,323,979,307	.004,094,786,306	.087,286,451,395	.025,588,646,716	.0^3,110,965,076	.$0^4$41,299,935
24 45	.012,690,321,024	.004,262,707,304	.088,991,197,748	.026,370,048,083	.0^3,116,569,821	.$0^4$43,869,046
25 00	.013,063,387,656	.004,435,684,566	.090,709,790,698	.027,166,888,734	.0^3,122,393,195	.$0^4$46,569,181
25 15	.013,443,217,122	.004,613,815,183	.092,442,127,637	.027,979,304,226	.0^3,128,441,132	.$0^4$49,405,567
25 30	.013,829,846,471	.004,797,196,996	.094,188,105,044	.028,807,429,514	.0^3,134,719,656	.$0^4$52,383,583
25 45	.014,223,311,862	.004,985,928,596	.095,947,618,478	.029,651,398,945	.0^3,141,234,865	.$0^4$55,508,759
26 00	.014,623,648,571	.005,180,109,309	.097,720,562,586	.030,511,346,253	.0^3,147,992,943	.$0^4$58,786,777
26 15	.015,030,890,972	.005,379,839,205	.099,506,831,110	.031,387,404,552	.0^3,155,000,150	.$0^4$62,223,486
26 30	.015,445,072,538	.005,585,219,074	.101,306,316,882	.032,279,706,332	.0^3,162,262,829	.$0^4$65,824,888
26 45	.015,866,225,834	.005,796,350,425	.103,118,911,835	.033,188,383,451	.0^3,169,787,405	.$0^4$69,597,148
27 00	.016,294,382,499	.006,013,335,489	.104,944,507,005	.034,113,567,137	.0^3,177,580,372	.$0^4$73,546,604
27 15	.016,729,573,256	.006,236,277,200	.106,782,992,531	.035,055,387,970	.0^3,185,648,312	.$0^4$77,679,758
27 30	.017,171,827,890	.006,465,279,188	.108,634,257,658	.036,013,975,890	.0^3,193,997,876	.$0^4$82,003,278
27 45	.017,621,175,248	.006,700,445,786	.110,498,190,750	.036,989,460,185	.0^3,202,635,792	.$0^4$86,524,018
28 00	.018,077,643,235	.006,941,882,005	.112,374,679,282	.037,981,969,484	.0^3,211,568,867	.$0^4$91,248,996
28 15	.018,541,258,798	.007,189,693,539	.114,263,609,848	.038,991,631,756	.0^3,220,803,974	.$0^4$96,185,416
28 30	.019,012,047,929	.007,443,986,755	.116,164,868,168	.040,018,574,304	.0^3,230,348,065	.0^3,101,340,663
28 45	.019,490,035,651	.007,704,868,686	.118,078,339,087	.041,062,923,756	.0^3,240,208,159	.0^3,106,722,307
29 00	.019,975,246,016	.007,972,447,017	.120,003,906,579	.042,124,806,067	.0^3,250,391,348	.0^3,112,338,098
29 15	.020,467,702,095	.008,246,830,094	.121,941,453,758	.043,204,346,505	.0^3,260,904,791	.0^3,118,195,988
29 30	.020,967,425,976	.008,528,126,896	.123,890,862,866	.044,301,669,653	.0^3,271,755,717	.0^3,124,304,108
29 45	.021,474,438,749	.008,816,447,048	.125,852,015,297	.045,416,899,402	.0^3,282,951,418	.0^3,130,670,797
30 00	.021,988,760,510	.009,111,900,796	.127,824,791,583	.046,550,158,942	.0^3,294,499,255	.0^3,137,304,580
30 15	.022,510,410,347	.009,414,599,013	.129,809,071,410	.047,701,570,763	.0^3,306,406,650	.0^3,144,214,191
30 30	.023,039,406,338	.009,724,653,183	.131,804,733,614	.048,871,256,644	.0^3,318,681,091	.0^3,151,408,564
30 45	.023,575,765,541	.010,042,175,396	.133,811,656,189	.050,059,337,654	.0^3,331,330,123	.0^3,158,896,832
31 00	.024,119,503,989	.010,367,278,343	.135,829,716,292	.051,265,934,142	.0^3,344,361,353	.0^3,166,688,348
31 15	.024,670,636,684	.010,700,075,306	.137,858,790,242	.052,491,165,735	.0^3,357,782,442	.0^3,174,792,670
31 30	.025,229,177,595	.011,040,680,148	.139,898,753,528	.053,735,151,330	.0^3,371,601,115	.0^3,183,219,567
31 45	.025,795,139,642	.011,389,207,314	.141,949,480,814	.054,998,009,093	.0^3,385,825,145	.0^3,191,979,033
32 00	.026,368,534,700	.011,745,771,807	.144,010,845,935	.056,279,856,451	.0^3,400,462,361	.0^3,201,081,266
32 15	.026,949,373,586	.012,110,489,203	.146,082,721,917	.057,580,810,087	.0^3,415,520,642	.0^3,210,536,705
32 30	.027,537,666,058	.012,483,475,618	.148,164,980,961	.058,900,985,938	.0^3,431,007,919	.0^3,220,355,996
32 45	.028,133,420,805	.012,864,847,720	.150,257,494,462	.060,240,499,186	.0^3,446,932,170	.0^3,230,550,020
33 00	.028,736,645,444	.013,254,722,712	.152,360,133,010	.061,599,464,258	.0^3,463,301,419	.0^3,241,129,888
33 15	.029,347,346,512	.013,653,218,325	.154,472,766,388	.062,977,994,816	.0^3,480,123,732	.0^3,252,106,942
33 30	.029,965,529,463	.014,060,452,810	.156,595,263,584	.064,376,203,756	.0^3,497,407,223	.0^3,263,492,755
33 45	.030,591,198,658	.014,476,544,931	.158,727,492,792	.065,794,203,202	.0^3,515,160,040	.0^3,275,299,143
34 00	.031,224,357,363	.014,901,613,956	.160,869,321,413	.067,232,104,501	.0^3,533,390,373	.0^3,287,538,157
34 15	.031,865,007,743	.015,335,779,648	.163,020,616,066	.068,690,018,217	.0^3,552,106,447	.0^3,300,222,094
34 30	.032,513,150,856	.015,779,162,261	.165,181,242,589	.070,168,054,129	.0^3,571,316,524	.0^3,313,363,497
34 45	.033,168,786,645	.016,231,882,524	.167,351,066,039	.071,666,321,224	.0^3,591,028,895	.0^3,326,975,151
35 00	.033,831,913,935	.016,694,061,642	.169,529,950,705	.073,184,927,696	.0^3,611,251,879	.0^3,341,070,100
35 15	.034,502,530,431	.017,165,821,277	.171,717,760,106	.074,723,980,934	.0^3,631,993,831	.0^3,355,661,630
35 30	.035,180,632,703	.017,647,283,550	.173,914,356,995	.076,283,587,526	.0^3,653,263,121	.0^3,370,763,292
35 45	.035,866,216,193	.018,138,571,026	.176,119,603,371	.077,863,853,247	.0^3,675,068,151	.0^3,386,388,889
36 00	.036,559,275,198	.018,639,806,708	.178,333,360,473	.079,464,883,060	.0^3,697,417,339	.0^3,402,552,489
36 15	.037,259,802,872	.019,151,114,027	.180,555,488,792	.081,086,781,111	.0^3,720,319,119	.0^3,419,268,418
36 30	.037,967,791,224	.019,672,616,836	.182,785,848,073	.082,729,650,714	.0^3,743,781,953	.0^3,436,551,272
36 45	.038,683,231,099	.020,204,439,397	.185,024,297,321	.084,393,594,367	.0^3,767,814,299	.0^3,454,415,912
37 00	.039,406,112,190	.020,746,706,376	.187,270,694,800	.086,078,713,726	.0^3,792,424,641	.0^3,472,877,469
37 15	.040,136,423,020	.021,299,542,836	.189,524,898,047	.087,785,109,616	.0^3,817,621,463	.0^3,491,951,351
37 30	.040,874,150,947	.021,863,074,219	.191,786,763,867	.089,512,882,017	.0^3,843,413,259	.0^3,511,653,235
37 45	.041,619,282,149	.022,437,426,352	.194,056,148,345	.091,262,130,065	.0^3,869,808,523	.0^3,531,999,082
38 00	.042,371,801,629	.023,022,725,423	.196,332,906,848	.093,032,952,046	.0^3,896,815,751	.0^3,553,005,130
38 15	.043,131,693,203	.023,619,097,982	.198,616,894,026	.094,825,445,391	.0^3,924,443,437	.0^3,574,687,899
38 30	.043,898,939,501	.024,226,670,927	.200,907,963,823	.096,639,706,672	.0^3,952,700,068	.0^3,597,064,193
38 45	.044,673,521,960	.024,845,571,501	.203,205,969,479	.098,475,831,595	.0^3,981,594,130	.0^3,620,151,109
39 00	.045,455,420,814	.025,475,927,277	.205,510,763,534	.100,333,915,006	.001,011,134,087	.0^3,643,966,030
39 15	.046,244,615,101	.026,117,866,149	.207,822,197,832	.102,214,050,871	.001,041,328,397	.0^3,668,526,627
39 30	.047,041,082,648	.026,771,516,327	.210,140,123,529	.104,116,332,285	.001,072,185,501	.0^3,693,850,871
39 45	.047,844,800,075	.027,437,006,326	.212,464,391,095	.106,040,851,460	.001,103,713,819	.0^3,719,957,026
40 00	.048,655,742,782	.028,114,464,958	.214,794,850,322	.107,987,699,725	.001,135,921,747	.0^3,746,863,661
40 15	.049,473,884,953	.028,804,021,318	.217,131,350,323	.109,956,967,520	.001,168,817,658	.0^3,774,589,636
40 30	.050,299,199,545	.029,505,804,780	.219,473,739,543	.111,948,744,393	.001,202,409,893	.0^3,803,154,120
40 45	.051,131,658,292	.030,219,944,988	.221,821,865,762	.113,963,118,993	.001,236,706,764	.0^3,832,576,590
41 00	.051,971,231,691	.030,946,571,844	.224,175,576,100	.116,000,179,070	.001,271,716,545	.0^3,862,876,827
41 15	.052,817,889,007	.031,685,815,496	.226,534,717,017	.118,060,011,467	.001,307,447,474	.0^3,894,074,920
41 30	.053,671,598,263	.032,437,806,338	.228,899,134,330	.120,142,702,120	.001,343,907,746	.0^3,926,191,276
41 45	.054,532,326,239	.033,202,674,989	.231,268,673,203	.122,248,336,050	.001,381,105,511	.0^3,959,246,609
42 00	.055,400,038,466	.033,980,552,292	.233,643,178,163	.124,376,997,363	.001,419,048,867	.0^3,993,261,952
42 15	.056,274,699,230	.034,771,569,305	.236,022,493,106	.126,528,769,240	.001,457,745,870	.001,028,258,662
42 30	.057,156,271,555	.035,575,857,283	.238,406,461,288	.128,703,733,940	.001,497,204,511	.001,064,258,406
42 45	.058,044,717,208	.036,393,547,675	.240,794,925,347	.130,901,972,794	.001,537,432,722	.001,101,283,179
43 00	.058,939,996,697	.037,224,772,115	.243,187,727,299	.133,123,566,197	.001,578,438,378	.001,139,355,298
43 15	.059,842,069,263	.038,069,662,411	.245,584,708,545	.135,368,593,610	.001,620,229,286	.001,178,497,409
43 30	.060,750,892,877	.038,928,350,533	.247,985,709,877	.137,637,133,552	.001,662,813,180	.001,218,732,484
43 45	.061,666,424,241	.039,800,968,606	.250,390,571,481	.139,929,263,596	.001,706,197,728	.001,260,083,824
44 00	.062,588,618,777	.040,687,648,902	.252,799,132,945	.142,245,060,371	.001,750,390,511	.001,302,575,065
44 15	.063,517,430,631	.041,588,523,824	.255,211,233,261	.144,584,599,552	.001,795,399,037	.001,346,230,172
44 30	.064,452,812,669	.042,503,725,901	.257,626,710,833	.146,947,955,857	.001,841,230,727	.001,391,073,446
44 45	.065,394,716,469	.043,433,387,783	.260,045,403,484	.149,335,203,046	.001,887,892,912	.001,437,129,531
45 00	.066,343,092,322	.044,377,642,218	.262,467,148,457	.151,746,413,917	.001,935,392,834	.001,484,423,405
° ′	+COS ϕ_0	−SIN ϕ_0	+COS ϕ_0	−SIN ϕ_0	+COS ϕ_0	−SIN ϕ_0
ϕ_1	$\int \frac{\phi_1^2}{2}$ COS $\phi\, d\phi_1$		$\int \phi_1$ COS $\phi\, d\phi_1$		$\int \left(\frac{\phi_1^2}{2}-\text{VERS } \phi_1\right)$ COS $\phi\, d\phi_1$	

TABLE 15—Continued

ϕ_1	$\int \frac{\Phi_1^2}{2}$ SIN $\phi\, d\phi_1$		$\int \Phi_1$ SIN $\phi\, d\phi_1$		$\int \left(\frac{\Phi_1^2}{2} - \text{VERS } \phi_1\right)$ SIN $\phi\, d\phi_1$	
° ′	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0
45 15	.067,297,889,228	.045,336,622,054	.264,891,782,418	.154,181,660,300	.001,983,737,634	.001,532,980,384
45 30	.068,259,054,893	.046,310,460,225	.267,319,141,472	.156,641,013,055	.002,032,934,357	.001,582,826,134
45 45	.069,226,535,730	.047,299,289,739	.269,749,061,159	.159,124,542,069	.002,082,989,943	.001,633,986,658
46 00	.070,200,276,846	.048,303,243,672	.272,181,376,460	.161,632,316,251	.002,133,911,220	.001,686,488,307
46 15	.071,180,222,051	.049,322,455,156	.274,615,921,808	.164,164,403,529	.002,185,704,910	.001,740,357,780
46 30	.072,166,313,852	.050,357,057,368	.277,052,531,088	.166,720,870,845	.002,238,377,617	.001,795,622,123
46 45	.073,158,493,441	.051,407,183,522	.279,491,037,644	.169,301,784,159	.002,291,935,818	.001,852,308,731
47 00	.074,156,700,707	.052,472,966,859	.281,931,274,287	.171,907,208,434	.002,346,385,871	.001,910,445,357
47 15	.075,160,874,227	.053,554,540,636	.284,373,073,298	.174,537,207,639	.002,401,734,008	.001,970,060,101
47 30	.076,170,951,261	.054,652,038,112	.286,816,266,430	.177,191,844,746	.002,457,986,323	.002,031,181,415
47 45	.077,186,867,751	.055,765 592 549	.289,260,684,922	.179,871,181,728	.002,515,148,769	.002,093,838,114
48 00	.078,208,558,324	.056,895,337,190	.291,706,159,498	.182,575,279,549	.002,573,227,167	.002,158,059,366
48 15	.079,235,956,282	.058,041,405,254	.294,152,520,375	.185,304,198,169	.002,632,227,183	.002,223,874,697
48 30	.080,268,993,609	.059,203,929,928	.296,599,597,267	.188,057,996,531	.002,692,154,339	.002,291,313,995
48 45	.081,307,600,959	.060,383,044,351	.299,047,219,393	.190,836,732,571	.002,753,013,997	.002,360,407,506
49 00	.082,351,707,662	.061,578,881,610	.301,495,215,481	.193,640,463,200	.002,814,811,363	.002,431,185,841
49 15	.083,401,241,715	.062,791,574,724	.303,943,413,772	.196,469,244,314	.002,877,551,475	.002,503,679,969
49 30	.084,456,129,789	.064,021,256,641	.306,391,642,030	.199,323,130,782	.002,941,239,206	.002,577,921,231
49 45	.085,516,297,222	.065,268,060,220	.308,839,727,546	.202,202,176,445	.003,005,879,257	.002,653,941,329
50 00	.086,581,668,015	.066,532,118,222	.311,287,497,139	.205,106,434,116	.003,071,476,148	.002,731,772,326
50 15	.087,652,164,834	.067,813,563,305	.313,734,777,167	.208,035,955,574	.003,138,034,216	.002,811,446,659
50 30	.088,727,709,009	.069,112,528,011	.316,181,393,535	.210,990,791,562	.003,205,557,612	.002,892,997,133
50 45	.089,808,220,533	.070,429,144,753	.318,627,171,693	.213,970,991,781	.003,274,050,300	.002,976,456,920
51 00	.090,893,618,054	.071,763,545,806	.321,071,936,646	.216,976,604,895	.003,343,516,039	.003,061,859,560
51 15	.091,983,818,881	.073,115,863,299	.323,515,512,962	.220,007,678,518	.003,413,958,391	.003,149,238,968
51 30	.093,078,738,985	.074,486,229,203	.325,957,724,775	.223,064,259,216	.003,485,380,718	.003,238,629,427
51 45	.094,178,292,985	.075,874,775,317	.328,398,395,789	.226,146,392,509	.003,557,786,157	.003,330,065,591
52 00	.095,282,394,161	.077,281,633,264	.330,837,349,287	.229,254,122,857	.003,631,177,642	.003,423,582,490
52 15	.096,390,954,445	.078,706,934,477	.333,274,408,137	.232,387,493,667	.003,705,557,879	.003,519,215,525
52 30	.097,503,884,424	.080,150,810,188	.335,709,394,797	.235,546,547,285	.003,780,929,354	.003,617,000,472
52 45	.098,621,093,333	.081,613,391,419	.338,142,131,319	.238,731,324,998	.003,857,294,314	.003,716,973,482
53 00	.099,742,489,063	.083,094,808,970	.340,572,439,356	.241,941,867,023	.003,934,654,779	.003,819,171,076
53 15	.100,867,978,156	.084,595,193,410	.343,000,140,172	.245,178,212,513	.004,013,012,526	.003,923,630,157
53 30	.101,997,465,797	.086,114,675,066	.345,425,054,640	.248,440,399,551	.004,092,369,079	.004,030,387,998
53 45	.103,130,855,831	.087,653,384,013	.347,847,003,257	.251,728,465,144	.004,172,725,725	.004,139,482,253
54 00	.104,268,050,743	.089,211,450,058	.350,265,806,139	.255,042,445,228	.004,254,083,480	.004,250,950,944
54 15	.105,408,951,671	.090,789,002,740	.352,681,283,040	.258,382,374,657	.004,336,443,111	.004,364,832,478
54 30	.106,553,458,402	.092,386,171,311	.355,093,253,347	.261,748,287,205	.004,419,805,114	.004,481,165,636
54 45	.107,701,469,365	.094,003,084,727	.357,501,536,091	.265,140,215,568	.004,504,169,710	.004,599,989,573
55 00	.108,852,881,645	.095,639,871,638	.359,905,949,952	.268,558,191,349	.004,589,536,851	.004,721,343,820
55 15	.110,007,590,967	.097,296,660,379	.362,306,313,266	.272,002,245,068	.004,675,906,200	.004,845,268,290
55 30	.111,165,491,707	.098,973,578,959	.364,702,444,032	.275,472,406,153	.004,763,277,138	.004,971,803,270
55 45	.112,326,476,888	.100,670,755,044	.367,094,159,912	.278,968,702,941	.004,851,648,750	.005,100,989,420
56 00	.113,490,438,182	.102,388,315,957	.369,481,278,245	.282,491,162,671	.004,941,019,827	.005,232,867,782
56 15	.114,657,265,903	.104,126,388,661	.371,863,616,050	.286,039,811,488	.005,031,388,852	.005,367,479,773
56 30	.115,826,849,018	.105,885,099,744	.374,240,990,026	.289,614,674,434	.005,122,754,003	.005,504,867,178
56 45	.116,999,075,141	.107,664,575,421	.376,613,216,572	.293,215,775,452	.005,215,113,143	.005,645,072,173
57 00	.118,173,830,534	.109,464,941,508	.378,980,111,777	.296,843,137,379	.005,308,463,818	.005,788,137,292
57 15	.119,351,000,106	.111,286,323,429	.381,341,491,442	.300,496,781,948	.005,402,803,245	.005,934,105,461
57 30	.120,530,467,418	.113,128,846,185	.383,697,171,071	.304,176,729,781	.005,498,128,313	.006,083,019,967
57 45	.121,712,114,678	.114,992,634,358	.386,046,965,888	.307,883,000,392	.005,594,435,575	.006,234,924,476
58 00	.122,895,822,749	.116,877,812,097	.388,390,690,838	.311,615,612,180	.005,691,721,246	.006,389,863,027
58 15	.124,081,471,138	.118,784,503,107	.390,728,160,599	.315,374,582,433	.005,789,981,186	.006,547,880,037
58 30	.125,268,938,011	.120,712,830,630	.393,059,189,575	.319,159,927,317	.005,889,210,912	.006,709,020,281
58 45	.126,458,100,186	.122,662,917,449	.395,383,591,921	.322,971,661,884	.005,989,405,581	.006,873,328,919
59 00	.127,648,833,131	.124,634,885,869	.397,701,181,533	.326,809,800,064	.006,090,559,982	.007,040,851,475
59 15	.128,841,010,971	.126,628,857,704	.400,011,772,063	.330,674,354,664	.006,192,668,538	.007,211,633,846
59 30	.130,034,506,488	.128,644,954,271	.402,315,176,924	.334,565,337,368	.006,295,725,299	.007,385,722,294
59 45	.131,229,191,120	.130,683,296,376	.404,611,209,291	.338,482,758,731	.006,399,723,934	.007,563,163,448
60 00	.132,424,934,960	.132,744,004,308	.406,899,682,116	.342,426,628,186	.006,504,657,720	.007,744,004,308
60 15	.133,621,606,768	.134,827,197,823	.409,180,408,128	.346,396,954,030	.006,610,519,552	.007,928,292,238
60 30	.134,819,073,959	.136,932,996,134	.411,453,199,840	.350,393,743,430	.006,717,301,922	.008,116,074,965
60 45	.136,017,202,614	.139,061,517,903	.413,717,869,558	.354,417,002,420	.006,824,996,923	.008,307,400,579
61 00	.137,215,857,476	.141,212,881,225	.415,974,229,382	.358,446,735,902	.006,933,596,234	.008,502,317,529
61 15	.138,414,901,952	.143,387,203,630	.418,222,091,224	.362,542,947,640	.007,043,091,120	.008,700,874,636
61 30	.139,614,198,124	.145,584,602,052	.420,461,266,797	.366,645,640,254	.007,153,472,437	.008,903,121,066
61 45	.140,813,606,732	.147,805,192,836	.422,691,567,636	.370,774,815,234	.007,264,730,597	.009,109,106,349
62 00	.142,012,987,196	.150,049,091,717	.424,912,805,098	.374,930,472,923	.007,376,855,594	.009,318,880,371
62 15	.143,212,197,603	.152,316,413,817	.427,124,790,370	.379,112,612,522	.007,489,836,978	.009,532,493,373
62 30	.144,411,094,721	.154,607,273,626	.429,327,334,473	.383,321,232,086	.007,603,663,863	.009,749,995,949
62 45	.145,609,533,988	.156,921,784,997	.431,520,248,273	.387,556,328,530	.007,718,324,904	.009,971,439,042
63 00	.146,807,369,528	.159,260,061,131	.433,703,342,481	.391,817,897,614	.007,833,808,312	.010,196,873,944
63 15	.148,004,454,137	.161,622,214,573	.435,876,427,666	.396,105,933,959	.007,950,101,823	.010,426,352,298
63 30	.149,200,639,306	.164,008,357,192	.438,039,314,258	.400,420,431,025	.008,067,192,724	.010,659,926,090
63 45	.150,395,775,203	.166,418,600,179	.440,191,812,554	.404,761,381,131	.008,185,067,816	.010,897,647,650
64 00	.151,589,710,687	.168,853,054,030	.442,333,732,724	.409,128,775,438	.008,303,713,428	.011,139,569,650
64 15	.152,782,293,309	.171,311,828,538	.444,464,884,823	.413,522,603,955	.008,423,115,402	.011,385,745,102
64 30	.153,973,369,314	.173,795,032,784	.446,585,078,789	.417,942,855,534	.008,543,259,096	.011,636,227,355
64 45	.155,162,783,641	.176,302,775,118	.448,694,124,455	.422,389,517,875	.008,664,129,367	.011,891,070,088
65 00	.156,350,379,927	.178,835,163,165	.450,791,831,558	.426,862,577,519	.008,785,710,568	.012,150,327,328
65 15	.157,536,000,514	.181,392,303,792	.452,878,009,732	.431,362,019,848	.008,907,986,552	.012,414,053,411
65 30	.158,719,486,447	.183,974,303,116	.454,952,468,535	.435,887,829,084	.009,030,940,654	.012,682,303,020
65 45	.159,900,677,477	.186,581,266,485	.457,015,017,438	.440,439,988,291	.009,154,555,690	.012,955,131,152
66 00	.161,079,412,068	.189,213,298,467	.459,065,465,841	.445,018,479,371	.009,278,813,952	.013,232,593,133
66 15	.162,255,527,396	.191,870,502,843	.461,103,623,074	.449,623,283,062	.009,403,697,203	.013,514,744,606
66 30	.163,428,859,354	.194,552,982,592	.463,129,298,409	.454,254,378,942	.009,529,186,662	.013,801,641,533
66 45	.164,599,242,556	.197,260,839,884	.465,142,301,062	.458,911,745,421	.009,655,263,013	.014,093,340,192
67 00	.165,766,510,339	.199,994,176,068	.467,142,440,202	.463,595,359,747	.009,781,906,389	.014,389,897,171
67 15	.166,930,494,766	.202,753,091,660	.469,129,524,956	.468,305,198,001	.009,909,096,368	.014,691,369,372
67 30	.168,091,026,631	.205,537,686,334	.471,103,364,417	.473,041,235,098	.010,036,811,965	.014,997,813,995
° ′ ϕ_1	+COS ϕ_0	−SIN ϕ_0	+COS ϕ_0	−SIN ϕ_0	+COS ϕ_0	−SIN ϕ_0
	$\int \frac{\Phi_1^2}{2}$ COS $\phi\, d\phi_1$		$\int \Phi_1$ COS $\phi\, d\phi_1$		$\int \left(\frac{\Phi_1^2}{2} - \text{VERS } \phi_1\right)$ COS $\phi\, d\phi_1$	

TABLE 15—Continued

ϕ_1	$\int \frac{\phi^2}{2}$ SIN $\phi\, d\, \phi_1$		$\int \phi_1$ SIN $\phi\, d\, \phi_1$		$\int \left(\frac{\phi^2}{2}-\text{VERS } \phi_1\right)$ SIN $\phi\, d\, \phi_1$	
° ′	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0	+SIN ϕ_0	+COS ϕ_0
67 45	.169,247,935,462	.208,348,058,911	.473,063,767,649	.477,803,444,786	.010,165,031,632	.015,309,288,552
68 00	.170,401,049,526	.211,184,307,350	.475,010,543,696	.482,591,799,641	.010,293,733,252	.015,625,850,851
68 15	.171,550,195,828	.214,046,528,731	.476,943,501,584	.487,406,271,077	.010,422,894,123	.015,947,558,994
68 30	.172,695,200,119	.216,934,819,254	.478,862,450,332	.492,246,829,335	.010,552,490,960	.016,274,471,383
68 45	.173,835,886,902	.219,849,274,223	.480,767,198,961	.497,113,443,484	.010,682,499,896	.016,606,646,710
69 00	.174,972,079,427	.222,789,988,032	.482,657,556,489	.502,006,081,428	.010,812,896,458	.016,944,143,946
69 15	.176,103,599,710	.225,757,054,164	.484,533,331,952	.506,924,709,892	.010,943,655,581	.017,287,022,359
69 30	.177,230,268,516	.228,750,565,171	.486,394,334,399	.511,869,294,439	.011,074,751,583	.017,635,341,485
69 45	.178,351,905,385	.231,770,612,668	.488,240,372,907	.516,839,799,451	.011,206,158,178	.017,989,161,145
70 00	.179,468,328,621	.234,817,287,326	.490,071,256,585	.521,836,188,143	.011,337,848,455	.018,348,541,432
70 15	.180,579,355,303	.237,890,678,849	.491,886,794,571	.526,858,422,556	.011,469,794,879	.018,713,542,699
70 30	.181,684,801,291	.240,990,875,981	.493,686,796,059	.531,906,463,556	.011,601,969,290	.019,084,225,579
70 45	.182,784,481,220	.244,117,966,481	.495,471,070,286	.536,980,270,838	.011,734,342,880	.019,460,650,957
71 00	.183,878,208,520	.247,272,037,121	.497,239,426,548	.542,079,802,920	.011,866,886,211	.019,842,879,979
71 15	.184,965,795,409	.250,453,173,669	.498,991,674,207	.547,205,017,149	.011,999,569,189	.020,230,974,045
71 30	.186,047,052,902	.253,661,460,887	.500,727,622,694	.552,355,869,694	.012,132,361,072	.020,624,994,804
71 45	.187,121,790,813	.256,896,982,514	.502,447,081,519	.557,532,315,554	.012,265,230,452	.021,025,004,152
72 00	.188,189,817,763	.260,159,821,256	.504,149,860,272	.562,734,308,550	.012,398,145,259	.021,431,064,225
72 15	.189,250,941,187	.263,450,058,779	.505,835,768,637	.567,961,801,328	.012,531,072,755	.021,843,237,396
72 30	.190,304,967,330	.266,767,775,698	.507,504,616,395	.573,214,745,362	.012,663,979,518	.022,261,586,274
72 45	.191,351,701,265	.270,113,051,564	.509,156,213,431	.578,493,090,946	.012,796,831,452	.022,686,173,695
73 00	.192,390,946,882	.273,485,964,855	.510,790,369,738	.583,796,787,206	.012,929,593,765	.023,117,062,717
73 15	.193,422,506,909	.276,886,592,968	.512,406,895,427	.589,125,782,087	.013,062,230,975	.023,554,316,619
73 30	.194,446,182,909	.280,315,012,205	.514,005,600,735	.594,480,022,360	.013,194,706,903	.023,997,998,895
73 45	.195,461,775,285	.283,771,297,768	.515,586,296,028	.599,859,453,624	.013,326,984,659	.024,448,173,252
74 00	.196,469,083,288	.287,255,523,742	.517,148,791,806	.605,264,020,301	.013,459,026,647	.024,904,903,598
74 15	.197,467,905,024	.290,767,763,092	.518,692,898,720	.610,693,665,641	.013,590,794,552	.025,368,254,046
74 30	.198,458,037,451	.294,308,087,644	.520,218,427,561	.616,148,331,716	.013,722,249,337	.025,838,288,898
74 45	.199,439,276,400	.297,876,568,087	.521,725,189,287	.621,627,959,425	.013,853,351,244	.026,315,072,655
75 00	.200,411,416,563	.301,473,273,950	.523,212,995,012	.627,132,488,495	.013,984,059,772	.026,798,663,999
75 15	.201,374,251,510	.305,098,273,604	.524,681,656,025	.632,661,857,481	.014,114,333,685	.027,289,145,795
75 30	.202,327,573,695	.308,751,634,242	.526,130,983,789	.638,216,003,760	.014,244,131,006	.027,786,565,081
75 45	.203,271,174,458	.312,433,421,876	.527,560,789,955	.643,794,863,538	.014,373,409,008	.028,290,993,071
76 00	.204,204,844,031	.316,143,701,321	.528,970,886,357	.649,398,371,849	.014,502,124,209	.028,802,495,136
76 15	.205,128,371,545	.319,882,536,189	.530,361,085,030	.655,026,462,558	.014,630,232,363	.029,321,136,809
76 30	.206,041,545,036	.323,649,988,883	.531,731,198,215	.660,679,068,354	.014,757,688,460	.029,846,983,786
76 45	.206,944,151,454	.327,446,120,575	.533,081,038,356	.666,356,120,757	.014,884,446,724	.030,380,101,897
77 00	.207,835,976,665	.331,270,991,212	.534,410,418,123	.672,057,550,117	.015,010,460,595	.030,920,557,131
77 15	.208,716,805,460	.335,124,659,490	.535,719,150,401	.677,783,285,614	.015,135,682,736	.031,468,415,600
77 30	.209,586,421,558	.339,007,182,855	.537,007,048,307	.683,533,255,260	.015,260,065,021	.032,023,743,552
77 45	.210,444,607,620	.342,918,617,494	.538,273,925,199	.689,307,385,897	.015,383,558,533	.032,586,607,369
78 00	.211,291,145,249	.346,859,018,315	.539,519,594,674	.695,105,603,197	.015,506,113,561	.033,157,073,544
78 15	.212,125,814,996	.350,828,438,950	.540,743,870,580	.700,927,831,672	.015,627,679,585	.033,735,208,687
78 30	.212,948,396,370	.354,826,931,736	.541,946,567,022	.706,773,994,661	.015,748,205,279	.034,321,079,516
78 45	.213,758,667,847	.358,854,547,708	.543,127,498,368	.712,644,014,339	.015,867,638,508	.034,914,752,852
79 00	.214,556,406,870	.362,911,336,593	.544,286,479,258	.718,537,811,720	.015,985,926,314	.035,516,295,612
79 15	.215,341,389,862	.366,997,346,796	.545,423,324,603	.724,455,306,651	.016,103,014,920	.036,125,774,801
79 30	.216,113,392,229	.371,112,625,389	.546,537,849,601	.730,396,417,816	.016,218,849,719	.036,743,257,505
79 45	.216,872,188,371	.375,257,218,109	.547,629,869,740	.736,361,062,738	.016,333,375,272	.037,368,810,895
80 00	.217,617,551,685	.379,431,169,344	.548,699,200,805	.742,349,157,782	.016,446,535,302	.038,002,502,207
80 15	.218,349,254,574	.383,634,522,119	.549,745,658,880	.748,360,618,148	.016,558,272,687	.038,644,398,741
80 30	.219,067,068,458	.387,867,318,097	.550,769,060,363	.754,395,357,880	.016,668,529,463	.039,294,567,858
80 45	.219,770,763,771	.392,129,597,559	.551,769,221,965	.760,453,289,865	.016,777,246,805	.039,953,076,964
81 00	.220,460,109,982	.396,421,399,402	.552,745,960,722	.766,534,325,833	.016,884,365,038	.040,619,993,516
81 15	.221,134,875,594	.400,742,761,128	.553,699,093,999	.772,638,376,356	.016,989,823,625	.041,295,385,005
81 30	.221,794,828,150	.405,093,718,832	.554,628,439,495	.778,765,350,855	.017,093,561,156	.041,979,318,953
81 45	.222,439,734,246	.409,474,307,197	.555,533,815,253	.784,915,157,599	.017,195,515,354	.042,671,862,905
82 00	.223,069,359,537	.413,884,559,480	.556,415,039,667	.791,087,703,701	.017,295,623,066	.043,373,084,425
82 15	.223,683,468,746	.418,324,507,508	.557,271,931,483	.797,282,895,126	.017,393,820,261	.044,083,051,084
82 30	.224,281,825,667	.422,794,181,667	.558,104,309,813	.803,500,636,692	.017,490,042,016	.044,801,830,460
82 45	.224,864,193,177	.427,293,610,891	.558,911,994,137	.809,740,832,065	.017,584,222,523	.045,529,490,122
83 00	.225,430,333,244	.431,822,822,655	.559,694,804,308	.816,003,383,769	.017,676,295,080	.046,266,097,629
83 15	.225,980,006,934	.436,381,842,966	.560,452,560,565	.822,288,193,181	.017,766,192,086	.047,011,720,524
83 30	.226,512,974,418	.440,970,696,353	.561,185,083,533	.828,595,160,535	.017,853,845,035	.047,766,426,318
83 45	.227,028,994,981	.445,589,405,858	.561,892,194,231	.834,924,184,923	.017,939,184,516	.048,530,282,490
84 00	.227,527,827,034	.450,237,993,031	.562,573,714,085	.841,275,164,298	.018,022,140,208	.049,303,356,482
84 15	.228,009,228,116	.454,916,477,915	.563,229,464,923	.847,647,995,474	.018,102,640,872	.050,085,715,683
84 30	.228,472,954,908	.459,624,879,043	.563,859,268,992	.854,042,574,125	.018,180,614,353	.050,877,427,425
84 45	.228,918,763,234	.464,363,213,425	.564,462,948,960	.860,458,794,795	.018,255,987,564	.051,678,558,980
85 00	.229,346,408,079	.469,131,496,540	.565,040,327,920	.866,896,550,889	.018,328,686,502	.052,489,177,541
85 15	.229,755,643,589	.473,929,742,333	.565,591,229,402	.873,355,734,686	.018,398,636,219	.053,309,350,230
85 30	.230,146,223,090	.478,757,963,197	.566,115,477,377	.879,836,237,328	.018,465,760,845	.054,139,144,074
85 45	.230,517,899,079	.483,616,169,971	.566,612,896,260	.886,337,948,836	.018,529,983,555	.054,978,626,006
86 00	.230,870,423,258	.488,504,371,935	.567,083,310,929	.892,860,758,098	.018,591,226,595	.055,827,862,865
86 15	.231,203,546,516	.493,422,576,789	.567,526,546,710	.899,404,552,882	.018,649,411,254	.056,686,921,363
86 30	.231,517,018,959	.498,370,790,653	.567,942,429,403	.905,969,219,829	.018,704,457,876	.057,555,868,098
86 45	.231,810,589,908	.503,349,018,064	.568,330,785,280	.912,554,644,464	.018,756,285,845	.058,434,769,546
87 00	.232,084,007,911	.508,357,261,959	.568,691,441,098	.919,160,711,190	.018,804,813,591	.059,323,692,044
87 15	.232,337,020,747	.513,395,523,663	.569,024,224,088	.925,787,303,295	.018,849,958,574	.060,222,701,776
87 30	.232,569,375,452	.518,463,802,895	.569,328,961,985	.932,434,302,950	.018,891,637,304	.061,131,864,783
87 45	.232,780,818,303	.523,562,097,749	.569,605,483,019	.939,101,591,216	.018,929,765,306	.062,051,246,942
88 00	.232,971,094,849	.528,690,404,690	.569,853,615,926	.945,789,048,040	.018,964,257,144	.062,980,913,958
88 15	.233,139,949,907	.533,848,718,543	.570,073,189,952	.952,496,552,268	.018,995,026,396	.063,920,931,359
88 30	.233,287,127,578	.539,037,032,489	.570,264,034,865	.959,223,981,631	.019,021,985,670	.064,871,364,486
88 45	.233,412,371,257	.544,255,338,053	.570,425,980,955	.965,971,212,760	.019,045,046,591	.065,832,278,484
89 00	.233,515,423,636	.549,503,625,099	.570,558,859,043	.972,738,121,186	.019,064,119,793	.066,803,738,290
89 15	.233,596,026,723	.554,781,881,824	.570,662,500,493	.979,524,581,337	.019,079,114,928	.067,785,808,639
89 30	.233,653,921,841	.560,090,094,742	.570,736,737,206	.986,330,466,546	.019,089,940,654	.068,778,554,029
89 45	.233,688,849,647	.565,428,248,686	.570,781,401,637	.993,155,649,052	.019,096,504,633	.069,782,038,737
90 00	.233,700,550,136	.570,796,326,795	.570,796,326,795	1.000,000,000,000	.019,098,713,533	.070,796,326,795
° ′ ϕ_1	+COS ϕ_0	− SIN ϕ_0	+COS ϕ_0	− SIN ϕ_0	+COS ϕ_0	− SIN ϕ_0
	$\int \frac{\phi^2}{2}$ COS $\phi\, d\, \phi_1$		$\int \phi_1$ COS $\phi\, d\, \phi_1$		$\int \left(\frac{\phi^2}{2}-\text{VERS } \phi_1\right)$ COS $\phi\, d\, \phi_1$	

LOAD CONSTANTS FOR CIRCULAR ARCHES

203. General Statement.—With the aid of tables 11 to 15, load constants, which are the trigonometric parts of the expressions for deflections due to applied loads, as stated in chapter V, may be computed and tabulated for use in trial load analyses. These quantities are designated D-terms, D_1 being used for twist deflections, D_2 for radial deflections, and D_3 for tangential deflections. D-terms, multiplied by proper factors, give deflections at an arch point due to applied load between the point and the abutment. The method of using tabulated load constants in arch analyses is described in chapter VI.

204. Evaluation.—The formulas in chapter V, together with the tabular values of trigonometric functions and integrals in tables 11 to 15, inclusive, give all quantities needed for computing trigonometric parts of load formulas, or D-terms. The tabular load constants are the trigonometric parts of the D-terms with $P = 1000$ included in the uniform loads and $\frac{P}{\phi_1} = \frac{1000}{\phi_1}$ included in the triangular loads. Forms and sample calculations for D-terms of several unit loads are shown in table 16.

Sheet 1 of table 16 gives values of trigonometric functions and integrals needed in evaluating load constants for an arch with a 48° half central angle, proper sine and cosine multipliers being shown beneath the quantities where needed. The values were copied from tables 11 to 15, inclusive, and are tabulated on sheet 1 for convenience. Since D-terms are determined for the quarter-points of the half arch, the quarter-values of ϕ are listed in descending order, and the quantities for these angles are tabulated.

Sample computations for uniform and triangular radial, tangential, and twist loads are shown on sheets 2, 3, and 4 of table 16. In computing values for uniform loads, columns having integrals underneath are 1000 times the values for the integrals on sheet 1, except that some values are multiplied by three as indicated. Formulas for calculating the remaining columns are given on the computation sheets.

Computations for triangular loads are complicated, since some points are outside the loaded portion of the arch while other points are under the load. For points outside the load, values are obtained from sheet 1, using ϕ_1 as the angle between the abutment and the end of the load, and ϕ_0 as the angle between the end of the load and the point considered. As ex-

TABLE 16—COMPUTATIONS FOR LOAD CONSTANTS

$\phi = 48°$

INTEGRALS FOR LOAD FORMULAS, SHEET 1 OF 4

ϕ DEG.	$\sin \phi_1$	$\cos \phi_1$	$\text{vers } \phi_1$	$\phi_1 - \sin \phi_1$	$\frac{\phi_1^2}{2} - \text{vers } \phi_1$	$\frac{\phi_1^2}{2}$	$\frac{\phi_1^3}{6}$	ϕ_1
48	.743,144,83	.669,130,61	.330,869,39	.094,613,215	.020,049,874	.350,919,26	.097,995,146	.837,758,04
36	.587,785,25	.809,016,99	.190,983,00	.040,533,278	.006,409,082,3	.197,392,08	.041,341,702	.628,318,53
24	.406,736,64	.913,545,46	.086,454,542	.012,142,377	.001,275,274,5	.087,729,817	.012,249,393	.418,879,02
12	.207,911,69	.978,147,60	.021,852,399	.001,527,819,4	.000,080,054,958	.021,932,454	.001,531,174,2	.209,439,51
				$\int_0^{\phi_1} \text{vers } \phi_1 \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin \phi_1) \, d\phi_1$	$\int_0^{\phi_1} \phi_1 \, d\phi$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1$	

ϕ DEG.	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers } \phi_1\right) d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin \phi_1) \sin \phi \, d\phi_1$		$\int_0^{\phi_1} \text{vers } \phi_1 \sin \phi \, d\phi_1$		$\int_0^{\phi_1} \sin \phi_1 \sin \phi \, d\phi_1$	
		$+\sin \phi_0$	$+\cos \phi_0$	$+\sin \phi_0$	$+\cos \phi_0$	$+\sin \phi_0$	$+\cos \phi_0$
48	.003,381,930,6	.015,574,044	.012,326,733	.075,635,331	.054,737,278	.276,132,11	.170,248,55
36	.000,808,423,81	.005,587,609,1	.003,069,746,8	.035,861,858	.018,237,254	.172,745,75	.076,395,136
24	.000,107,015,85	.001,201,555,9	.000,418,312,75	.011,501,926	.003,737,194,0	.082,717,348	.023,653,303
12	.000,003,354,735,5	.000,078,887,841	.000,013,342,029	.001,507,774,9	.000,238,763,68	.021,613,636	.003,035,594,4
		$+\cos \phi_0$	$-\sin \phi_0$	$+\cos \phi_0$	$-\sin \phi_0$	$+\cos \phi_0$	$-\sin \phi_0$
		$\int_0^{\phi_1} (\phi_1 - \sin \phi_1) \cos \phi \, d\phi_1$		$\int_0^{\phi_1} \text{vers } \phi_1 \cos \phi \, d\phi_1$		$\int_0^{\phi_1} \sin \phi_1 \cos \phi \, d\phi_1$	

ϕ DEG.	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin \phi \, d\phi_1$		$\int_0^{\phi_1} \phi_1 \sin \phi \, d\phi_1$		$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers } \phi_1\right) \sin \phi \, d\phi_1$	
	$+\sin \phi_0$	$+\cos \phi_0$	$+\sin \phi_0$	$+\cos \phi_0$	$+\sin \phi_0$	$+\cos \phi_0$
48	.078,208,558	.056,895,337	.291,706,16	.182,575,27	.002,573,227,2	.002,158,059,4
36	.036,559,275	.018,639,806	.178,333,36	.079,464,883	.000,697,417,34	.000,402,552,49
24	.011,611,315	.003,773,728,6	.083,918,904	.024,071,617	.000,100,388,14	.000,036,534,612
12	.001,511,077,3	.000,239,345,95	.021,692,523	.003,048,936,4	.000,003,302,334,2	.000,000,582,276,82
	$+\cos \phi_0$	$-\sin \phi_0$	$+\cos \phi_0$	$-\sin \phi_0$	$+\cos \phi_0$	$-\sin \phi_0$
	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \cos \phi \, d\phi_1$		$\int_0^{\phi_1} \phi_1 \cos \phi \, d\phi_1$		$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers } \phi_1\right) \cos \phi \, d\phi_1$	

TABLE 16—Continued

$\phi = 48°$

UNIFORM RADIAL LOAD, SHEET 2 OF 4

POINT	D_2 2ND TERM	D_3 2ND TERM		D_1	D_2		D_3	
	2ND PART	1ST PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	828.396,33	75.635,331	510.745,62	94.613,215	54.737,277	883.133,61	18.977,884	435.110,29
1/4	518.237,25	35.861,857	229.185,41	40.533,278	18.237,254	536.474,50	4.671,420	193.323,55
1/2	248.152,04	11.510,926	70.959,912	12.142,377	3.737,193,9	251.889,23	.631,451	59.448,986
3/4	64.840,905	1.507,774,9	9.106,782,9	1.527,819,4	.238,763,67	65.079,669	.020,044,5	7.599,008
	$\int_0^{\phi_1} \sin\phi_1 \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} \sin\phi_1 \sin\phi \, d\phi_1$	$\int_0^{\phi_1} \text{vers}\,\phi_1 \, d\phi_1$	$\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$			

UNIFORM LOAD INCLUDES FACTOR, P = 1000

TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\phi_1} = \frac{1000}{\phi_1}$

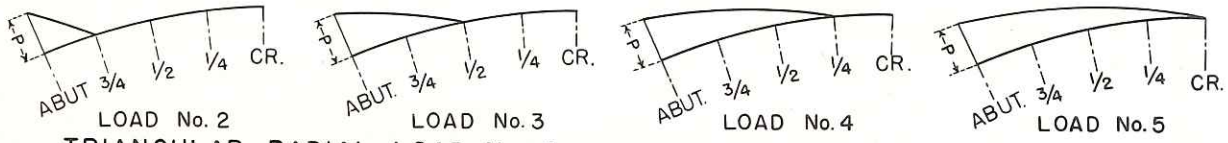
D_2 2ND TERM = (D_2 1ST TERM) + (D_2 2ND TERM, 2ND PART)
D_3 1ST TERM = D_1 − (D_3 2ND TERM, 1ST PART)
D_3 2ND TERM = (D_3 2ND TERM, 2ND PART) − (D_3 2ND TERM, 1ST PART)

TRIANGULAR RADIAL LOAD No. 2

POINT	D_2 2ND TERM	D_3 2ND TERM		D_1	D_2		D_3	
	2ND PART	1ST PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	15.462,322	.267,281,74	15.461,434	.382,234,27	.272,933,41	15.735,256	.114,952,53	15.194,152
1/4	18.339,046	.318,187,05	11.908,767	.382,234,27	.211,398,16	18.550,445	.064,047,23	11.590,580
1/2	20 414,266	.355,186,04	7.835,629,6	.382,234,27	.140,623,79	20.554,890	.027,048,237	7.480,443,5
3/4	21.597,285	.376,661,71	3.420,037,7	.382,234,27	.063,703,496	21.660,988	.005,572,569	3.043,376,0
	$3\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \, d\phi$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$			

FRACTION OF UNIFORM LOAD TO INCLUDE IN TRIANGULAR LOAD

POINT \ LOAD	No. 2	No. 3	No 4	No. 5
CROWN	0	0	0	0
1/4	0	0	0	1/4
1/2	0	0	1/3	1/2
3/4	0	1/2	2/3	3/4



LOAD No. 2 LOAD No. 3 LOAD No. 4 LOAD No. 5

TRIANGULAR RADIAL LOAD No. 4

POINT	D_2 2ND TERM	D_3 2ND TERM		D_1	D_2		D_3	
	2ND PART	1ST PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	149.381,80	7.682,838,9	120.773,90	10.200,371	6.627,840,2	156.009,64	2.517,532,1	113.091,06
1/4	171.227,75	8.892,955,9	87.076,474	10.200,371	4.885,653,9	176.113,40	1.307,415,1	78.183,518
1/2	137.677,97	5.749,310,8	41.497,089	6.077,121,4	1.911,496,8	139.589,47	.327,810,6	35.747,778
3/4	50.426,365	1.130,737,2	7.211,201,2	1.145,957,7	.180,410,29	50.606,775	.015,220,5	6.080,464,0
	$3\int_0^{\phi_1} \text{vers}\,\phi_1 \cos\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} \text{vers}\,\phi_1 \sin\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$			

TABLE 16—Continued

$\phi = 48°$

UNIFORM TANGENTIAL LOAD, SHEET 3 OF 4

POINT	D_2 2ND TERM		D_3 2ND TERM		D_3 1ST TERM	D_1	D_2		D_3	
	1ST PART	2ND PART	1ST PART	2ND PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	170.248,55	226.905,99	276.132,12	164.211,83	15.574,044	20.049,874	12.326,733	56.657,44	4.475,830	440.343,95
1/4	76.395,136	107.585,57	172.745,75	54.711,762	5.587,609,1	6.409,082,3	3.069,746,8	31.190,438	.821,473,2	227.457,51
1/2	23.653,303	34.532,778	82.717,348	11.211,582	1.201,555,9	1.275,274,5	.418,312,75	10.879,475	.073,718,6	93.928,93
3/4	3.035,594,4	4.523,324,7	21.613,636	.716,291,04	.078,887,842	.080,054,958	.013,342,029	1.487,730	.001,167,116	22.329,93
	$\int_0^{\phi_1} \sin\phi_1 \sin\phi \, d\phi_1$	$3\int_0^{\phi_1} \text{vers}\phi_1 \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \sin\phi_1 \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} \text{vers}\phi_1 \sin\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \, d\phi_1$	$\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$			

UNIFORM LOAD INCLUDES FACTOR, P = 1000

D_2 2ND TERM = (D_2 2ND TERM, 2ND PART) − (D_2 2ND TERM, 1ST PART)
D_3 1ST TERM = D_1 − (D_3 1ST TERM, 2ND PART)
D_3 2ND TERM = (D_3 2ND TERM, 1ST PART) + (D_3 2ND TERM, 2ND PART)

TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\phi_1} = \frac{1000}{\phi_1}$

TRIANGULAR TANGENTIAL LOAD No. 3

POINT	D_2 2ND TERM		D_3 2ND TERM		D_3 1ST TERM	D_1	D_2		D_3	
	1ST PART	2ND PART	1ST PART	2ND PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	19.327,805	6.642,964,5	21.475,653	6.237,107,7	.183,463,87	.255,481,52	.177,157,50	-12.684,840	.072,017,65	27.712,761
1/4	14.440,406	7.794,567,3	25.024,836	4.719,662,0	.216,287,86	.255,481,52	.135,141,91	-6.645,839,2	.039,193,66	29.744,498
1/2	8.921,893,2	8.605,510,2	27.480,312	2.995,944,4	.239,659,03	.255,481,52	.087,219,962	−.316,383,0	.015,822,49	30.476,256
3/4	2.087,803,5	2.826,654,9	14.406,365	.453,700,7	.047,327,663	.048,036,321	.008,061,098	+.738,851,45	.000,708,658	14.860,066
	$\int_0^{\phi_1} \text{vers}\phi_1 \sin\phi \, d\phi_1$	$3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \text{vers}\phi_1 \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) \sin\phi \, d\phi_1$			

TRIANGULAR TANGENTIAL LOAD No. 5

POINT	D_2 2ND TERM		D_3 2ND TERM		D_3 1ST TERM	D_1	D_2		D_3	
	1ST PART	2ND PART	1ST PART	2ND PART	2ND PART		1ST TERM	2ND TERM	1ST TERM	2ND TERM
CROWN	65.337,813	55.770,437	90.283,026	44.141,861	3.071,563,7	4.036,882,3	2.575,993,7	− 9.567,376	.965,318,58	134.424,89
1/4	40.867,902	46.905,543	85.993,377	24.670,661	2.229,382,7	2.567,255,4	1.247,948,3	+ 6.037,641	.337,872,7	110.664,04
1/2	16.287,598	21.569,144	55.098,830	7.103,763,2	.720,607,46	.765,378,01	.252,766,36	+ 5.281,546	.044,770,55	62.202,593
3/4	2.561,698,9	3.674,989,9	18.010,000	.584,995,91	.063,107,752	.064,045,639	.010,701,564	+ 1.113,291,0	.000,937,887	18.594,996
	$\int_0^{\phi_1} \text{vers}\phi_1 \sin\phi \, d\phi_1$	$3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \text{vers}\phi_1 \cos\phi \, d\phi_1$	$3\int_0^{\phi_1} (\phi_1 - \sin\phi_1) \sin\phi \, d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) d\phi_1$	$\int_0^{\phi_1} \left(\frac{\phi_1^2}{2} - \text{vers}\phi_1\right) \sin\phi \, d\phi_1$			

TABLE 16—Continued

$\phi = 48°$

UNIFORM TWIST LOAD, SHEET 4 OF 4

UNIFORM LOAD INCLUDES FACTOR, P = 1000

POINT	D_3 2ND PART	D_1	D_2	D_3
CROWN	291.706,16	350.919,26	182.575,27	59.213,10
1/4	178.333,36	197.392,08	79.464,883	19.058,72
1/2	83.918,904	87.729,817	24.071,617	3.810,913
3/4	21.692,523	21.932,454	3.048,936 4	.239,931
	$\int_0^{\phi_1} \phi_1 \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \phi_1 \, d\phi_1$	$\int_0^{\phi_1} \phi_1 \sin\phi \, d\phi_1$	

$D_3 = D_1 - D_3$ 2ND PART

TRIANGULAR TWIST LOAD No. 2

POINT	D_3 2ND PART	D_1	D_2	D_3
CROWN	5.165,229,8	7.310,818,3	5.165,328,6	2.145,588,5
1/4	6.126,289,4	7.310,818,3	3.978,542,1	1.184,528,9
1/2	6.819,600,6	7.310,818,3	2.617,874,2	.491,217,7
3/4	7.214,862,7	7.310,818,3	1.142,792,7	.095,955,6
	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1$	

TRIANGULAR TWIST LOAD No. 3

POINT	D_3 2ND PART	D_1	D_2	D_3
CROWN	21.659,119	29.243,272	19.504,962	7.584,153
1/4	25.241,125	29.243,272	14.575,548	4.002,147
1/2	27.719,973	29.243,272	9.009,113,2	1.523,299
3/4	14.453,693	14.621,636	2.095,864,5	0.167,943
	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1$	

TRIANGULAR LOAD INCLUDES FACTOR, $\frac{P}{\phi_1} = \frac{1000}{\phi_1}$

TRIANGULAR TWIST LOAD No. 4

POINT	D_3 2ND PART	D_1	D_2	D_3
CROWN	50.746,448	65.797,362	41.115,423	15.050,915
1/4	58.185,893	65.797,362	29.666,173	7.611,469,0
1/2	46.452,950	48.738,787	14.029,948	2.285,837,0
3/4	16.866,636	17.058,575	2.413,555,2	.191,939,2
	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1$	

TRIANGULAR TWIST LOAD No. 5

POINT	D_3 2ND PART	D_1	D_2	D_3
CROWN	93.354,589	116.973,09	67.913,806	23.618,500
1/4	88.222,760	98.696,042	42.115,850	10.473,282
1/2	55.819,438	58.486,544	16.540,365	2.667,106
3/4	18.073,108	18.277,045	2.572,400,5	0.203,937
	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \cos\phi \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \, d\phi_1$	$\int_0^{\phi_1} \frac{\phi_1^2}{2} \sin\phi \, d\phi_1$	

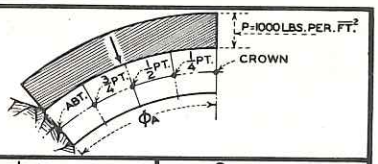
plained before, these values are multiplied by the factor $1000/\phi_1$. In calculating D-terms for points under the load, only deflections produced by the load between the point and the abutment are considered. This load is divided into a triangular load and a uniform load. Values for the triangular part of the load are equal to the indicated integrals for the angle between the abutment and the point considered. These values are multiplied by $1000/\phi_1$, which automatically corrects for the fact that the ordinate of the triangular load at the abutment is a fraction of P. The value of the uniform load to be included with the triangular part is equal to a fractional part of the 1000-pound uniform load values, the fraction being indicated on sheet 2.

205. Tabulations.—Values of load constants calculated for circular arches of uniform thickness, for all angles between 10 and 90 degrees, are tabulated in tables 17 to 21, inclusive. Tables 17 and 18 give load constants for radial loads, tables 19 and 20 give load constants for tangential loads, and table 21 gives load constants for twist loads.

All D-terms for the crown point are shown on the first sheets for radial and tangential loads. This was done because all D-terms are needed in determining the moment, thrust, and shear at the crown. The D_2 and D_3 terms for radial and tangential loads are divided into two parts, since each part has a differential multiplier. The first part is used in calculating the deflection due to bending, and the second part, in calculating the deflection due to rib-shortening and shear.

TABLE 17—CONSTANTS FOR RADIAL DEFLECTIONS, RADIAL LOADS

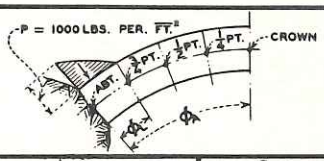
UNIT RADIAL LOAD NO. 1
RADIAL DEFLECTIONS



Φ °	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_2		D_2		D_2	
		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.884,747,5	.115,402,2	45.345,94	.008,068,29	4.407,601	.036,595,20	25.592,23	.007,240,160	11.401,42	$.0^3$452,941,1	2.854,429
11	1.177,222	.168,780,2	54.780,89	.012,984,1	5.860,143	.053,546,90	30.938,74	.010,597,50	13.790,21	$.0^3$663,1069	3.453,514
12	1.527,819	.238,763,7	65.079,67	.020,044,5	7.599,008	.075,788,48	36.783,40	.015,004,82	16.404,30	$.0^3$939,087,5	4.109,518
13	1.941,748	.328,446,8	76.232,91	.029,882,1	9.648,978	.104,313,7	43.123,20	.020,660,52	19.243,11	.001,293,362	4.822,402
14	2.424,200	.441,171,9	88.230,48	.043,242,4	12.034,51	.140,199,1	49.954,88	.027,779,93	22.305,99	.001,739,488	5.592,126
15	2.980,343	.580,524,7	101.061,5	.060,991,5	14.779,73	.184,602,7	57.274,95	.036,595,20	25.592,23	.002,292,105	6.418,646
16	3.615,325	.750,328,1	114.714,3	.084,124,9	17.908,37	.238,763,7	65.079,67	.047,355,24	29.101,08	.002,966,925	7.301,915
17	4.334,268	.954,637,2	129.176,5	.113,775,6	21.443,79	.304,000,6	73.365,04	.060,325,63	32.831,76	.003,780,741	8.241,883
18	5.142,271	1.197,732	144.435,0	.151,222,4	25.408,91	.381,710,7	82.126,82	.075,788,48	36.783,40	.004,751,416	9.238,496
19	6.044,403	1.484,113	160.476,1	.197,896,7	29.826,22	.473,368,5	91.360,53	.094,042,36	40.955,11	.005,897,886	10.291,70
20	7.045,707	1.818,490	177.285,2	.255,391,4	34.717,75	.580,524,7	101.061,5	.115,402,2	45.345,94	.007,240,160	11.401,42
21	8.151,193	2.205,780	194.847,2	.325,466,9	40.105,03	.704,804,6	111.224,7	.140,199,1	49.954,88	.008,799,316	12.567,62
22	9.365,842	2.651,096	213.146,2	.410,058,9	46.009,09	.847,907,1	121.845,0	.168,780,2	54.780,89	.010,597,50	13.790,21
23	10.694,60	3.159,739	232.166,0	.511,285	52.450,43	1.011,603	132.917,0	.201,508,7	59.822,87	.012,657,90	15.069,13
24	12.142,38	3.737,194	251.889,2	.631,451	59.448,99	1.197,732	144.435,0	.238,763,7	65.079,67	.015,004,82	16.404,30
25	13.714,05	4.389,115	272.298,4	.773,057	67.024,14	1.408,206	156.393,2	.280,939,6	70.550,10	.017,663,56	17.795,66
26	15.414,46	5.121,323	293.375,2	.938,803	75.194,69	1.644,999	168.785,5	.328,446,8	76.232,91	.020,660,52	19.243,11
27	17.248,40	5.939,789	315.100,9	1.131,596	83.978,80	1.910,156	181.605,7	.381,710,7	82.126,82	.024,023,14	20.746,58
28	19.220,63	6.850,633	337.455,9	1.354,554	93.394,03	2.205,780	194.847,2	.441,171,9	88.230,48	.027,779,93	22.305,99
29	21.335,86	7.860,109	360.420,7	1.611,008	103.457,3	2.534,040	208.503,3	.507,286,4	94.542,51	.031,960,42	23.921,23
30	23.598,78	8.974,596	383.974,6	1.904,515	114.184,9	2.897,163	222.567,1	.580,524,7	101.061,5	.036,595,20	25.592,23
31	26.013,99	10.200,59	408.096,9	2.238,850	125.592,3	3.297,434	237.031,5	.661,372,0	107.785,9	.041,715,92	27.318,88
32	28.586,10	11.544,69	432.766,3	2.618,024	137.694,4	3.737,194	251.889,2	.750,328,1	114.714,3	.047,355,24	29.101,08
33	31.319,62	13.013,59	457.961,1	3.046,274	150.505,5	4.218,838	267.132,8	.847,907,1	121.845,0	.053,546,90	30.938,74
34	34.219,04	14.614,08	483.659,1	3.528,075	164.039,1	4.744,813	282.754,5	.954,637,2	129.176,5	.060,325,63	32.831,76
35	37.288,80	16.352,99	509.837,9	4.068,140	178.307,7	5.317,616	298.746,5	1.071,060	136.707,0	.067,727,22	34.780,01
36	40.533,28	18.237,25	536.474,5	4.671,420	193.323,6	5.939,789	315.100,9	1.197,732	144.435,0	.075,788,48	36.783,40
37	43.956,80	20.273,83	563.545,8	5.343,112	209.097,8	6.613,922	331.809,2	1.335,222	152.358,6	.084,547,24	38.841,81
38	47.563,64	22469,72	591.028,3	6.088,654	225.640,9	7.342,646	348.863,4	1.484,113	160.476,1	.094,042,36	40.955,11
39	51.358,02	24.831,96	618.898,2	6.913,730	242.962,6	8.128,634	366.254,7	1.644,999	168.785,5	.104,313,7	43.123,20
40	55.344,09	27.367,60	647.131,5	7.824,269	261.071,9	8.974,596	383.974,6	1.818,490	177.285,2	.115,402,2	45.345,94
41	59.525,96	30.083,70	675.703,9	8.826,449	279.976,9	9.883,279	402.014,2	2.005,206	185.973,0	.127,349,7	47.623,21
42	63.907,68	32.987.29	704.590,9	9.926,690	299.685,0	10.857,46	420.364,6	2.205,780	194.847,2	.140,199,1	49.954,88
43	68.493,22	36.085,42	733.768,1	11.131,66	320.202,8	11.899,95	439.016,6	2.420,857	203.905,6	.153,994,3	52.340,82
44	73.286,50	39.385,07	763.210,5	12448,27	341.536,0	13.013,59	457.961,1	2.651,096	213.1462	.168,780,2	54.780,89
45	78.291,38	42.893,22	792.893,2	13.883,68	363.689,5	14.201,25	477.188,7	2.897,163	222.567,1	.184,602,7	57.274,95
46	83.511,66	46.616,76	822.791,5	15.445,29	386.667,7	15.465,80	496.689,8	3.159,739	232.166,0	.201,508,7	59.822,87
47	88.951,05	50.562,52	852.879,9	17.140,73	410473,8	16.810,16	516.455,0	3.439,516	241.940,8	.219,546,1	62.424,49
48	94.613,22	54.737,28	883.133,6	18.977,88	435.110,3	18.237,25	536.474,5	3.737,194	251.889,2	.238,763,7	65.079,67
49	100.501,8	59.147,70	913.527,5	20.964,86	460.579,0	19.750,02	556.738,5	4.053,486	262.009,2	.259,211,3	67.788,26
50	106.620,2	63.800,35	944.036,5	23.110,00	486.880,9	21.351,42	577.237,1	4.389,115	272.298,4	.280,939,6	70.550,10
51	112.972,0	68.701,69	974.635,4	25.421,86	514.016,1	23.044,41	597.960,4	4.744,813	282.754,5	.304,000,6	73.365,04
52	119.560,5	73.858,05	1,005.299	27.909,24	541.983,8	24.831,96	618.898,2	5.121,323	293.375,2	.328,446,8	76.232,91
53	126.389,0	79.275,64	1,036.004	30.581,16	570.782,7	26.717,05	640.040,4	5.519,394	304.158,1	.354,331,9	79.153,56
54	133.460,8	84.960,50	1,066.723	33.446,84	600.410,3	28.702,65	661.376,8	5.939,789	315.100,9	.381,710,7	82.126,82
55	140.779,1	90.918,53	1,097.434	36.515,71	630.863,8	30.791,74	682.897,1	6.383,276	326.200,9	.410,638,4	85.152,51
56	148.346,8	97.155,45	1,128.110	39.797,39	662.139,3	32.987,29	704.590,9	6.850,633	337.455,9	.441,171,9	88.230,48
57	156.167,1	103.676,8	1,158.729	43.301,73	694.232,0	35.292,26	726.448,0	7.342,646	348.863,4	.473,368,5	91.360,53
58	164.242,9	110.488,0	1,189.266	47.038,77	727.136,8	37.709,62	748.457,7	7.860,109	360.420,7	.507,286,4	94.542,51
59	172.577,0	117.594.0	1,219.698	51.018,69	760.847,4	40.242,29	770.609,6	8.403,823	372.125,3	.542,985,1	97.776,21
60	181.172,2	125.000,0	1,250.000	55.251,87	795.357,0	42.893,22	792.893,2	8.974,596	383.974,6	.580,524,7	101.061,5
61	190.031,1	132.710,6	1,280.150	59.748,87	830.657,9	45.665,30	815.298,0	9.573,245	395.966,0	.619,966,3	104.398,1
62	199.156,6	140.730,2	1,310.125	64.520,41	866.741,9	48.561,44	837.813,4	10.200,59	408.096,9	.661,372,0	107.785,9
63	208.550,9	149.063,2	1,339.902	69.577,35	903.599,8	51.584,48	860.428,8	10.857,46	420.364,6	.704,804,6	111.224,7
64	218.216,7	157.713,5	1,369.460	74.930,67	941.222,0	54.737,28	883.133,6	11.544,69	432.766,3	.750,328,1	114.714,3
65	228.156,2	166.684,8	1,398.776	80.591,55	979.598,0	58.022,64	905.917,3	12.263,12	445.299,4	.798,007,1	118.254,4
66	238.371,9	175.980,7	1,427.829	86.571,25	1,018.717	61.443,34	928.769,2	13.013,59	457.961,1	.847,907,1	121.845,0
67	248.865,8	185.604,3	1,456.598	92.881,15	1,058.567	65.002,12	951.678,8	13.796,96	470.748,6	.900,094,7	125.485,7
68	259.640,0	195.558,5	1,485.063	99.532,72	1,099.135	68.701,69	974.635,4	14.614,08	483.659,1	.954,637,2	129.176,5
69	270.696,8	205.845,8	1,513.204	106.537,6	1,140409	72.544,71	997.628,7	15.465,80	496.689,8	1.011,603	132.917,0
70	282.037,9	216.468,7	1,541.002	113.907,4	1,182.375	76.533,81	1,020.648	16.352,99	509.837,9	1.071,060	136.707,0
71	293.665,2	227.429,2	1,568.437	121.653,9	1,225.018	80.671,56	1,043.683	17.276,52	523.100,4	1.133,080	140.546,4
72	305.580,5	238.728,8	1,595.492	129.788,9	1,268.325	84.960,50	1,066.723	18.237,25	536.474,5	1.197,732	144.435,0
73	317.785,6	250.368,9	1,622.147	138.324,2	1,312.280	89.403,10	1,089.758	19.236,07	549.957,3	1.265,089	148.372,5
74	330.282,0	262.350,6	1,648.387	147.271,9	1,356.866	94.001,78	1,112.778	20.273,83	563.545,8	1.335,222	152.358,6
75	343.071,1	274.674,6	1,674.194	156.643,8	1,402.068	98.758,91	1,135.772	21.351,42	577.237,1	1.408,206	156,393,2
76	356.154,5	287.341,2	1,699.552	166.451,8	1,447.869	103.676,8	1,158.729	22.469,72	591.028,3	1.484,113	160.476,1
77	369.533,5	300.350,4	1,724.446	176.707,9	1,494.251	108.757,7	1,181.641	23.629,61	604.916,3	1.563,019	164.606,9
78	383.209,2	313.701,9	1,748.861	187.424,2	1,541.198	114.003,8	1,204.497	24.831,96	618.898,2	1.644,999	168.785,5
79	397.182,9	327.395,0	1,772.783	198.612,5	1,588.690	119.417,2	1,227.286	26.077,67	632.970,9	1.730,131	173.011,7
80	411.455,7	341.428,7	1,796.198	210.284,6	1,636.709	125.000,0	1,250.000	27.367,60	647.131,5	1.818,490	177.285,2
81	426.028,4	355.801,4	1,819.094	222.452,6	1,685.237	130.754,2	1,272.627	28.702,65	661.376,8	1.910,156	181.605,7
82	440.901,9	370.511,5	1,841.458	235.128,2	1,734.253	136.681,5	1,295.161	30.083,70	675.703,9	2.005,206	185.973,0
83	456.077,2	385.556,7	1,863.279	248.323,1	1,783.740	142.783,9	1,317.589	31.511,62	690.109,6	2.103,721	190.386,9
84	471.554,7	400.934,6	1,884.545	262.049,0	1,833.675	149.063,2	1,339.902	32.987,29	704.590,9	2.205,780	194.847,2
85	487.335,2	416.642,3	1,905.248	276.317,4	1,884.041	155.521,0	1,362.092	34.511,60	719.144,8	2.311,465	199.353,5
86	503.419,1	432.676,5	1,925.378	291.139,9	1,934.816	162.158,8	1,384.149	36.085,42	733.768,1	2.420,857	203.905,6
87	519.806,9	449.033,6	1,944.925	306.527,7	1,985.979	168.978,3	1,406.064	37.709,62	748.457,7	2.534,040	208.503,3
88	536.498,9	465.709,5	1,963.883	322.492,1	2,037.510	175.980,7	1,427.829	39.385,07	763.210,5	2.651,096	213.146,2
89	553.495,3	482.699,9	1,982.243	339.044,0	2,089.389	183.167,5	1,449.433	41.112,65	778.023,4	2.772,108	217.834,3
90	570.796,3	500.000,0	2,000.000	356.194,5	2,141.593	190.539,9	1,470.870	42.893,22	792.893,2	2.897,163	222.567,1

TABLE 17—Continued

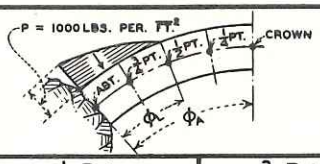
UNIT RADIAL LOAD NO. 2
RADIAL DEFLECTIONS



ϕ °	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_2		D_2		D_2	
		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.003,461,094	$.0^3$571,230,3	.939,663,1	$.0^4$47,553,5	.151,630,9	$.0^3$421,790,1	.945,382,8	$.0^3$271,547,0	.949,302,9	$.0^3$120,767,0	.951,416,0
11	.004,606,654	$.0^3$835,516,7	1.133,884	$.0^4$76,546,9	.201,634,6	$.0^3$617,208,5	1.142,252	$.0^3$397,478,6	1.147,990	$.0^3$176,833,3	1.151,083
12	.005,980,602	.001,182,066	1.345,368	$.0^3$118,204,3	.261,512,2	$.0^3$873,631,6	1.357,211	$.0^3$562,802,9	1.365,333	$.0^3$250,431,5	1.369,714
13	.007,603,689	.001,626,228	1.573,781	$.0^3$176,272,8	.332,124,5	.001,202,532	1.590,079	$.0^3$774,967,0	1.601,262	$.0^3$344,909,7	1.607,295
14	.009,496,661	.002,184,593	1.818,761	$.0^3$255,170,3	.414,323,3	.001,616,339	1.840,662	.001,042,055	1.855,697	$.0^3$463,884,2	1.863,810
15	.011,680,26	.002,874,968	2.079,917	$.0^3$360,036	.508,949,7	.002,128,434	2.108,750	.001,372,786	2.128,554	$.0^3$611,259,1	2.139,243
16	.014,175,24	.003,716,346	2.356,831	$.0^3$496,787	.616,834,0	.002,753,133	2.394,118	.001,776,507	2.419,741	$.0^3$791,225,8	2.433,576
17	.017,002,31	.004,728,889	2.649,059	$.0^3$672,161	.738,795,0	.003,505,683	2.696,526	.002,263,197	2.729,162	.001,008,264	2.746,790
18	.020,182,23	.005,933,890	2.956,128	$.0^3$893,778	.875,638,4	.004,402,244	3.015,717	.002,843,456	3.056,714	.001,267,137	3.078,864
19	.023,735,71	.007,353,745	3.277,540	.001,170,180	1.028,157	.005,459,878	3.351,424	.003,528,507	3.402,286	.001,572,898	3.429,778
20	.027,683,48	.009,011,923	3.612,770	.001,510,887	1.197,130	.006,696,539	3.703,359	.004,330,189	3.765,763	.001,930,885	3.799,508
21	.032,046,25	.010,932,93	3.961,265	.001,926,434	1.383,321	.008,131,053	4.071,224	.005,260,956	4.147,024	.002,346,719	4.188,030
22	.036,844,74	.013,142,27	4.322,451	.002,428,435	1.587,479	.009,783,111	4.454,705	.006,333,871	4.545,941	.002,826,311	4.595,320
23	.042,099,66	.015,666,42	4.695,724	.003,029,610	1.810,337	.011,673,24	4.853,472	.007,562,593	4.962,379	.003,375,845	5.021,350
24	.047,831,71	.018,532,76	5.080,459	.003,743,841	2.052,609	.013,822,80	5.267,184	.008,961,396	5.396,200	.004,001,806	5.466,095
25	.054,061,57	.021,769,59	5.476,003	.004,586,214	2.314,995	.016,253,97	5.695,482	.010,545,14	5.847,256	.004,710,950	5.929,523
26	.060,809,95	.025,406,02	5.881,682	.005,573,052	2.598,175	.018,989,71	6.137,995	.012,329,27	6.315,397	.005,510,315	6.411,606
27	.068,097,52	.029,471,98	6.296,797	.006,721,967	2.902,809	.022,053,77	6.594,339	.014,329,83	6.800,463	.006,407,230	6.912,312
28	.075,944,95	.033,998,13	6.720,626	.008,051,894	3.229,540	.025,470,63	7.064,114	.016,563,43	7.302,291	.007,409,296	7.431,609
29	.084,372,92	.039,015,87	7.152,426	.009,583,134	3.578,988	.029,265,54	7.546,907	.019,047,26	7.820,712	.008,524,401	7.969,462
30	.093,402,09	.044,557,24	7.591,431	.011,337,37	3.951,755	.033,464,45	8.042,293	.021,799,07	8.355,548	.009,760,710	8.525,839
31	.103,053,1	.050,654,88	8.036,851	.013,337,75	4.348,420	.038,094,17	8.549,831	.024,837,19	8.906,620	.011,126,67	9.100,701
32	.113,346,6	.057,341,95	8.487,882	.015,608,84	4.769,537	.043,181,48	9.069,069	.028,180,48	9.473,738	.012,630,99	9.694,012
33	.124,303,3	.064,652,34	8.943,689	.018,176,77	5.215,651	.048,754,91	9.599,542	.031,848,39	10.056,71	.014,282,69	10.305,73
34	.135,943,7	.072,620,06	9.403,426	.021,069,12	5.687,261	.054,842,85	10.140,77	.035,860,86	10.655,33	.016,091,04	10.935,82
35	.148,288,5	.081,279,78	9.866,229	.024,315,11	6.184,864	.061,474,56	10.692,26	.040,238,39	11.269,41	.018,065,60	11.584,25
36	.161,358,3	.090,666,37	10.331,21	.027,945,47	6.708,918	.068,679,76	11.253,52	.045,002,02	11.898,73	.020,216,19	12.250,96
37	.175,173,6	.100,815,1	10.797,45	.031,992,62	7.259,863	.076,488,81	11.824,02	.050,173,30	12.543,07	.022,552,92	12.935,91
38	.189,755,2	.111,761,3	11.264,05	.036,490,54	7.838,113	.084,932,59	12.403,23	.055,774,29	13.202,21	.025,086,16	13.639,07
39	.205,123,6	.123,540,7	11.730,06	.041,474,91	8.444,055	.094,042,47	12.990,63	.061,827,55	13.875,92	.027,826,57	14.360,37
40	.221,299,2	.136,189,0	12.194,52	.046,983,06	9.078,050	.103,850,3	13.585,65	.068,356,16	14.563,98	.030,785,06	15.099,79
41	.238,302,8	.149,742,0	12.656,47	.053,054,05	9.740,432	.114,388,4	14.187,73	.075,383,69	15.266,13	.033,972,83	15.857,27
42	.256,154,9	.164,235,5	13.114,92	.059,728,64	10.431,51	.125,689,5	14.796,30	.082,934,16	15.982,15	.037,401,34	16.632,75
43	.274,875,9	.179,705,1	13.568,87	.067,049,26	11.151,56	.137,786,7	15.410,77	.091,032,10	16.711,77	.041,082,32	17.426,20
44	.294,486,5	.196,186,6	14.017,30	.075,060,18	11.900,83	.150,713,5	16.030,55	.099,702,47	17.454,75	.045,027,76	18.237,56
45	.315,007,1	.213,715,1	14.459,20	.083,807,36	12.679,54	.164,503,8	16.655,03	.108,970,7	18.210,81	.049,249,94	19.066,77
46	.336,458,3	.232,326,0	14.893,53	.093,338,54	13.487,90	.179,191,7	17.283,59	.118,862,7	18.979,71	.053,761,38	19.913,78
47	.358,860,5	.252,054,1	15.319,23	.103,703,3	14.326,06	.194,811,8	17.915,61	.129,405,1	19.761,16	.058,575,10	20.778,54
48	.382,234,3	.272,933,4	15.735,26	.114,952,6	15.194,15	.211,398,2	18.550,44	.140,623,8	20.554,89	.063,703,50	21.660,99
49	.406,600,0	.294,998,3	16.140,53	.127,139,8	16.092,29	.228,986,2	19.187,45	.152,546,7	21.360,62	.069,160,53	22.561,08
50	.431,978,3	.318,282,2	16.533,98	.140,319,4	17.020,54	.247,611,1	19.825,97	.165,201,3	22.178,06	.074,959,59	23.478,73
51	.458,389,4	.342,817,9	16.914,51	.154,548,4	17.978,94	.267,307,8	20.465,34	.178,615,4	23.006,92	.081,114,50	24.413,90
52	.485,853,9	.368,637,5	17.281,04	.169,884,9	18.967,51	.288,111,7	21.104,89	.192,817,4	23.846,91	.087,639,34	25.366,53
53	.514,392,2	.395,772,6	17.632,47	.186,389,0	19.986,23	.310,058,4	21.743,93	.207,836,2	24.697,71	.094,548,51	26.336,55
54	.544,024,6	.424,253,9	17.967,68	.204,122,7	21.035,04	.333,183,2	22.381,77	.223,700,8	25.559,03	.101,856,6	27.323,90
55	.574,771,6	.454,111,2	18.285,58	.223,149,7	22.113,87	.357,521,7	23.017,71	.240,440,6	26.430,56	.109,578,5	28.328,52
56	.606,653,5	.485,373,7	18.585,04	.243,535,0	23.222,60	.383,109,7	23.651,04	.258,085,7	27.311,97	.117,729,2	29.350,33
57	.639,690,7	.518,069,0	18.864,95	.265,346,0	24.361,08	.409,982,3	24.281,06	.276,666,0	28.202,95	.126,324,3	30.389,28
58	.673,903,6	.552,224,2	19.124,19	.288,651,5	25.529,12	.438,175,1	24.907,02	.296,212,2	29.103,16	.135,379,2	31.445,29
59	.709,312,5	.587,864,9	19.361,64	.313,521,9	26.726,53	.467,723,3	25.528,22	.316,755,0	30.012,29	.144,909,9	32.518,30
60	.745,937,6	.625,015,9	19.576,17	.340,029,5	27.953,05	.498,662,2	26.143,91	.338,325,5	30.929,99	.154,932,4	33.608,23
61	.783,799,4	.663,700,3	19.766,66	.368,247,7	29.208,41	.531,026,6	26.753,35	.360,955,1	31.855,92	.165,463,3	34.715,03
62	.822,918,1	.703,940,3	19.931,99	.398,252,4	30.492,28	.564,851,4	27.355,78	.384,675,6	32.789,74	.176,518,9	35.838,60
63	.863,313,9	.745,756,6	20.071,04	.430,120,3	31.804,34	.600,171,1	27.950,46	.409,519,0	33.731,10	.188,116,3	36.978,88
64	.905,007,1	.789,168,2	20.182,69	.463,930,1	33.144,21	.637,019,9	28.536,63	.435,517,4	34.679,65	.200,272,5	38.135,80
65	.948,018,1	.834,192,8	20.265,83	.499,762,0	34.511,47	.675,431,7	29.113,52	.462,703,3	35.635,03	.213,004,8	39.309,28
66	.992,366,9	.880,846,7	20.319,35	.537,697,5	35.905,67	.715,440,2	29.680,36	.491,109,5	36.596,87	.226,330,9	40.499,25
67	1.038,074	.929,144,0	20.342,14	.577,819,8	37.326,36	.757,078,2	30.236,37	.520,768,8	37.564,82	.240,268,4	41.705,62
68	1.085,159	.979,097,7	20.333,10	.620,213,4	38.773,00	.800,378,9	30.780,77	.551,714,6	38.538,49	.254,835,7	42.928,32
69	1.133,643	1.030,719	20.291,15	.664,963,9	40.245,07	.845,374,2	31.312,81	.583,979,8	39.517,53	.270,050,5	44.167,27
70	1.183,545	1.084,016	20.215,19	.712,159,3	41.741,99	.892,096,1	31.831,63	.617,598,2	40.501,53	.285,931,8	45.422,38
71	1.234,886	1.138,997	20.104,15	.761,887,3	43.263,15	.940,575,6	32.336,50	.652,603,6	41.490,14	.302,498,2	46.693,57
72	1.287,685	1.195,667	19.956,97	.814,238,3	44.807,91	.990,843,7	32.826,61	.689,029,7	42.482,96	.319,768,7	47.980,77
73	1.341,964	1.254,029	19.772,59	.869,303,2	46.375,59	1.042,930	33.301,16	.726,910,6	43.479,59	.337,762,4	49.283,89
74	1.397,742	1.314,085	19.549,96	.927,174,2	47.965,49	1.096,865	33.759,36	.766,280,4	44.479,64	.356,498,9	50.602,84
75	1.455,039	1.375,834	19.288,05	.987,944,7	49.576,87	1.152,676	34.200,40	.807,173,3	45.482,73	.375,997,6	51.937,53
76	1.513,875	1.439,272	18.985,84	1.051,709	51.208,97	1.210,391	34.623,47	.849,623,8	46.488,43	.396,278,6	53.287,88
77	1.574,270	1.504,394	18.642,33	1.118,563	52.860,97	1.270,039	35.027,78	.893,666,3	47.496,36	.417,361,9	54.653,80
78	1.636,244	1.571,193	18.256,51	1.188,602	54.532,04	1.331,645	35.412,51	.939,335,3	48.506,10	.439,267,8	56.035,21
79	1.699,818	1.639,658	17.827,42	1.261,923	56.221,32	1.395,235	35.776,87	.986,665,6	49.517,24	.462,016,8	57.432,00
80	1.765,010	1.709,778	17.354,10	1.338,626	57.927,91	1.460,834	36.120,03	1.035,692	50.529,36	.485,629,8	58.844,09
81	1.831,842	1.781,538	16.835,60	1.418,809	59.650,89	1.528,465	36.441,20	1.086,448	51.542,04	.510,127,5	60.271,39
82	1.900,332	1.854,920	16.270,99	1.502,570	61.389,29	1.598,153	36.739,57	1.138,970	52.554,87	.535,531,2	61.713,80
83	1.970,502	1.929,905	15.659,37	1.590,011	63.142,13	1.669,917	37.014,32	1.193,292	53.567,42	.561,862,2	63.171,23
84	2.042,370	2.006,470	14.999,85	1.681,232	64.908,38	1.743,780	37.264,65	1.249,449	54.579,25	.589,142,3	64.643,59
85	2.115,956	2.084,590	14.291,57	1.776,334	66.687,01	1.819,763	37.489,76	1.307,476	55.589,94	.617,393,0	66.130,76
86	2.191,281	2.164,238	13.533,68	1.875,420	68.476,93	1.897,882	37.688,85	1.367,407	56.599,06	.646,636,4	67.632,67
87	2.268,365	2.245,384	12.725,35	1.978,590	70.277,04	1.978,156	37.861,10	1.429,276	57.606,16	.676,894,7	69.149,20
88	2.347,226	2.327,994	11.865,78	2.085,947	72.086,20	2.060,602	38.005,73	1.493,119	58.610,81	.708,190,4	70.680,27
89	2.427,885	2.412,033	10.954,20	2.197,594	73.903,25	2.145,235	38.121,92	1.558,971	59.612,56	.740,545,8	72.225,76
90	2.510,362	2.497,463	9.989,850	2.313,633	75.727,01	2.232,069	38.208,86	1.626,865	60.610,97	.773,984,0	73.785,58

TABLE 18—Continued

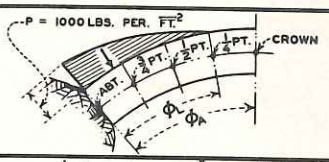
UNIT RADIAL LOAD NO. 3
RADIAL DEFLECTIONS



φ °	Crown D_1	Crown D_2 1ST Term	Crown D_2 2ND Term	Crown D_3 1ST Term	Crown D_3 2ND Term	¼ Point D_2 1ST Term	¼ Point D_2 2ND Term	½ Point D_2 1ST Term	½ Point D_2 2ND Term	¾ Point D_2 1ST Term	¾ Point D_2 2ND Term
10	.027,683,48	.004,330,189	3.765,763	.0³,343,603	.551,592,0	.003,133,519	3.786,239	.001,930,885	3.799,508	.0³,286,864,0	1.902,923
11	.036,844,74	.006,333,871	4.545,941	.0³,553,096	.733,550,1	.004,585,372	4.575,900	.002,826,311	4.595,320	.0³,419,970,1	2.302,299
12	.047,831,71	.008,961,396	5.396,200	.0³,854,090	.951,468,0	.006,490,496	5.438,601	.004,001,806	5.466,095	.0³,594,759,5	2.739,616
13	.060,809,95	.012,329,27	6.315,397	.001,273,658	1.208,493	.008,934,161	6.373,752	.005,510,315	6.411,606	.0³,819,135,8	3.214,848
14	.075,944,95	.016,563,43	7.302,291	.001,843,716	1.507,742	.012,008,76	7.380,717	.007,409,296	7.431,609	.001,101,686	3.727,968
15	.093,402,09	.021,799,07	8.355,548	.002,601,396	1.852,296	.015,813,75	8.458,805	.009,760,710	8.525,839	.001,451,682	4.278,944
16	.113,346,6	.028,180,48	9.473,738	.003,589,45	2.245,200	.020,455,57	9.607,278	.012,630,99	9.694,012	.001,879,075	4.867,746
17	.135,943,7	.035,860,86	10.655,33	.004,856,55	2.689,460	.026,047,58	10.825,35	.016,091,04	10.935,82	.002,394,502	5.494,336
18	.161,358,3	.045,002,02	11.898,73	.006,457,71	3.188,042	.032,709,94	12.112,18	.020,216,19	12.250,96	.003,009,276	6.158,680
19	.189,755,2	.055,774,29	13.202,21	.008,454,70	3.743,865	.040,569,56	13.466,89	.025,086,16	13.639,07	.003,735,392	6.860,737
20	.221,299,2	.068,356,16	14.563,98	.010,916,23	4.359,804	.049,759,96	14.888,54	.030,785,06	15.099,79	.004,585,523	7.600,466
21	.256,154,9	.082,934,16	15.982,15	.013,918,44	5.038,685	.060,421,22	16.376,15	.037,401,34	16.632,75	.005,573,018	8.377,824
22	.294,486,5	.099,702,47	17.454,75	.017,545,19	5.783,281	.072,699,81	17.928,69	.045,027,76	18.237,56	.006,711,903	9.192,765
23	.336,458,3	.118,862,7	18.979,71	.021,888,37	6.596,312	.086,748,53	19.545,08	.053,761,38	19.913,78	.008,016,875	10.045,24
24	.382,234,3	.140,623,8	20.554,89	.027,048,23	7.480,444	.102,726,4	21.224,21	.063,703,50	21.660,99	.009,503,311	10.935,20
25	.431,978,3	.165,201,3	22.178,06	.033,133,70	8.438,282	.120,798,4	22.964,89	.074,959,59	23.478,73	.011,187,25	11.862,59
26	.485,853,9	.192,817,4	23.846,91	.040,262,80	9.472,375	.141,135,6	24.765,92	.087,639,34	25.366,53	.013,085,42	12.827,36
27	.544,024,6	.223,700,8	25.559,03	.048,562,54	10.585,20	.163,914,7	26.626,02	.101,856,6	27.323,90	.015,215,19	13.829,45
28	.606,653,5	.258,085,7	27.311,97	.058,169,75	11.779,19	.189,318,6	28.543,91	.117,729,2	29.350,33	.017,594,61	14.868,80
29	.673,903,6	.296,212,2	29.103,16	.069,230,92	13.056,68	.217,534,9	30.518,22	.135,379,2	31.445,29	.020,242,41	15.945,35
30	.745,937,6	.338,325,5	30.929,99	.081,902,86	14.419,97	.248,757,1	32.547,56	.154,932,4	33.608,23	.023,177,95	17.059,03
31	.822,918,1	.384,675,6	32.789,74	.096,352,42	15.871,25	.283,183,9	34.630,49	.176,518,9	35.838,60	.026,421,29	18.209,79
32	.905,007,1	.435,517,4	34.679,65	.112,757,0	17.412,68	.321,019,1	36.765,53	.200,272,5	38.135,80	.029,993,12	19.397,55
33	.992,366,9	.491,109,5	36.596,87	.131,305,7	19.046,32	.362,471,2	38.951,35	.226,330,9	40.499,25	.033,914,79	20.622,24
34	1.085,159	.551,714,6	38.538,49	.152,197,2	20.774,15	.407,753,9	41.185,80	.254,835,7	42.928,32	.038,208,33	21.883,79
35	1.183,545	.617,598,2	40.501,53	.175,642,3	22.598,08	.457,084,8	43.467,86	.285,931,8	45.422,38	.042,896,41	23.182,13
36	1.287,685	.689,029,7	42.482,96	.201,863,2	24.519,95	.510,686,6	45.795,69	.319,768,7	47.980,77	.048,002,34	24.517,18
37	1.397,742	.766,280,4	44.479,64	.231,093,8	26.541,49	.568,785,9	48.167,59	.356,498,9	50.602,84	.053,550,08	25.888,86
38	1.513,875	.849,623,8	46.488,43	.263,579,2	28.664,36	.631,613,4	50.581,86	.396,278,6	53.287,88	.059,564,26	27.297,09
39	1.636,244	.939,335,3	48.506,10	.299,577,1	30.890,16	.699,403,7	53.036,71	.439,267,8	56.035,21	.066,070,14	28.741,78
40	1.765,010	1.035,692	50.529,36	.339,357,0	33.220,36	.772,395,3	55.530,36	.485,629,8	58.844,09	.073,093,62	30.222,87
41	1.900,332	1.138,970	52.554,87	.383,200,5	35.656,36	.850,829,4	58.060,95	.535,531,2	61.713,80	.080,661,25	31.740,24
42	2.042,370	1.249,449	54.579,25	.431,402,0	38.199,47	.934,951,8	60.626,62	.589,142,3	64.643,59	.088,800,21	33.293,82
43	2.191,281	1.367,407	56.599,06	.484,267,7	40.850,91	1.025,010	63.225,44	.646,636,4	67.632,67	.097,538,30	34.883,51
44	2.347,226	1.493,119	58.610,81	.542,117,0	43.611,81	1.121,256	65.855,49	.708,190,4	70.680,27	.106,904,0	36.509,22
45	2.510,362	1.626,865	60.610,97	.605,281,3	46.483,19	1.223,942	68.514,77	.773,984,0	73.785,58	.116,926,3	38.170,86
46	2.680,847	1.768,917	62.595,97	.674,105,1	49.465,99	1.333,325	71.201,27	.844,200,3	76.947,80	.127,635,1	39.868,32
47	2.858,839	1.919,551	64.562,20	.748,945,5	52.561,11	1.449,665	73.912,95	.919,025,3	80.166,08	.139,060,6	41.601,51
48	3.044,494	2.079,036	66.505,99	.830,172,4	55.769,09	1.573,221	76.647,73	.998,648,1	83.439,58	.151,233,6	43.370,33
49	3.237,969	2.247,641	68.423,68	.918,168,1	59.090,77	1.704,255	79.403,49	1.083,261	86.767,44	.164,185,9	45.174,66
50	3.439,421	2.425,632	70.311,53	1.013,329	62.526,62	1.843,032	82.178,10	1.173,058	90.148,78	.177,949,6	47.014,41
51	3.649,006	2.613,269	72.165,80	1.116,027	66.077,05	1.989,817	84.969,40	1.268,237	93.582,70	.192,557,5	48.889,47
52	3.866,879	2.810,809	73.982,70	1.226,755	69.742,49	2.144,877	87.775,18	1.368,999	97.068,31	.208,043,1	50.799,72
53	4.093,196	3.018,505	75.758,44	1.345,942	73.523,06	2.308,479	90.593,21	1.475,546	100.604,6	.224,440,2	52.745,06
54	4.328,110	3.236,605	77.489,18	1.473,970	77.418,94	2.480,891	93.421,24	1.588,083	104.190,8	.241,783,6	54.725,36
55	4.571,776	3.465,349	79.171,10	1.611,330	81.430,14	2.662,383	96.257,01	1.706,820	107.825,9	.260,108,4	56.740,51
56	4.824,348	3.704,973	80.800,31	1.758,495	85.556,58	2.853,222	99.098,21	1.831,966	111.508,8	.279,450,6	58.790,40
57	5.085,980	3.955,706	82.372,95	1.915,947	89.798,07	3.053,679	101.942,5	1.963,733	115.238,7	.299,846,4	60.874,91
58	5.356,824	4.217,771	83.885,13	2.084,183	94.154,32	3.264,021	104.787,5	2.102,337	119.014,5	.321,332,8	62.993,90
59	5.637,032	4.491,380	85.332,93	2.263,711	98.624,92	3.484,517	107.631,0	2.247,995	122.835,2	.343,947,5	65.147,26
60	5.926,757	4.776,741	86.712,47	2.455,053	103.209,4	3.715,434	110.470,4	2.400,927	126.699,9	.367,728,6	67.334,85
61	6.226,150	5.074,050	88.019,84	2.658,738	107.907,0	3.957,039	113.303,3	2.561,352	130.607,4	.392,714,8	69.556,56
62	6.535,362	5.383,498	89.251,12	2.875,313	112.717,2	4.209,597	116.127,5	2.729,494	134.556,8	.418,945,5	71.812,25
63	6.854,543	5.705,262	90.402,39	3.105,332	117.639,0	4.473,372	118.940,2	2.905,578	138.546,9	.446,460,5	74.101,78
64	7.183,843	6.039,513	91.469,78	3.349,362	122.671,6	4.748,626	121.739,3	3.089,832	142.576,8	.475,300,3	76.425,03
65	7.523,413	6.386,410	92.449,36	3.607,980	127.813,8	5.035,620	124.522,0	3.282,483	146.645,3	.505,506,0	78.781,86
66	7.873,400	6.746,100	93.337,27	3.881,776	133.064,5	5.334,611	127.286,0	3.483,761	150.751,3	.537,119,0	81.172,12
67	8.233,954	7.118,720	94.129,61	4.171,348	138.422,4	5.645,855	130.028,6	3.693,897	154.893,8	.570,181,6	83.595,67
68	8.605,221	7.504,397	94.822,55	4.477,306	143.886,2	5.969,604	132.747,5	3.913,125	159.071,6	.604,736,4	86.052,39
69	8.987,350	7.903,241	95.412,22	4.800,269	149.454,4	6.306,109	135.440,0	4.141,679	163.283,6	.640,826,6	88.542,11
70	9.380,488	8.315,355	95.894,81	5.140,868	155.125,4	6.655,617	138.103,6	4.379,794	167.528,7	.678,496,0	91.064,70
71	9.784,780	8.740,825	96.266,54	5.499,740	160.897,4	7.018,370	140.735,7	4.627,706	171.805,7	.717,789,0	93.620,01
72	10.200,37	9.179,725	96.523,62	5.877,534	166.768,7	7.394,607	143.333,8	4.885,654	176.113,4	.758,750,5	96.207,88
73	10.627,41	9.632,116	96.662,32	6.274,907	172.737,3	7.784,566	145.895,2	5.153,875	180.450,7	.801,425,7	98.828,17
74	11.066,04	10.098,04	96.678,94	6.692,523	178.801,2	8.188,477	148.417,4	5.432,609	184.816,5	.845,860,6	101.480,7
75	11.516,40	10.577,54	96.569,78	7.131,056	184.958,6	8.606,566	150.897,8	5.722,097	189.209,5	.892,101,6	104.165,4
76	11.978,63	11.070,61	96.331,23	7.591,187	191.206,5	9.039,056	153.333,7	6.022,579	193.628,5	.940,195,7	106.882,0
77	12.452,89	11.577,27	95.959,67	8.073,604	197.543,2	9.486,165	155.722,6	6.334,297	198.072,3	.990,190,4	109.630,4
78	12.939,31	12.097,50	95.451,57	8.579,004	203.966,1	9.948,104	158.061,9	6.657,492	202.539,8	1.042,134	112.410,4
79	13.438,03	12.631,26	94.803,40	9.108,085	210.472,6	10.425,08	160.348,9	6.992,407	207.029,7	1.096,074	115.221,8
80	13.949,20	13.178,51	94.011,72	9.661,558	217.060,1	10.917,29	162.581,0	7.339,285	211.540,7	1.152,060	118.064,6
81	14.472,95	13.739,19	93.073,10	10.240,14	223.725,8	11.424,94	164.755,7	7.698,368	216.071,6	1.210,142	120.938,5
82	15.009,43	14.313,21	91.984,19	10.844,54	230.466,9	11.948,21	166.870,2	8.069,900	220.621,2	1.270,368	123.843,4
83	15.558,77	14.900,47	90.741,70	11.475,48	237.280,4	12.487,28	168.922,1	8.454,124	225.188,3	1.332,791	126.779,1
84	16.121,11	15.500,85	89.342,37	12.133,70	244.163,4	13.042,33	170.908,6	8.851,282	229.771,5	1.397,461	129.745,4
85	16.696,59	16.114,22	87.783,03	12.819,93	251.112,5	13.613,53	172.827,2	9.261,617	234.369,6	1.464,429	132.742,1
86	17.285,35	16.740,43	86.060,56	13.534,90	258.124,7	14.201,04	174.675,3	9.685,371	238.981,3	1.533,747	135.769,1
87	17.887,52	17.379,29	84.171,91	14.279,34	265.196,5	14.805,02	176.450,2	10.122,79	243.605,4	1.605,467	138.826,2
88	18.503,23	18.030,62	82.114,08	15.054,00	272.324,6	15.425,60	178.149,4	10.574,11	248.240,5	1.679,643	141.913,3
89	19.132,63	18.694,22	79.884,18	15.859,62	279.505,4	16.062,93	179.770,4	11.039,57	252.885,4	1.756,327	145.030,0
90	19.775,85	19.369,84	77.479,35	16.696,94	286.735,4	16.717,15	181.310,5	11.519,42	257.538,7	1.835,573	148.176,3

TABLE 17—Continued

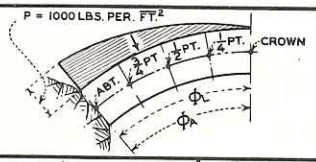
UNIT RADIAL LOAD NO. 4
RADIAL DEFLECTIONS



Φ °	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_2		D_2		D_2	
		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.093,402,09	.013,802,32	8.485,191	.001,047,005	1.117,013	.009,760,710	8.525,839	.003,700,643	6.333,480	$.0^3$,342,223,1	2.220,092
11	.124,303,3	.020,188,92	10.246,26	.001,685,29	1.485,502	.014,282,69	10.305,73	.005,416,706	7.660,283	$.0^3$,501.015,7	2.686,037
12	.161,358,3	.028,563,98	12.166,79	.002,602,28	1.926,801	.020,216,19	12.250,96	.007,669,476	9.112,164	$.0^3$,709,535,5	3.196,250
13	.205,123,6	.039,298,80	14.244,54	.003,880,45	2.447,347	.027,826,57	14.360,37	.010,560,38	10.688,77	$.0^3$,977,211,7	3.750,699
14	.256,154,9	.052,794,79	16.477,08	.005,616,91	3.053,393	.037,401,34	16.632,75	.014,199,51	12.389,73	.001,314,287	4.349,354
15	.315,007,1	.069,482,86	18.861,81	.007,924,69	3.751,206	.049,249,94	19.066,77	.018,705,54	14.214,64	.001,731,823	4.992,178
16	.382,234,3	.089,822,91	21.395,93	.010,933,83	4.546,957	.063,703,50	21.660,99	.024,205,74	16.163,04	.002,241,692	5.679,135
17	.458,389,4	.114,303,1	24.076,46	.014,792,45	5.446,741	.081,114,50	24.413,90	.030,835,91	18.234,47	.002,856,582	6.410,185
18	.544,024,6	.143,439,3	26.900,25	.019,667,98	6.456,569	.101,856,6	27.323,90	.038,740,29	20.428,44	.003,589,990	7.185,285
19	.639,690,7	.177,774,3	29.863,96	.025,747,95	7.582,365	.126,324,3	30.389,28	.048,071,56	22.744,41	.004,456,224	8.004,390
20	.745,937,6	.217,877,1	32.964,09	.033,241,50	8.829,960	.154,932,4	33.608,23	.058,990,77	25.181,84	.005,470,402	8.867,452
21	.863,313,9	.264,342,1	36.196,95	.042,379,78	10.205,09	.188,116,3	36.978,88	.071,667,25	27.740,13	.006,648,451	9.774,422
22	.992,366,9	.317,788,4	39.558,69	.053,417,65	11.713,37	.226,330,9	40.499,25	.086,278,57	30.418,67	.008,007,100	10.725,24
23	1.133,643	.378,858,2	43.045,30	.066,635,2	13.360,35	.270,050,5	44.167,27	.103,010,5	33.216,81	.009,563,885	11.719,87
24	1.287,685	.448,217,6	46.652,59	.082,333,7	15.151,43	.319,768,7	47.980,77	.122,056,9	36.133,88	.011,337,15	12.758,23
25	1.455,039	.526,553,9	50.376,21	.100,846,9	17.091,90	.375,997,6	51.937,53	.143,619,6	39.169,19	.013,346,02	13.840,28
26	1.636,244	.614,575,6	54.211,67	.122,531,2	19.186,95	.439,267,8	56.035,21	.167,908,5	42.321,99	.015,610,45	14.965,94
27	1.831,842	.713,010,8	58.154,30	.147,772,6	21.441,63	.510,127,5	60.271,39	.195,141,3	45.591,54	.018,151,17	16.135,16
28	2.042,370	.822,607,0	62.199,30	.176,985,5	23.860,87	.589,142,3	64.643,58	.225,543,5	48.977,05	.020,989,72	17.347,86
29	2.268,365	.944,129,2	66.341,72	.210,613,6	26.449,45	.676,894,7	69.149,20	.259,348,3	52.477,69	.024,148,41	18.603,97
30	2.510,362	1.078,359	70.576,44	.249,131,7	29.212,04	.773,984,0	73.785,58	.296,796,5	56.092,65	.027,650,37	19.903,43
31	2.768,895	1.226,095	74.898,23	.293,045,1	32.153,15	.881,024,9	78.549,99	.338,136,6	59.821,03	.031,519,50	21.246,15
32	3.044,494	1.388,148	79.301,71	.342,891,2	35.277,15	.998,648,1	83.439,58	.383,624,4	63.661,95	.035,780,49	22.632,06
33	3.337,688	1.565,344	83.781,37	.399,239,9	38.588,28	1.127,499	88.451,48	.433,522,9	67.614,49	.040,458,83	24.061,07
34	3.649,006	1.758,520	88.331,56	.462,693,6	42.090,59	1.268,237	93.582,70	.488,102,8	71.677,70	.045,580,76	25.533,11
35	3.978,972	1.968,525	92.946,52	.533,888,4	45.788,02	1.421,536	98.830,20	.547,641,3	75.850,59	.051,173,35	27.048,09
36	4.328,110	2.196,216	97.620,36	.613,495,8	49.684,31	1.588,083	104.190,8	.612423,2	80.132,18	.057,264,38	28.605,92
37	4.696,940	2.442,461	102.347,1	.702,220,4	53.783,07	1.768,579	109.661,4	.682,740,0	84.521,42	.063,882,47	30.206,51
38	5.085,980	2.708,132	107.120,5	.800,802,8	58.087,74	1.963,733	115.238,7	.758,890,0	89.017,27	.071,056,96	31.849,76
39	5.495,748	2.994,109	111.934,5	.910,018,9	62.601,57	2.174,271	120.919,3	.841,178,3	93.618,65	.078,818,00	33.535,59
40	5.926,757	3.301,275	116.782,8	1.030,680	67.327,66	2.400,927	126.699,9	.929,916,6	98.324,45	.087,196,47	35.263,89
41	6.379,519	3.630,517	121.658,7	1.163,636	72.268,95	2.644,444	132.576,9	1.025,423	103.133,5	.096,224,06	37.034,56
42	6.854,543	3.982,724	126.556,0	1.309,770	77.428,16	2.905,578	138.546,9	1.128,022	108.044,8	.105,933,2	38.847,50
43	7.352,335	4.358,784	131.467,8	1.470,004	82.807,88	3.185,093	144.606,3	1.238,043	113.057,0	.116,357,0	40.702,61
44	7.873,400	4.759,586	136.387,6	1.645,297	88.410,49	3.483,761	150.751,3	1.355,825	118.168,9	.127,529,4	42.599,78
45	8.418,239	5.186,014	141.308,6	1.836,644	94.238,18	3.802,360	156.978,4	1.481,710	123.379,4	.139,485,1	44.538,89
46	8.987,350	5.638,949	146.223,9	2.045,079	100.293,0	4.141,679	163.283,6	1.616,047	128.687,2	.152,259,6	46.519,84
47	9.581,230	6.119,268	151.126,6	2.271,673	106.576,7	4.502,510	169.663,3	1.759,189	134.091,0	.165,889,1	48.542,51
48	10.200,37	6.627,840	156.009,6	2.517,532	113.091,1	4.885,654	176.113,4	1.911,497	139.589,5	.180410,3	50.606,78
49	10.845,26	7.165,526	160.866,1	2.783,803	119.837,4	5.291,913	182.630,1	2.073,336	145.181,4	.195,860,2	52.712,53
50	11.516,40	7.733,178	165.688,7	3.071,668	126.817,1	5.722,097	189.209,5	2.245,077	150.865,3	.212,279,6	54.859,64
51	12.214,25	8.331,636	170.470,5	3.382,348	134.031,1	6.177,018	195.847,4	2.427,096	156.640,0	.229,705,2	57.047,99
52	12.939,31	8.961,727	175.204,1	3.717,098	141.480,3	6.657,492	202.539,8	2.619,773	162.503,9	.248,177,6	59.277,45
53	13.692,05	9.624,266	179.882,4	4.077,212	149.165,4	7.164,336	209.282,6	2.823,495	168.455,8	.267,737,5	61.547,89
54	14.472,95	10.320,05	184.498,1	4.464,022	157.086,8	7.698,368	216.071,6	3.038,652	174.494,2	.288,426,0	63.859,18
55	15.282,48	11.049,86	189.043,9	4.878,895	165.244,9	8.260,410	222.902,7	3.265,639	180.617,6	.310,285,1	66.211,18
56	16.121,11	11.814,47	193.512,4	5.323,233	173.639,7	8.851,282	229.771,5	3.504,855	186.824,5	.333,357,7	68.603,76
57	16.989,30	12.614,60	197.896,3	5.798,470	182.271,1	9.471,801	236.673,8	3.756,704	193.113,6	.357,687,1	71.036,78
58	17.887,52	13.450,99	202.188,2	6.306,088	191.138,8	10.122,79	243.605,4	4.021,595	199.483,2	.383,317,3	73.510,10
59	18.816,21	14.324,34	206.380,9	6.847,587	200.242,1	10.805,06	250.561,8	4.299,938	205.931,9	.410,293,3	76.023,57
60	19.775,85	15.235,31	210.466,8	7.424,515	209.579,4	11.519,42	257.538,7	4.592,150	212.458,1	.438,660,6	78.577,06
61	20.766,90	16.184,57	214.436,6	8.038,460	219.152,8	12.266,69	264.531,7	4.898,649	219.060,3	.468,465,3	81.170,40
62	21.789,75	17.172,73	218.289,1	8.690,985	228.958,2	13.047,67	271.536,4	5.219,859	225.736,8	.499,754,3	83.803,46
63	22.844,90	18.200,38	222.010,8	9.383,777	238.995,1	13.863,16	278.548,3	5.556,206	232.486,2	.532,575,2	86.476,08
64	23.932,77	19.268,10	225.596,3	10.118,49	249.262,1	14.713,95	285.563,0	5.908,118	239.306,6	.566,976,2	89.188,10
65	25.053,81	20.376,41	229.038,5	10.896,84	259.757,3	15.600,84	292.576,0	6.276,028	246.196,7	.603,006,3	91.939,38
66	26.208,44	21.525,82	232.330,0	11.720,55	270.479,0	16.524,60	299.582,7	6.660,371	253.154,6	.640,715,1	94.729,73
67	27.397,09	22.716,79	235.463,5	12.591,38	281.424,8	17.486,00	306.578,7	7.061,585	260.178,7	.680,152,6	97.559,02
68	28.620,19	23.949,76	238.432,0	13.511,12	292.592,6	18.485,80	313.559,4	7.480,109	267.267,4	.721,370,0	100.427,1
69	29.878,16	25.225,12	241.228,1	14.481,60	303.979,7	19.524,77	320.520,3	7.916,385	274.419,0	.764,418,6	103.333,7
70	31.171,42	26.543,23	243.844,9	15.504,66	315.583,5	20.603,64	327.456,8	8.370,860	281.631,8	.809,350,8	106.278,8
71	32.500,39	27.904,42	246.275,2	16.582,16	327.400,9	21.723,15	334.364,3	8.843,978	288.903,9	.856,219,3	109.262,2
72	33.865,47	29.308,96	248.512,1	17.716,00	339.428,9	22.884,02	341.238,4	9.336,187	296.233,8	.905,077,8	112.283,6
73	35.267,06	30.757,09	250.548,6	18.908,10	351.664,2	24.086,94	348.074,3	9.847,939	303.619,6	.955,980,1	115.342,9
74	36.705,58	32.249,00	252.377,9	20.160,40	364.103,3	25.332,63	354.867,6	10.379,68	311.059,6	1.008,981	118.440,0
75	38.181,41	33.784,85	253.993,3	21.474,86	376.742,4	26.621,75	361.613,8	10.931,87	318.552,0	1.064,136	121.574,7
76	39.694,95	35.364,76	255.387,9	22.853,45	389.577,7	27.954,98	368.308,1	11.504,96	326.095,1	1.121,501	124.746,7
77	41.246,60	36.988,77	256.555,2	24.298,18	402.605,1	29.332,96	374.946,1	12.099,40	333.687,0	1.181,133	127.955,9
78	42.836,72	38.656,91	257.488,7	25.811,06	415.820,4	30.756,33	381.523,2	12.715,65	341.325,9	1.243,089	131.202,1
79	44.465,71	40.369,15	258.181,9	27.394,12	429.219,2	32.225,70	388.034,9	13.354,16	349.010,1	1.307,426	134.485,1
80	46.133,94	42.125,41	258.628,5	29.049,41	442.796,8	33.741,68	394.476,7	14.015,39	356.737,6	1.374,203	137.804,8
81	47.841,79	43.925,55	258.822,4	30.778,98	456.548,5	35.304,84	400.844,0	14.699,80	364.506,7	1.443,480	141.160,9
82	49.589,62	45.769,40	258.757,2	32.584,92	470.469,1	36.915,75	407.132,4	15.407,83	372.315,4	1.515,314	144.553,3
83	51.377,78	47.656,73	258.427,0	34.469,29	484.553,8	38.574,95	413.337,3	16.139,95	380.162,1	1.589,768	147.981,7
84	53.206,65	49.587,24	257.826,1	36.434,18	498.797,2	40.282,96	419.454,3	16.896,62	388.044,6	1.666,901	151.446,0
85	55.076,57	51.560,62	256.948,6	38.481,70	513.193,8	42.040,29	425.479,1	17.678,28	395.961,3	1.746,774	154.945,9
86	56.987,90	53.576,46	255.788,8	40.613,94	527.738,0	43.847,41	431.407,0	18.485,39	403.910,2	1.829,450	158.481,3
87	58.940,97	55.634,33	254.341,3	42.833,01	542.424,0	45.704,77	437.233,9	19.318,40	411.889,5	1.914,992	162.051,9
88	60.936,13	57.733,72	252.600,7	45.141,02	557.246,0	47.612,82	442.955,3	20.177,76	419.897,2	2.003,460	165.657,6
89	62.973,71	59.874,09	250.561,7	47.540,07	572.197,7	49.571,97	448.566,8	21.063,93	427.931,4	2.094,921	169.298,1
90	65.054,05	62.054,81	248.219,2	50.032,28	587.273,0	51.582,59	454.064,3	21.977,35	435.990,2	2.189,437	172.973,2

TABLE 18—Continued

UNIT RADIAL LOAD NO. 5
RADIAL DEFLECTIONS

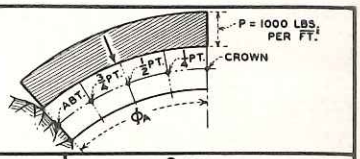


ϕ °	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_2		D_2		D_2	
		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.221,299,2	.030,785,06	15.099,79	.002,242,50	1.764,561	.016469,33	12.792,44	.004,585,523	7.600,466	$.0^3$369,902,6	2.378,676
11	.294,486,5	.045,027,76	18.237,56	.003,609,25	2.346,503	.024,098,74	15.463,99	.006,711,903	9.192,765	$.0^3$541,538,5	2.877,906
12	.382,234,3	.063,703,50	21.660,99	.005,572,56	3.043,376	.034,109,26	18.384,07	.009,503,311	10.935,20	$.0^3$766,923,5	3.424,567
13	.485,853,9	.087,639,34	25.366,53	.008,308,77	3.865,212	.046,948,35	21.551,08	.013,085,42	12.827,36	.001,056,249	4.018,625
14	.606,653,5	.117,729,2	29.350,33	.012,025,49	4.821,934	.063,100,77	24.963,29	.017,594,61	14.868,80	.001,420,587	4.660,047
15	.745,937,6	.154,932,4	33.608,23	.016,964,25	5.923,349	.083,088,14	28.618,81	.023,177,95	17.059,03	.001,871,894	5.348,795
16	.905,007,1	.200,272,5	38.135,80	.023402,75	7.179,140	.107,468,5	32.515,66	.029,993,12	19.397,55	.002,423,000	6.084,830
17	1.085,159	.254,835,7	42.928,32	.031,657,4	8.598,854	.136,836,0	36.651,69	.038,208,33	21.883,79	.003,087,622	6.868,110
18	1.287,685	.319,768,7	47.980,77	.042,085,2	10.191,90	.171,820,1	41.024,63	.048,002,34	24.517,18	.003,880,346	7.698,588
19	1.513,875	.396,278,6	53.287,88	.055,086,6	11.967,54	.213,085,3	45.632,09	.059,564,26	27.297,09	.004,816,640	8.576,217
20	1.765,010	.485,629,8	58.844,09	.071,106,9	13.934,87	.261,330,5	50.471,54	.073,093,62	30.222,87	.005,912,842	9.500,945
21	2.042,370	.589,142,3	64.643,59	.090,639,2	16.102,82	.317,288,4	55.540,33	.088,800,21	33.293,82	.007,186,167	10.472,72
22	2.347,226	.708,190,4	70.680,27	.114,225,7	18.480,17	.381,724,9	60.835,68	.106,904,0	36.509,22	.008,654,699	11.491,49
23	2.680,847	.844,200,3	76.947,80	.142,460,3	21.075,49	.455,438,5	66.354,69	.127,635,1	39.868,32	.010,337,39	12.557,19
24	3.044,494	.998,648,1	83.439,58	.175,990,2	23.897,18	.539,259,6	72.094,32	.151,233,6	43.370,33	.012,254,06	13.669,75
25	3.439,421	1.173,058	90.148,78	.215,518,3	26.953,43	.634,049,6	78.051,45	.177,949,6	47.014,41	.014,425,41	14.829,12
26	3.866,879	1.368,999	97.068,31	.261,803,8	30.252,25	.740,700,7	84.222,79	.208,043,1	50.799,72	.016,872,97	16.035,24
27	4.328,110	1.588,083	104.190,8	.315,664,9	33.801,43	.860,134,5	90.604,97	.241,783,6	54.725,36	.019,619,17	17.288,02
28	4.824,348	1.831,966	111.508,8	.377,980,6	37.608,53	.993,301,7	97.194,48	.279,450,6	58.790,40	.022,687,27	18.587,40
29	5.356,824	2.102,337	119.014,5	.449,692,1	41.680,91	1.141,181	103.987,7	.321,332,8	62.993,90	.026,101,41	19.933,30
30	5.926,757	2.400,927	126.699,9	.531,802,4	46.025,69	1.304,779	110.980,9	.367,728,6	67.334,85	.029,886,58	21.325,63
31	6.535,362	2.729,494	134.556,8	.625,361,4	50.649,77	1.485,127	118.170,4	.418,945,5	71.812,25	.034,068,60	22.764,33
32	7.183,843	3.089,832	142.576,8	.731,564,2	55.559,79	1.683,285	125.552,0	.475,300,3	76.425,03	.038,674,18	24.249,32
33	7.873,400	3.483,761	150.751,3	.851,552,8	60.762,15	1.900,334	133.121,8	.537,119,0	81.172,12	.043,730,85	25.780,49
34	8.605,221	3.913,125	159.071,6	.986,618,8	66.263,00	2.137,381	140.875,7	.604,736,4	86.052,39	.049,266,97	27.357,77
35	9.380,488	4.379,794	167.528,7	1.138,103	72.068,25	2.395,556	148.809,3	.678,496,0	91.064,70	.055,311,81	28.981,07
36	10.200,37	4.885,654	176.113,4	1.307,415	78.183,52	2.676,010	156.918,3	.758,750,5	96.207,88	.061,895,41	30.650,29
37	11.066,04	5.432,609	184.816,5	1.496,039	84.614,18	2.979,914	165.198,4	.845,860,6	101.480,7	.069,048,66	32.365,33
38	11.978,63	6.022,579	193.628,5	1.705,530	91.365,33	3.308,461	173.644,9	.940,195,7	106.882,0	.076,803,07	34.126,10
39	12.939,31	6.657,492	202.539,8	1.937,513	98.441,78	3.662,862	182.253,2	1.042,134	112.410,4	.085,191,93	35.932,49
40	13.949,20	7.339,285	211.540,7	2.193,688	105.848,1	4.044,344	191.018,6	1.152,060	118.064,6	.094,247,90	37.784,40
41	15.009,43	8.069,900	220.621,2	2.475,834	113.588,5	4.454,153	199.936,3	1.270,368	123.843,4	.104,005,5	39.681,72
42	16.121,11	8.851,282	229.771,5	2.785,797	121.667,0	4.893,549	209.001,3	1.397,461	129.745,4	.114,499,6	41.624,35
43	17.285,35	9.685,371	238.981,3	3.125,498	130.087,3	5.363,808	218.208,9	1.533,747	135.769,1	.125,766,3	43.612,16
44	18.503,23	10.574,11	248.240,5	3.496,937	138.852,7	5.866,219	227.553,8	1.679,643	141.913,3	.137,842,1	45.645,06
45	19.775,85	11.519,42	257.538,7	3.902,186	147.966,4	6.402,082	237.030,9	1.835,573	148.176,3	.150,764,5	47.722,91
46	21.104,28	12.523,23	266.865,6	4.343,392	157.431,1	6.972,709	246.635,2	2.001,970	154.556,9	.164,571,9	49.845,59
47	22.489,57	13.587,45	276.210,5	4.822,773	167.249,3	7.579,423	256.361,2	2.179,270	161.053,4	.179,303,4	52.014,00
48	23.932,77	14.713,95	285.563,0	5.342,629	177.423,3	8.223,554	266.203,7	2.367,921	167.664,4	.194,998,6	54.225,00
49	25.434,93	15.904,62	294.912,5	5.905,325	187.954,9	8.906,441	276.157,2	2.568,374	174.388,3	.211,698,6	56.481,46
50	26.997,07	17.161,30	304.248,2	6.513,305	198.845,7	9.629,428	286.216,4	2.781,087	181.223,6	.229,444,6	58.782,26
51	28.620,19	18.485,80	313.559,4	7.169,085	210.096,9	10.393,87	296.375,6	3.006,525	188.168,6	.248,279,1	61.127,25
52	30.305,31	19.879,93	322.835,4	7.875,252	221.709,6	11.201,11	306.629,4	3.245,161	195.221,8	.268,244,9	63.516,32
53	32.053,41	21.345,42	332.065,3	8.634,462	233.684,4	12.052,51	316.972,1	3.497,470	202.381,4	.289,386,1	65.949,31
54	33.865,47	22.884,02	341.238,4	9.449,447	246.021,7	12.949,44	327.397,9	3.763,937	209.645,8	.311,747,2	68.426,09
55	35.742,44	24.497,39	350.343,7	10.323,00	258.721,4	13.893,24	337.901,3	4.045,048	217.013,4	.335,373,4	70.946,51
56	37.685,29	26.187,18	359.370,6	11.257,99	271.783,3	14.885,28	348.476,4	4.341,299	224.482,4	.360,311,3	73.510,44
57	39.694,95	27.954,98	368.308,1	12.257,36	285.206,8	15.926,92	359.117,4	4.653,190	232.051,0	.386,607,4	76.117,72
58	41.772,34	29.802,34	377.145,5	13.324,09	298.991,1	17.019,50	369.818,5	4.981,223	239.717,6	.414,309,6	78.768,20
59	43.918,37	31.730,76	385.871,9	14.461,26	313.134,8	18.164,37	380.573,7	5.325,909	247.480,2	.443,466,3	81.461,73
60	46.133,94	33.741,68	394.476,7	15.671,99	327.636,7	19.362,87	391.377,3	5.687,761	255.337,2	.474,126,6	84.198,16
61	48.419,94	35.836,48	402.949,1	16.959,47	342494,7	20.616,34	402.223,3	6.067,298	263.286,7	.506,340,5	86.977,33
62	50.777,22	38.016,49	411.278,5	18.326,95	357.706,8	21.926,11	413.105,7	6.465,041	271.326,8	.540,158,7	89.799,07
63	53.206,65	40.282,96	419.454,3	19.777,72	373.270,7	23.293,49	424.018,5	6.881,520	279.455,8	.575,632,6	92.663,24
64	55.709,07	42.637,10	427.466,0	21.315,16	389.183,4	24.719,78	434.955,7	7.317,261	287.671,6	.612,814,2	95.569,64
65	58.285,29	45.080,04	435.303,1	22.942,67	405.442,2	26.206,29	445.911,3	7.772,801	295.972,4	.651,756,5	98.518,14
66	60.936,13	47.612,82	442.955,3	24.663,72	422.043,6	27.754,28	456.879,3	8.248,676	304.356,2	.692,513,1	101.508,5
67	63.662,39	50.236,43	450.412,2	26.481,84	438.984,0	29.365,03	467.853,7	8.745,429	312.821,2	.735,138,1	104.540,7
68	66.464,85	52.951,79	457.663,8	28.400,57	456.259,6	31.039,77	478.828,4	9.263,600	321.365,4	.779,686,8	107.614,4
69	69.344,27	55.759,71	464.700,1	30.423,53	473.866,1	32.779,76	489.797,4	9.803,739	329.986,7	.826,214,6	110.729,5
70	72.301,40	58.660,94	471.511,0	32.554,38	491.799,2	34.586,18	500.754,6	10.366,39	338.683,3	.874,778,2	113.885,9
71	75.336,98	61.656,14	478.086,7	34.796,79	510.054,1	36.460,26	511.694,0	10.952,11	347.453,0	.925,434,5	117.083,2
72	78.451,73	64.745,90	484.417,6	37.154,51	528.625,7	38.403,14	522.609,6	11.561,45	356.294,0	.978,241,4	120.321,4
73	81.646,34	67.930,69	490.494,2	39.631,29	547.508,8	40.415,98	533.495,3	12.194,97	365.204,0	1.033,257	123.600,3
74	84.921,52	71.210,92	496.307,0	42.230,92	566.697,9	42.499,91	544.345,2	12.853,22	374.181,2	1.090,541	126.919,7
75	88.277,93	74.586,90	501.846,9	44.957,24	586.187,1	44.656,04	555.153,2	13.536,76	383.223,3	1.150,154	130.279,3
76	91.716,22	78.058,83	507.104,8	47.814,09	605.970,5	46.885,43	565.913,4	14.246,15	392.328,4	1.212,154	133.679,0
77	95.237,05	81.626,82	512.071,8	50.805,35	626.041,5	49.189,14	576.619,8	14.981,95	401.494,3	1.276,605	137.118,6
78	98.841,01	85.290,91	516.739,2	53.934,90	646.393,8	51.568,20	587.266,6	15.744,73	410.719,0	1.343,566	140.598,0
79	102.528,7	89.050,99	521.098,5	57.206,67	667.020,5	54.023,58	597.847,8	16.535,04	420.000,3	1.413,102	144.116,8
80	106.300,8	92.906,89	525.141,3	60.624,57	687.914,6	56.556,26	608.357,5	17.353,44	429.336,1	1.485,275	147.674,9
81	110.157,8	96.858,32	528.859,5	64.192,55	709.068,8	59.167,16	618.789,9	18.200,51	438.724,2	1.560,149	151.272,1
82	114.100,3	100.904,9	532.245,2	67.914,54	730475,7	61.857,18	629.139,5	19.076,80	448.162,6	1.637,787	154.908,2
83	118.128,7	105.046,1	535.290,6	71.794,49	752.127,4	64.627,19	639.400,1	19.982,87	457.648,9	1.718,256	158.583,0
84	122.243,8	109.281,3	537.988,2	75.836,36	774.016,2	67.478,02	649.566,3	20.919,29	467.181,2	1.801,620	162.296,3
85	126.445,8	113.609,9	540.330,8	80.044,07	796.134,0	70.410,45	659.632,3	21.886,61	476.757,2	1.887,947	166.047,8
86	130.735,4	118.030,9	542.311,2	84.421,60	818.472,4	73.425,26	669.592,5	22.885,39	486.374,7	1.977,302	169.837,4
87	135.113,0	122.543,6	543.922,8	88.972,85	841.022,8	76.523,14	679.441,5	23.916,20	496.031,5	2.069,754	173.664,7
88	139.579,1	127.146,7	545.158,8	93.701,74	863.776,8	79.704,79	689.173,6	24.979,59	505.725,5	2.165,369	177.529,8
89	144.134,1	131.839,2	546.012,9	98.612,18	886.725,3	82.970,85	698.783,5	26.076,11	515.454,4	2.264,218	181.432,1
90	148.778,4	136.619,8	546.479,1	103.708,1	909,859,3	86.321,91	708.265,7	27.206,32	525.216,0	2.366,368	185.371,7

TABLE 18—CONSTANTS FOR TWIST AND TANGENTIAL DEFLECTIONS, RADIAL LOADS

UNIT RADIAL LOAD NO. 1

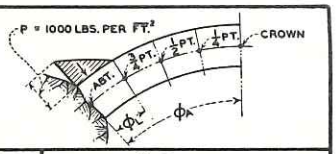
TWIST AND TANGENTIAL DEFLECTIONS



ϕ °	Crown D_1	Crown D_3 1ST Term	Crown D_3 2ND Term	¼ Point D_1	¼ Point D_3 1ST Term	¼ Point D_3 2ND Term	½ Point D_1	½ Point D_3 1ST Term	½ Point D_3 2ND Term	¾ Point D_1	¾ Point D_3 1ST Term	¾ Point D_3 2ND Term
10	.884,747,5	.008,068,29	4.407,601	.373,501,7	.001,917,68	1.863,673	.110,719,9	$.0^3$252,82	.553,093,6	.013,843,94	$.0^5$7,907	.069,203,86
11	1.177,222	.012,984,1	5.860,143	.497,041,3	.003,087,13	2.479,032	.147,356,3	$.0^3$407,09	.735,967,5	.018,425,91	$.0^4$12,733	.092,104,07
12	1.527,819	.020,044,5	7.599,008	.645,167,6	.004,767,50	3.216,303	.191,291,9	$.0^3$628,85	.955,201,6	.023,921,32	$.0^4$19,671	.119,567,2
13	1.941,748	.029,882,1	9.648,978	.820,098,2	.007,110,21	4.086,271	.243,187,6	$.0^3$938,12	1.214,062	.030,413,13	$.0^4$29,350	.152,006,9
14	2.424,200	.043,242,4	12.034,51	1.024,046	.010,293,6	5.099,643	.303,704,2	.001,358,54	1.515,804	.037,984,29	$.0^4$42,511	.189,836,4
15	2.980,343	.060,991,5	14.779,73	1.259,219	.014,525,3	6.267,043	.373,501,7	.001,917,68	1.863,673	.046,717,72	$.0^4$60,020	.233,468,6
16	3.615,325	.084,124,9	17.908,37	1.527,819	.020,044,5	7.599,008	.453,239,2	.002,647,26	2.260,901	.056,696,34	$.0^4$82,872	.283,315,9
17	4.334,268	.113,775,6	21.443,79	1.832,045	.027,123,6	9.105,976	.543,575,3	.003,583,55	2.710,709	.068,003,01	$.0^3$112,206	.339,790,7
18	5.142,271	.151,222,4	25.408,91	2.174,085	.036,070,8	10.798,28	.645,167,6	.004,767,50	3.216,303	.080,720,61	$.0^3$149,314	.403,304,4
19	6.044,403	.197,896,7	29.826,22	2.556,125	.047,231,6	12.686,16	.758,673,1	.006,245,30	3.780,875	.094,931,96	$.0^3$195,646	.474,268,5
20	7.045,707	.255,391,4	34.717,75	2.980,343	.060,991,5	14.779,73	.884,747,5	.008,068,29	4.407,601	.110,719,9	$.0^3$252,82	.553,093,6
21	8.151,193	.325,466,9	40.105,03	3.448,907	.077,777,2	17.088,98	1.024,046	.010,293,6	5.099,643	.128,167,1	$.0^3$322,65	.640,190,1
22	9.365,842	.410,058,9	46.009,09	3.963,982	.098,059,2	19.623,79	1.177,222	.012,984,1	5.860,143	.147,356,3	$.0^3$407,09	.735,967,5
23	10.694,60	.511,285	52.450,43	4.527,721	.122,353,2	22.393,90	1.344,930	.016,209,3	6.692,230	.168,370,4	$.0^3$508,37	.840,835,2
24	12.142,38	.631,451	59.448,99	5.142,271	.151,222,4	25.408,91	1.527,819	.020,044,5	7.599,008	.191,291,9	$.0^3$628,85	.955,201,6
25	13.714,05	.773,057	67.024,14	5.809,769	.185,278,7	28.678,29	1.726,543	.024,572,3	8.583,568	.216,203,4	$.0^3$771,16	1.079,475
26	15.414,46	.938,803	75.194,69	6.532,345	.225,185,5	32.211,35	1.941,748	.029,882,1	9.648,978	.243,187,6	$.0^3$938,12	1.241,062
27	17.248,40	1.131,596	83.978,80	7.312,117	.271,658,3	36.017,27	2.174,085	.036,070,8	10.798,28	.272,327,1	.001,132,81	1.359,370
28	19.220,63	1.354,554	93.394,03	8.151,193	.325,466,9	40.105,03	2.424,200	.043,242,4	12.034,51	.303,704,2	.001,358,54	1.515,804
29	21.335,86	1.611,008	103.457,3	9.051,675	.387,437,4	44.483,50	2.692,738	.051,509,3	13.360,67	.337,401,6	.001,618,89	1.683,770
30	23.598,78	1.904,515	114.184,9	10.015,65	.458,452,8	49.161,34	2.980,343	.060,991,5	14.779,73	.373,501,7	.001,917,68	1.863,673
31	26.013,99	2.238,850	125.592,3	11.045,20	.539,457	54.147,07	3.287,658	.071,817,6	16.294,65	.412,086,7	.002,258,95	2.055,916
32	28.586,10	2.618,024	137.694,4	12.142,38	.631,451	59.448,99	3.615,325	.084,124,9	17.908,37	.453,239,2	.002,647,26	2.260,901
33	31.319,62	3.046,274	150.505,5	13.309,25	.735,501	65.075,26	3.963,982	.098,059,2	19.623,79	.497,041,3	.003,087,13	2.479,032
34	34.219,04	3.528,075	164.039,1	14.547,86	.852,736	71.033,84	4.334,268	.113,775,6	21.443,79	.543,575,3	.003,583,55	2.710,709
35	37.288,80	4.068,140	178.307,7	15.860,24	.984,347	77.332,50	4.726,820	.131,438,7	23.371,22	.592,923,4	.004,141,82	2.956,333
36	40.533,28	4.671,420	193.323,6	17.248,40	1.131,596	83.978,80	5.142,271	.151,222,4	25.408,91	.645,167,6	.004,767,50	3.216,303
37	43.956,80	5.343,112	209.097,8	18.714,35	1.295,808	90.980,12	5.581,255	.173,310,4	27.559,66	.700,390,2	.005,466,60	3.491,018
38	47.563,64	6.088,654	225.640,9	20.260,08	1.478,377	98.343,63	6.044,403	.197,896,7	29.826,22	.758,673,1	.006,245,30	3.780,875
39	51.358,02	6.913,730	242.962,6	21.887,57	1.680,768	106.076,3	6.532,345	.225,185,5	32.211,35	.820,098,2	.007,110,21	4.086,271
40	55.344,09	7.824,269	261.071,9	23.598,78	1.904,515	114.184,9	7.045,707	.255,391,4	34.717,75	.884,747,5	.008,068,29	4.407,601
41	59.525,96	8.826,449	279.976,9	25.395,66	2.151,224	122.675,8	7.585,115	.288,739,7	37.348,10	.952,702,9	.009,126,87	4.745,261
42	63.907,68	9.926,690	299.685,0	27.280,15	2.422,574	131.555,6	8.151,193	.325,466,9	40.105,03	1.024,046	.010,293,6	5.099,643
43	68.493,22	11.131,66	320.202,8	29.254,17	2.720,315	140.830,2	8.744,563	.365,820,4	42.991,17	1.098,859	.011,576,3	5.471,140
44	73.286,50	12.448,27	341.536,0	31.319,62	3.046,274	150.505,5	9.365,842	.410,058,9	46.009,09	1.177,222	.012,984,1	5.860,143
45	78.291,38	13.883,68	363.689,5	33.478,39	3.402,351	160.587,2	10.015,65	.458,452,8	49.161,34	1.259,219	.014,525,3	6.267,043
46	83.511,66	15.445,29	386.667,7	35.732,39	3.790,520	171.080,7	10.694,60	.511,285	52.450,43	1.344,930	.016,209,3	6.692,230
47	88.951,05	17.140,73	410.473,8	38.083,37	4.212,834	181.991,2	11.403,31	.568,849	55.878,83	1.434,436	.018,045,5	7.136,089
48	94.613,22	18.977,88	435.110,3	40.533,28	4.671,420	193.323,6	12.142,38	.631,451	59.448,99	1.527,819	.020,044,5	7.599,008
49	100.501,8	20.964,86	460.579,0	43.083,90	5.168,483	205.082,5	12.912,42	.699,410	63.163,30	1.625,161	.022,216,4	8.081,375
50	106.620,2	23.110,00	486.880,9	45.737,04	5.706,302	217.272,6	13.714,05	.773,057	67.024,14	1.726,543	.024,572,3	8.583,568
51	112.972,0	25.421,86	514.016,1	48.494,49	6.287,240	229.898,0	14.547,86	.852,735	71.033,84	1.832,045	.027,123,6	9.105,976
52	119.560,5	27.909,24	541.983,8	51.358,02	6.913,730	242.962,6	15.414,46	.938,803	75.194,69	1.941,748	.029,882,1	9.648,978
53	126.389,0	30.581,16	570.782,7	54.329,38	7.588,289	256.470,3	16.314,44	1.031,629	79.508,93	2.055,735	.032,860,3	10.212,95
54	133.460,8	33.446,84	600.410,3	57.410,30	8.313,509	270.424,5	17.248,40	1.131,596	83.978,80	2.174,085	.036,070,8	10.798,28
55	140.779,1	36.515,71	630.863,8	60.602,50	9.092,059	284.828,4	18.216,93	1.239,101	88.606,45	2.296,880	.039,526,9	11.405,35
56	148.346,8	39.797,39	662.139,3	63.907,68	9.926,690	299.685,0	19.220,63	1.354,554	93.394,03	2.424,200	.043,242,4	12.034,51
57	156.167,1	43.301,73	694.232,0	67.327,51	10.820,23	314.997,1	20.260,08	1.478,377	98.343,63	2.556,125	.047,231,6	12.686,16
58	164.242,9	47.038,77	727.136,8	70.863,65	11.775,57	330.767,1	21.335,86	1.611,008	103.457,3	2.692,738	.051,509,3	13.360,67
59	172.577,0	51.018,69	760.847,4	74.517,73	12.795,70	346.997,3	22.448,57	1.752,899	108.737,0	2.834,117	.056,090,7	14.058,40
60	181.172,2	55.251,87	795.357,0	78.291,38	13.883,68	363.689,5	23.598,78	1.904,515	114.184,9	2.980,343	.060,991,5	14.779,73
61	190.031,1	59.748,87	830.657,9	82.186,19	15.042,64	380.845,7	24.787,06	2.066,334	119.802,6	3.131,497	.066,228,2	15.525,03
62	199.156,6	64.520,41	866.741,9	86.203,73	16.275,80	398.467,0	26.013,99	2.238,850	125.592,3	3.287,658	.071,817,6	16.294,65
63	208.550,9	69.577,35	903.599,8	90.345,56	17.586,42	416.555,0	27.280,15	2.422,574	131.555,6	3.448,907	.077,777,2	17.088,98
64	218.216,7	74.930,67	941.222,0	94.613,22	18.977,88	435.110,3	28.586,10	2.618,024	137.694,4	3.615,325	.084,124,9	17.908,37
65	228.156,2	80.591,55	979.598,0	99.008,20	20.453,62	454.133,8	29.932,40	2.825,741	144.010,5	3.786,989	.090,879,2	18.753,19
66	238.371,9	86.571,25	1,018.717	103.532,0	22.017,12	473.625,8	31.319,62	3.046,274	150.505,5	3.963,982	.098,059,2	19.623,79
67	248.865,8	92.881,15	1,058.567	108.186,1	23.671,99	493.586,6	32.748,31	3.280,192	157.181,2	4.146,382	.105,684,6	20.520,54
68	259.640,0	99.532,72	1,099.135	112.972,0	25.421,86	514.016,1	34.219,04	3.528,075	164.039,1	4.334,268	.113,775,6	21.443,79
69	270.696,8	106.537,6	1,140.409	117.891,0	27.270,45	534.913,9	35.732,35	3.790,520	171.080,7	4.527,721	.122,353,2	22.393,90
70	282.037,9	113.907,4	1,182.375	122.944,5	29.221,56	556.279,5	37.288,80	4.068,140	178.307,7	4.726,820	.131,438,7	23.371,22
71	293.665,2	121.653,9	1,225.018	128.134,0	31.279,05	578.112,0	38.888,93	4.361,558	185.721,5	4.931,643	.141,054,2	24.376,11
72	305.580,5	129.788,9	1,268.325	133.460,8	33.446,84	600.410,3	40.533,28	4.671,420	193.323,6	5.142,271	.151,222,4	25.408,91
73	317.785,6	138.324,2	1,312.280	138.926,2	35.728,91	623.173,2	42.222,39	4.998,381	201.115,2	5.358,782	.161,966,4	26.469,98
74	330.282,0	147.271,9	1,356.866	144.531,5	38.129,33	646.399,1	43.956,80	5.343,112	209.097,8	5.581,255	.173,310,4	27.559,66
75	343.071,1	156.643,8	1,402.068	150.278,1	40.652,21	670.086,0	45.737,04	5.706,302	217.272,6	5.809,769	.185,278,7	28.678,29
76	356.154,5	166.451,8	1,447.869	156.167,1	43.301,73	694.232,0	47.563,64	6.088,654	225.640,9	6.044,403	.197,896,7	29.826,22
77	369.533,5	176.707,9	1,494.251	162.199,8	46.082,14	718.834,8	49.437,13	6.490,886	234.203,9	6.285,236	.211,190,1	31.003,80
78	383.209,2	187.424,2	1,541.198	168.377,5	48.997,72	743.891,8	51.358,02	6.913,730	242.962,6	6.532,345	.225,185,5	32.211,35
79	397.182,9	198.612,5	1,588.690	174.701,2	52.052,83	769.400,2	53.326,83	7.357,937	251.918,3	6.785,809	.239,910,0	33.449,23
80	411.455,7	210.284,6	1,636.709	181.172,2	55.251,87	795.357,0	55.344,09	7.824,269	261.071,9	7.045,707	.255,391,4	34.717,75
81	426.028,4	222.452,6	1,685.237	187.791,5	58.599,30	821.759,0	57.410,30	8.313,509	270.424,5	7.312,117	.271,658,3	36.017,27
82	440.901,9	235.128,2	1,734.253	194.560,4	62.099,65	848.602,6	59.525,96	8.826,449	279.976,9	7.585,115	.288,739,7	37.348,10
83	456.077,2	248.323,1	1,783.740	201.479,8	65.757,46	875.884,2	61.691,59	9.363,900	289.730,2	7.864,782	.306,665,7	38.710,58
84	471.554,7	262.049,0	1,833.675	208.550,9	69.577,35	903.599,8	63.907,68	9.926,690	299.685,0	8.151,193	.325,466,9	40.105,03
85	487.335,2	276.317,4	1,884.041	215.774,7	73.563,95	931.745,4	66.174,72	10.515,66	309.842,3	8.444,428	.345,174,4	41.531,79
86	503.419,1	291.139,9	1,934.816	223.152,1	77.721,98	960.316,5	68.493,22	11.131,66	320.202,8	8.744,563	.365,820,4	42.991,17
87	519.806,9	306.527,7	1,985.979	230.684,2	82.056,16	989.308,5	70.863,65	11.775,57	330.767,1	9.051,675	.387,437,4	44.483,50
88	536.498,9	322.492,1	2,037.510	238.371,9	86.571,25	1,018.717	73.286,50	12.448,27	341.536,0	9.365,842	.410,058,9	46.009,09
89	553.495,3	339.044,0	2,089.389	246.216,1	91.272,09	1,048.536	75.762,25	13.150,67	352.509,9	9.687,141	.433,719,2	47.568,27
90	570.796,3	356.194,5	2,141.593	254.217,7	96.163,51	1,078.762	78.291,38	13.883,68	363.689,5	10.015,65	.458,452,8	49.161,34

TABLE 18—Continued

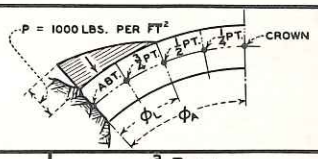
UNIT RADIAL LOAD NO. 2
TWIST AND TANGENTIAL DEFLECTIONS



φ	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_3		D_1	D_3		D_1	D_3		D_1	D_3	
°		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.003,461,094	$.0^4$47,553,5	.151,630,9	.003,461,094	$.0^4$25,885,8	.110,499,1	.003,461,094	$.0^4$10,757,0	.069,156,87	.003,461,094	$.0^5$2,196,3	.027,683,04
11	.004,606,654	$.0^4$76,546,9	.201,634,6	.004,606,654	$.0^4$41,677,3	.147,000,8	.004,606,654	$.0^4$17,321,8	.092,028,41	.004,606,654	$.0^5$3,536,7	.036,844,04
12	.005,980,602	$.0^3$1,118,204,3	.261,512,2	.005,980,602	$.0^4$64,374,0	.190,742,7	.005,980,602	$.0^4$26,759,7	.119,450,3	.005,980,602	$.0^5$5,464,3	.047,830,61
13	.007,603,689	$.0^3$1,176,272,8	.332,124,5	.007,603,689	$.0^4$96,023,1	.242,368,3	.007,603,689	$.0^4$39,923,2	.151,832,5	.007,603,689	$.0^5$8,153,1	.060,808,32
14	.009,496,661	$.0^3$1,255,170,3	.414,323,3	.009,496,661	$.0^3$1,139,042	.302,517,8	.009,496,661	$.0^4$57,821,0	.189,583,8	.009,496,661	$.0^4$11,810	.075,942,59
15	.011,680,26	$.0^3$1,360,036	.508,949,7	.011,680,26	$.0^3$1,196,242	.371,826,9	.011,680,26	$.0^4$81,624	.233,112,0	.011,680,26	$.0^4$16,673	.093,398,76
16	.014,175,24	$.0^3$1,496,787	.616,834,0	.014,175,24	$.0^3$1,270,868	.450,927,2	.014,175,24	$.0^3$1,112,690	.282,823,6	.014,175,24	$.0^4$23,022	.113,342,0
17	.017,002,31	$.0^3$1,672,161	.738,795,0	.017,002,31	$.0^3$1,366,615	.540,445,6	.017,002,31	$.0^3$1,152,560	.339,124,1	.017,002,31	$.0^4$31,171	.135,937,5
18	.020,182,23	$.0^3$1,893,778	.875,638,4	.020,182,23	$.0^3$1,487,670	.641,003,9	.020,182,23	$.0^3$1,202,986	.402,417,5	.020,182,23	$.0^4$41,480	.161,350,0
19	.023,735,71	.001,170,180	1.028,157	.023,735,71	$.0^3$1,638,731	.753,218,7	.023,735,71	$.0^3$1,265,935	.473,106,4	.023,735,71	$.0^4$54,352	.189,744,3
20	.027,683,48	.001,510,887	1.197,130	.027,683,48	$.0^3$1,825,041	.877,700,9	.027,683,48	$.0^3$1,343,603	.551,592,0	.027,683,48	$.0^4$70,239	.221,285,2
21	.032,046,25	.001,926,434	1.383,321	.032,046,25	.001,052,408	1.015,056	.032,046,25	$.0^3$1,438,425	.638,274,0	.032,046,25	$.0^4$89,636	.256,136,9
22	.036,844,74	.002,428,435	1.587,479	.036,844,74	.001,327,250	1.165,882	.036,844,74	$.0^3$1,553,096	.733,550,1	.036,844,74	$.0^3$1,113,101	.294,463,9
23	.042,099,66	.003,029,610	1.810,337	.042,099,66	.001,656,602	1.330,772	.042,099,66	$.0^3$1,690,572	.837,816,6	.042,099,66	$.0^3$1,141,239	.336,430,0
24	.047,831,71	.003,743,841	2.052,609	.047,831,71	.002,048,159	1.510,312	.047,831,71	$.0^3$1,854,090	.951,468,0	.047,831,71	$.0^3$1,174,717	.382,199,3
25	.054,061,57	.004,586,214	2.314,995	.054,061,57	.002,510,291	1.705,080	.054,061,57	.001,047,175	1.074,897	.054,061,57	$.0^3$1,214,258	.431,935,4
26	.060,809,95	.005,573,052	2.598,175	.060,809,95	.003,052,080	1.915,648	.060,809,95	.001,273,658	1.208,493	.060,809,95	$.0^3$1,260,653	.485,801,8
27	.068,097,52	.006,721,967	2.902,809	.068,097,52	.003,683,335	2.142,579	.068,097,52	.001,537,683	1.352,646	.068,097,52	$.0^3$1,314,754	.543,961,6
28	.075,944,95	.008,051,894	3.229,540	.075,944,95	.004,414,629	2.386,429	.075,944,95	.001,843,716	1.507,742	.075,944,95	$.0^3$1,377,481	.606,577,9
29	.084,372,92	.009,583,134	3.578,988	.084,372,92	.005,257,311	2.647,745	.084,372,92	.002,196,563	1.674,164	.084,372,92	$.0^3$1,449,823	.673,813,5
30	.093,402,09	.011,337,37	3.951,755	.093,402,09	.006,223,558	2.927,067	.093,402,09	.002,601,396	1.852,296	.093,402,09	$.0^3$1,532,858	.745,831,0
31	.103,053,1	.013,337,75	4.348,420	.103,053,1	.007,326,362	3.224,924	.103,053,1	.003,063,729	2.042,515	.103,053,1	$.0^3$1,627,72	.822,792,4
32	.113,346,6	.015,608,84	4.769,537	.113,346,6	.008,579,557	3.541,839	.113,346,6	.003,589,45	2.245,200	.113,346,6	$.0^3$1,735,63	.904,859,8
33	.124,303,3	.018,176,77	5.215,651	.124,303,3	.009,997,893	3.878,323	.124,303,3	.004,184,82	2.460,724	.124,303,3	$.0^3$1,857,87	.992,195,1
34	.135,943,7	.021,069,12	5.687,261	.135,943,6	.011,596,99	4.234,877	.135,943,6	.004,856,55	2.689,460	.135,943,6	$.0^3$1,995,86	1.084,960
35	.148,288,5	.024,315,11	6.184,864	.148,288,5	.013,393,42	4.611,997	.148,288,5	.005,611,67	2.931,777	.148,288,5	.001,151,02	1.183,314
36	.161,358,3	.027,945,47	6.708,918	.161,358,3	.015,404,66	5.010,163	.161,358,3	.006,457,71	3.188,042	.161,358,3	.001,324,92	1.287,420
37	.175,173,6	.031,992,62	7.259,863	.175,173,6	.017,649,22	5.429,848	.175,173,6	.007,402,61	3.458,617	.175,173,6	.001,519,25	1.397,438
38	.189,755,2	.036,490,54	7.838,113	.189,755,2	.020,146,54	5.871,513	.189,755,2	.008,454,70	3.743,865	.189,755,2	.001,735,71	1.513,527
39	.205,123,6	.041,474,91	8.444,055	.205,123,6	.022,917,08	6.335,610	.205,123,6	.009,622,81	4.044,142	.205,123,6	.001,976,15	1.635,848
40	.221,299,2	.046,983,06	9.078,050	.221,299,2	.025,982,34	6.822,577	.221,299,2	.010,916,23	4.359,804	.221,299,2	.002,242,50	1.764,561
41	.238,302,8	.053,054,05	9.740,432	.238,302,8	.029,364,87	7.332,844	.238,302,8	.012,344,69	4.691,202	.238,302,8	.002,536,78	1.899,824
42	.256,154,9	.059,728,64	10.431,51	.256,154,9	.033,088,27	7.866,826	.256,154,9	.013,918,44	5.038,685	.256,154,9	.002,861,16	2.041,796
43	.274,875,9	.067,049,26	11.151,56	.274,875,9	.037,177,21	8.424,930	.274,875,9	.015,648,19	5.402,598	.274,875,9	.003,217,86	2.190,636
44	.294,486,5	.075,060,18	11.900,83	.294,486,5	.041,657,51	9.007,548	.294,486,5	.017,545,19	5.783,281	.294,486,5	.003,609,25	2.346,503
45	.315,007,1	.083,807,36	12.679,54	.315,007,1	.046,556,05	9.615,060	.315,007,1	.019,621,15	6.181,074	.315,007,1	.004,037,78	2.509,553
46	.336,458,3	.093,338,54	13.487,90	.336,458,3	.051,900,88	10.247,83	.336,458,3	.021,888,37	6.596,312	.336,458,3	.004,506,02	2.679,943
47	.358,860,5	.103,703,3	14.326,06	.358,860,5	.057,721,28	10.906,23	.358,860,5	.024,359,71	7.029,326	.358,860,5	.005,016,74	2.857,832
48	.382,234,3	.114,952,6	15.194,15	.382,234,3	.064,047,23	11.590,58	.382,234,3	.027,048,23	7.480,444	.382,234,3	.005,572,56	3.043,376
49	.406,600,0	.127,139,8	16.092,29	.406,600,0	.070,910,75	12.301,22	.406,600,0	.029,968,25	7.949,989	.406,600,0	.006,176,74	3.236,730
50	.431,978,3	.140,319,4	17.020,54	.431,978,3	.078,344,04	13.038,47	.431,978,3	.033,133,70	8.438,282	.431,978,3	.006,831,83	3.438,051
51	.458,389,4	.154,548,4	17.978,94	.458,389,4	.086,381,35	13.802,63	.458,389,4	.036,560,05	8.945,641	.458,389,4	.007,541,42	3.647,493
52	.485,853,9	.169,884,9	18.967,51	.485,853,9	.095,057,74	14.593,98	.485,853,9	.040,262,80	9.472,375	.485,853,9	.008,308,77	3.865,212
53	.514,392,2	.186,389,0	19.986,23	.514,392,2	.104,409,4	15.412,81	.514,392,2	.044,257,94	10.018,79	.514,392,2	.009,137,14	4.091,362
54	.544,024,6	.204,122,7	21.035,04	.544,024,6	.114,474,1	16.259,37	.544,024,6	.048,562,54	10.585,20	.544,024,6	.010,030,28	4.326,096
55	.574,771,6	.223,149,7	22.113,87	.574,771,6	.125,290,7	17.133,91	.574,771,6	.053,193,99	11.171,90	.574,771,6	.010,991,96	4.569,569
56	.606,653,5	.243,535,0	23.222,60	.606,653,5	.136,898,7	18.036,66	.606,653,5	.058,169,75	11.779,19	.606,653,5	.012,025,49	4.821,934
57	.639,690,7	.265,346,0	24.361,08	.639,690,7	.149,339,9	18.967,84	.639,690,7	.063,509,05	12.407,35	.639,690,7	.013,135,37	5.083,343
58	.673,903,6	.288,651,5	25.529,12	.673,903,6	.162,656,8	19.927,65	.673,903,6	.069,230,92	13.056,68	.673,903,6	.014,325,56	5.353,947
59	.709,312,5	.313,521,9	26.726,53	.709,312,5	.176,893,3	20.916,27	.709,312,5	.075,355,36	13.727,46	.709,312,5	.015,600,42	5.633,899
60	.745,937,6	.340,029,5	27.953,05	.745,937,6	.192,094,4	21.933,89	.745,937,6	.081,902,86	14.419,97	.745,937,6	.016,964,25	5.923,349
61	.783,799,4	.368,247,7	29.208,41	.783,799,4	.208,306,5	22.980,65	.783,799,4	.088,894,57	15.13448	.783,799,4	.018,421,55	6.222,449
62	.822,918,1	.398,252,4	30.492,28	.822,918,1	.225,577,4	24.056,70	.822,918,1	.096,352,42	15.871,25	.822,918,1	.019,977,19	6.531,348
63	.863,313,9	.430,120,3	31.804,34	.863,313,9	.243,955,9	25.162,16	.863,313,9	.104,298,9	16.630,57	.863,313,9	.021,635,91	6.850,195
64	.905,007,1	.463,930,1	33.144,21	.905,007,1	.263,492,5	26.297,15	.905,007,1	.112,757,0	17.412,68	.905,007,1	.023,402,75	7.179,140
65	.948,018,1	.499,762,0	34.511,47	.948,018,1	.284,238,6	27.461,76	.948,018,1	.121,751,2	18.217,85	.948,018,1	.025,282,92	7.518,330
66	.992,366,9	.537,697,5	35.905,67	.992,366,9	.306,247,1	28.656,06	.992,366,9	.131,305,7	19.046,32	.992,366,9	.027,281,75	7.867,915
67	1.038,074	.577,819,8	37.326,36	1.038,074	.329,571,9	29.880,12	1.038,074	.141,445,6	19.898,34	1.038,074	.029,404,60	8.228,041
68	1.085,159	.620,213,4	38.773,00	1.085,159	.354,268,9	31.133,98	1.085,159	.152,197,2	20.774,15	1.085,159	.031,657,4	8.598,854
69	1.133,643	.664,963,9	40.245,07	1.133,642	.380,394,2	32.417,68	1.133,642	.163,586,8	21.673,99	1.133,642	.034,045,7	8.980,302
70	1.183,545	.712,159,3	41.741,99	1.183,545	.408,006,5	33.731,22	1.183,545	.175,642,3	22.598,08	1.183,545	.036,575,7	9.373,128
71	1.234,886	.761,887,3	43.263,15	1.234,886	.437,164,5	35.074,60	1.234,886	.188,391,4	23.546,66	1.234,886	.039,253,4	9.776,880
72	1.287,685	.814,238,3	44.807,91	1.287,685	.467,929,0	36.447,81	1.287,685	.201,863,2	24.519,95	1.287,685	.042,085,2	10.191,90
73	1.341,964	.869,303,2	46.375,59	1.341,964	.500,361,9	37.850,80	1.341,964	.216,087,1	25.518,15	1.341,964	.045,077,7	10.618,33
74	1.397,742	.927,174,2	47.965,49	1.397,742	.534,526,1	39.283,52	1.397,742	.231,093,8	26.541,49	1.397,742	.048,237,5	11.056,32
75	1.455,039	.987,944,7	49.576,87	1.455,039	.570,486,1	40.745,90	1.455,039	.246,913,8	27.590,16	1.455,039	.051,571,5	11.506,01
76	1.513,875	1.051,709	51.208,97	1.513,875	.608,307,5	42.237,85	1.513,875	.263,579,2	28.664,36	1.513,875	.055,086,6	11.967,54
77	1.574,270	1.118,563	52.860,97	1.574,270	.648,057,0	43.759,26	1.574,270	.281,122,5	29.764,31	1.574,270	.058,790,2	12.441,05
78	1.636,244	1.188,602	54.532,04	1.636,244	.689,802,8	45.310,01	1.636,244	.299,577,1	30.890,16	1.636,244	.062,689,7	12.926,68
79	1.699,818	1.261,923	56.221,32	1.699,818	.733,614,3	46.889,97	1.699,818	.318,976,9	32.042,12	1.699,818	.066,792,7	13.424,57
80	1.765,010	1.338,626	57.927,91	1.765,010	.779,561,7	48.498,98	1.765,010	.339,357,0	33.220,36	1.765,010	.071,106,9	13.934,87
81	1.831,842	1.418,809	59.650,89	1.831,842	.827,717,0	50.136,86	1.831,842	.360,752,6	34.425,05	1.831,842	.075,640,4	14.457,70
82	1.900,332	1.502,570	61.389,29	1.900,332	.878,152,9	51.803,42	1.900,332	.383,200,5	35.656,36	1.900,332	.080,401,3	14.993,21
83	1.970,502	1.590,011	63.142,13	1.970,502	.930,943,7	53.498,45	1.970,502	.406,737,6	36.914,45	1.970,502	.085,398,0	15.541,54
84	2.042,370	1.681,232	64.908,38	2.042,370	.986,164,3	55.221,73	2.042,370	.431,402,0	38.199,47	2.042,370	.090,639,2	16.102,82
85	2.115,956	1.776,334	66.687,01	2.115,956	1.043,891	56.973,01	2.115,956	.457,232,1	39.511,58	2.115,956	.096,133,4	16.677,19
86	2.191,281	1.875,420	68.476,93	2.191,281	1.104,202	58.752,03	2.191,281	.484,267,7	40.850,91	2.191,281	.101,889,7	17.264,78
87	2.268,365	1.978,590	70.277,04	2.268,365	1.167,175	60.558,51	2.268,365	.512,549,0	42.217,61	2.268,365	.107,917,4	17.865,73
88	2.347,226	2.085,947	72.086,20	2.347,226	1.232,890	62.392,16	2.347,226	.542,117,0	43.611,81	2.347,226	.114,225,7	18.480,17
89	2.427,885	2.197,594	73.903,25	2.427,885	1.301,428	64.252,66	2.427,885	.573,013,4	45.033,63	2.427,885	.120,824,2	19.108,23
90	2.510,362	2.313,633	75.727,01	2.510,362	1.372,871	66.139,68	2.510,362	.605,281,3	46.483,19	2.510,362	.127,722,8	19.750,05

TABLE 17—Continued

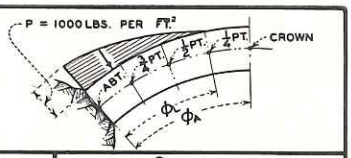
UNIT RADIAL LOAD NO. 3
TWIST AND TANGENTIAL DEFLECTIONS



φ	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_3		D_1	D_3		D_1	D_3		D_1	D_3	
°		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.027,683,48	$.0^3$343,603	.551,592,0	.027,683,48	$.0^3$180,744	.386,806,7	.027,683,48	$.0^4$70,239	.221,285,1	.008,652,514	$.0^5$5,052	.048,443,45
11	.036,844,74	$.0^3$553,096	.733,550,1	.036,844,74	$.0^3$291,002	.514,599,6	.036,844,74	$.0^3$1 13,101	.294,463,9	.011,516,28	$.0^5$8,135	.064,474,06
12	.047,831,71	$.0^3$854,090	.951,468,0	.047,831,71	$.0^3$449,468	.667,748,8	.047,831,71	$.0^3$1 74,717	.382,199,3	.014,950,96	$.0^4$12,568	.083,698,93
13	.060,809,95	.001,273,658	1.208,493	.060,809,95	$.0^3$670,432	.848,512,2	.060,809,95	$.0^3$260,653	.485,801,8	.019,008,41	$.0^4$18,751	.106,407,6
14	.075,944,95	.001,843,716	1.507,742	.075,944,95	$.0^3$970,757	1.059,136	.075,944,95	$.0^3$377,481	.606,577,9	.023,740,47	$.0^4$27,160	.132,889,5
15	.093,402,09	.002,601,396	1.852,296	.093,402,09	.001,370,08	1.301,851	.093,402,09	$.0^3$532,858	.745,831,0	.029,198,92	$.0^4$38,347	.163,433,7
16	.113,346,6	.003,589,45	2.245,200	.113,346,6	.001,891,04	1.578,876	.113,346,6	$.0^3$735,63	.904,859,8	.035,435,79	$.0^4$52,947	.198,329,0
17	.135,943,7	.004,856,55	2.689,460	.135,943,7	.002,559,42	1.892,414	.135,943,7	$.0^3$995,86	1.084,960	.042,502,66	$.0^4$71,689	.237,864,0
18	.161,358,3	.006,457,71	3.188,042	.161,358,3	.003,404,40	2.244,650	.161,358,3	.001,324,92	1.287,420	.050,451,42	$.0^4$95,397	.282,327,2
19	.189,755,2	.008,454,70	3.743,865	.189,755,2	.004,458,81	2.637,755	.189,755,2	.001,735,71	1.513,527	.059,333,83	$.0^3$1,24,998	.332,006,4
20	.221,299,2	.010,916,23	4.359,804	.221,299,2	.005,759,17	3.073,879	.221,299,2	.002,242,50	1.764,561	.069,201,66	$.0^3$1,61,529	.387,189,4
21	.256,154,9	.013,918,44	5.038,685	.256,154,9	.007,346,03	3.555,155	.256,154,9	.002,861,16	2.041,796	.080,106,66	$.0^3$2,06,143	.448,163,5
22	.294,486,5	.017,545,19	5.783,281	.294,486,5	.009,264,11	4.083,692	.294,486,5	.003,609,25	2.346,503	.092,100,54	$.0^3$2,60,095	.515,215,7
23	.336,458,3	.021,888,37	6.596,312	.336,458,3	.011,562,83	4.661,583	.336,458,3	.004,506,02	2.679,943	.105,235,0	$.0^3$3,24,80	.588,632,6
24	.382,234,3	.027,048,23	7.480,444	.382,234,3	.014,294,78	5.290,894	.382,234,3	.005,572,56	3.043,376	.119,561,8	$.0^3$4,01,79	.668,700,5
25	.431,978,3	.033,133,70	8.438,282	.431,978,3	.017,519,37	5.973,672	.431,978,3	.006,831,83	3.438,051	.135,132,5	$.0^3$4,92,71	.755,705,0
26	.485,853,9	.040,262,80	9.472,375	.485,853,9	.021,299,53	6.711,939	.485,853,9	.008,308,77	3.865,212	.151,998,8	$.0^3$5,99,38	.849,931,7
27	.544,024,6	.048,562,54	10.585,20	.544,024,6	.025,703,65	7.507,690	.544,024,6	.010,030,28	4.326,096	.170,212,3	$.0^3$7,23,78	.951,665,6
28	.606,653,5	.058,169,75	11.779,19	.606,653,5	.030,805,32	8.362,897	.606,653,5	.012,025,49	4.821,934	.189,824,6	$.0^3$8,68,01	1.061,191
29	.673,903,6	.069,230,92	13.056,68	.673,903,6	.036,683,64	9.279,505	.673,903,6	.014,325,56	5.353,947	.210,887,3	.001,034,35	1.178,792
30	.745,937,6	.081,902,86	14.419,97	.745,937,6	.043,423,47	10.259,43	.745,937,6	.016,964,25	5.923,349	.233,451,9	.001,225,27	1.304,752
31	.822,918,1	.096,352,42	15.871,25	.822,918,1	.051,115,08	11.304,56	.822,918,1	.019,977,19	6.531,348	.257,569,9	.001,443,34	1.439,354
32	.905,007,1	.112,757,0	17.412,68	.905,007,1	.059,854,87	12.416,75	.905,007,1	.023,402,75	7.179,140	.283,292,9	.001,691,44	1.582,881
33	.992,366,9	.131,305,7	19.046,32	.992,366,9	.069,745,85	13.597,84	.992,366,9	.027,281,75	7.867,915	.310,672,3	.001,972,50	1.735,614
34	1.085,159	.152,197,2	20.774,15	1.085,159	.080,896,32	14.849,61	1.085,159	.031,657,4	8.598,854	.339,759,4	.002,289,70	1.897,834
35	1.183,545	.175,642,3	22.598,08	1.183,545	.093,421,60	16.173,85	1.183,545	.036,575,7	9.373,128	.370,605,9	.002,646,42	2.069,824
36	1.287,685	.201,863,2	24.519,95	1.287,685	.107,443,5	17.572,27	1.287,685	.042,085,2	10.191,90	.403,262,9	.003,046,21	2.251,862
37	1.397,742	.231,093,8	26.541,49	1.397,742	.123,090,5	19.046,58	1.397,742	.048,237,5	11.056,32	.437,781,9	.003,492,93	2.444,228
38	1.513,875	.263,579,2	28.664,36	1.513,875	.140,497,8	20.598,45	1.513,875	.055,086,6	11.967,54	.474,214,1	.003,990,51	2.647,201
39	1.636,244	.299,577,1	30.890,16	1.636,244	.159,807,8	22.229,50	1.636,244	.062,689,7	12.926,68	.512,610,9	.004,543,18	2.861,059
40	1.765,010	.339,357,0	33.220,36	1.765,010	.181,169,8	23.941,33	1.765,010	.071,106,9	13.934,87	.553,023,4	.005,155,40	3.086,081
41	1.900,332	.383,200,5	35.656,36	1.900,332	.204,740,5	25.735,51	1.900,332	.080,401,3	14.993,21	.595,502,8	.005,831,83	3.322,542
42	2.042,370	.431,402,0	38.199,47	2.042,370	.230,683,7	27.613,54	2.042,370	.090,639,2	16.102,82	.640,100,4	.006,577,38	3.570,719
43	2.191,281	.484,267,7	40.850,91	2.191,281	.259,170,9	29.576,91	2.191,281	.101,889,7	17.264,78	.686,867,3	.007,397,23	3.830,888
44	2.347,226	.542,117,0	43.611,81	2.347,226	.290,381,3	31.627,07	2.347,226	.114,225,7	18.480,17	.735,854,4	.008,296,67	4.103,323
45	2.510,362	.605,281,3	46.483,19	2.510,362	.324,501,3	33.765,42	2.510,362	.127,722,8	19.750,05	.787,112,9	.009,281,54	4.388,298
46	2.680,847	.674,105,1	49.465,99	2.680,847	.361,725,6	35.993,32	2.680,847	.142,460,3	21.075,49	.840,693,9	.010,357,63	4.686,086
47	2.858,839	.748,945,5	52.561,11	2.858,839	.402,256,7	38.312,09	2.858,839	.158,520,8	22.457,52	.896,648,2	.011,531,12	4.996,960
48	3.044,494	.830,172,4	55.769,09	3.044,494	.446,304,8	40.723,03	3.044,494	.175,990,2	23.897,18	.955,026,8	.012,808,54	5.321,192
49	3.237,969	.918,168,1	59.090,77	3.237,969	.494,088,1	43.227,36	3.237,969	.194,958,4	25.395,48	1.015,891	.014,196,6	5.659,052
50	3.439,421	1.013,329	62.526,62	3.439,421	.545,833,8	45.826,29	3.439,421	.215,518,3	26.953,43	1.079,260	.015,702,0	6.010,809
51	3.649,006	1.116,027	66.077,05	3.649,006	.601,776,6	48.520,97	3.649,006	.237,766,7	28.572,03	1.145,217	.017,332,5	6.376,735
52	3.866,879	1.226,755	69.742,49	3.866,879	.662,159,4	51.312,50	3.866,879	.261,803,8	30.252,25	1.213,801	.019,095,4	6.757,095
53	4.093,196	1.345,942	73.523,06	4.093,196	.727,233,7	54.201,96	4.093,196	.287,733,8	31.995,07	1.285,064	.020,998,7	7.152,158
54	4.328,110	1.473,970	77.418,94	4.328,110	.797,259,8	57.190,36	4.328,110	.315,664,9	33.801,43	1.359,055	.023,050,5	7.562,190
55	4.571,776	1.611,330	81.430,14	4.571,776	.872,506,0	60.278,67	4.571,776	.345,708,6	35.672,27	1.435,826	.025,259,4	7.987,457
56	4.824,348	1.758,495	85.556,58	4.824,348	.953,249,4	63.467,82	4.824,348	.377,980,6	37.608,53	1.515,427	.027,633,9	8.428,224
57	5.085,980	1.915,947	89.798,07	5.085,980	1.039,776	66.758,69	5.085,980	.412,600,8	39.611,11	1.597,908	.030,183,5	8.884,753
58	5.356,824	2.084,183	94.154,32	5.356,824	1.132,379	70.152,13	5.356,824	.449,692,1	41.680,91	1.683,321	.032,917,4	9.357,308
59	5.637,032	2.263,711	98.624,92	5.637,032	1.231,362	73.648,90	5.637,032	.489,381,9	43.818,82	1.771,714	.035,845,6	9.846,150
60	5.926,757	2.455,053	103.209,4	5.926,757	1.337,037	77.249,75	5.926,757	.531,802,4	46.025,69	1.863,140	.038,977,8	10.351,54
61	6.226,150	2.658,738	107.907,0	6.226,150	1.449,723	80.955,37	6.226,150	.577,089,0	48.302,40	1.957,648	.042,324,8	10.873,74
62	6.535,362	2.875,313	112.717,2	6.535,362	1.569,750	84.766,40	6.535,362	.625,381,4	50.649,77	2.055,288	.045,897,4	11.413,00
63	6.854,543	3.105,332	117.639,0	6.854,543	1.697,456	88.683,42	6.854,543	.676,823,7	53.068,63	2.156,111	.049,706,6	11.969,59
64	7.183,843	3.349,362	122.671,6	7.183,843	1.833,188	92.706,99	7.183,843	.731,564,2	55.559,79	2.260,166	.053,763,8	12.543,76
65	7.523,413	3.607,980	127.813,8	7.523,413	1.977,299	96.837,59	7.523,413	.789,754,8	58.124,04	2.367,504	.058,081,0	13.135,76
66	7.873,400	3.881,776	133.064,5	7.873,400	2.130,156	101.075,6	7.873,400	.851,552,8	60.762,15	2.478,174	.062,670,5	13.745,85
67	8.233,954	4.171,348	138.422,4	8.233,954	2.292,130	105.421,6	8.233,954	.917,119,0	63.474,89	2.592,228	.067,544,6	14.374,29
68	8.605,221	4.477,306	143.886,2	8.605,221	2.463,603	109.875,7	8.605,221	.986,618,8	66.263,00	2.709,713	.072,716,5	15.021,32
69	8.987,350	4.800,269	149.454,4	8.987,350	2.644,964	114.438,3	8.987,350	1.060,222	69.127,22	2.830,682	.078,199,5	15.687,20
70	9.380,488	5.140,868	155.125,4	9.380,488	2.836,615	119.109,6	9.380,488	1.138,103	72.068,25	2.955,182	.084,007,2	16.372,17
71	9.784,780	5.499,740	160.897,4	9.784,780	3.038,961	123.889,8	9.784,780	1.220,440	75.086,79	3.083,264	.090,153,8	17.076,49
72	10.200,37	5.877,534	166.768,7	10.200,37	3.252,418	128.779,0	10.200,37	1.307,415	78.183,52	3.214,978	.096,653,8	17.800,41
73	10.627,41	6.274,907	172.737,3	10.627,41	3.477,411	133.777,3	10.627,41	1.399,218	81.359,10	3.350,373	.103,522,1	18.544,16
74	11.066,04	6.692,523	178.801,2	11.066,04	3.714,374	138.884,8	11.066,04	1.496,039	84.614,18	3.489,499	.110,774,0	19.307,99
75	11.516,40	7.131,056	184.958,6	11.516,40	3.963,748	144.101,3	11.516,40	1.598,076	87.949,39	3.632,404	.118,425,1	20.092,15
76	11.978,63	7.591,187	191.206,5	11.978,63	4.225,983	149.426,9	11.978,63	1.705,530	91.365,33	3.779,139	.126,491,7	20.896,88
77	12.452,89	8.073,604	197.543,2	12.452,89	4.501,537	154.861,3	12.452,89	1.818,605	94.862,60	3.929,753	.134,990,2	21.722,42
78	12.939,31	8.579,004	203.966,1	12.939,31	4.790,876	160.404,5	12.939,31	1.937,513	98.441,78	4.084,295	.143,937,6	22.569,02
79	13.438,03	9.108,085	210.472,6	13.438,03	5.094,475	166.056,1	13.438,03	2.062,467	102.1034	4.242,813	.153,351,3	23.436,90
80	13.949,20	9.661,558	217.060,1	13.949,20	5.412,817	171.815,8	13.949,20	2.193,688	105.848,1	4.405,359	.163,249,2	24.326,31
81	14.472,95	10.240,14	223.725,8	14.472,95	5.746,392	177.683,4	14.472,95	2.331,402	109.676,3	4.571,979	.173,649,4	25.237,48
82	15.009,43	10.844,54	230.466,9	15.009,43	6.095,698	183.658,4	15.009,43	2.475,834	113.588,5	4.742,724	.184,570,5	26.170,66
83	15.558,77	11.475,48	237.280,4	15.558,77	6.461,243	189.740,3	15.558,77	2.627,221	117.585,3	4.917,642	.196,031,9	27.126,06
84	16.121,11	12.133,70	244.163,4	16.121,11	6.843,538	195.928,7	16.121,11	2.785,797	121.667,0	5.096,781	.208,053,0	28.103,93
85	16.696,59	12.819,93	251.1125	16.696,59	7.243,106	202.222,9	16.696,59	2.951,807	125.834,2	5.280,192	.220,653,9	29.104,49
86	17.285,35	13.534,90	258.124,7	17.285,35	7.660,475	208.622,4	17.285,35	3.125,498	130.087,3	5.467,922	.233,855,1	30.127,98
87	17.887,52	14.279,34	265.196,5	17.887,52	8.096,183	215.126,5	17.887,52	3.307,124	134.426,6	5.660,020	.247,677,3	31.174,62
88	18.503,23	15.054,00	272.324,6	18.503,23	8.550,768	221.734,5	18.503,23	3.496,937	138.852,7	5.856,534	.262,142,3	32.244,63
89	19.132,63	15.859,62	279.505,4	19.132,63	9.024,785	228.445,6	19.132,63	3.695,204	143.365,8	6.057,513	.277,271,7	33.338,25
90	19.775,85	16.696,94	286.735,4	19.775,85	9.518,790	235.258,9	19.775,85	3.902,186	147.966,4	6.263,006	.293,087,8	34.455,70

TABLE 18—Continued

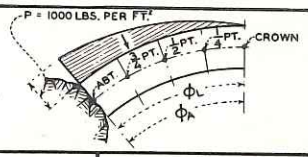
UNIT RADIAL LOAD NO. 4
TWIST AND TANGENTIAL DEFLECTIONS



Φ °	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_3		D_1	D_3		D_1	D_3		D_1	D_3	
		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.093,402,09	.001,047,005	1.117,013	.093,402,09	$.0^3$532,858	.745,831,0	.055,362,27	$.0^3$131,099	.331,888,0	.010,382,99	$.0^5$6,003,5	.055,363,59
11	.124,303,3	.001,685,29	1.485,502	.124,303,3	$.0^3$857,87	.992,195,1	.073,681,94	$.0^3$211,097	.441,631,7	.013,819,49	$.0^5$9,668	.073,684,06
12	.161,358,3	.002,602,28	1.926,801	.161,358,3	.001,324,92	1.287,420	.095,651,75	$.0^3$326,094	.573,200,1	.017,941,08	$.0^4$14,936	.095,655,03
13	.205,123,6	.003,880,45	2.447,347	.205,123,6	.001,976,15	1.635,848	.121,602,5	$.0^3$486,47	.728,555,1	.022,809,98	$.0^4$22,284	.121,607,4
14	.256,154,9	.005,616,91	3.053,393	.256,154,9	.002,861,16	2.041,796	.151,864,7	$.0^3$704,50	.909,653,3	.028,488,41	$.0^4$32,278	.151,871,8
15	.315,007,1	.007,924,69	3.751,206	.315,007,1	.004,037,78	2.509,553	.186,768,6	$.0^3$994,47	1.118,445	.035,038,57	$.0^4$45,571	.186,778,6
16	.382,234,3	.010,933,83	4.546,957	.382,234,3	.005,572,56	3.043,376	.226,644,2	.001,372,84	1.356,874	.042,522,64	$.0^4$62,922	.226,658,0
17	.458,389,4	.014,792,45	5.446,741	.458,389,4	.007,541,42	3.647,493	.271,820,9	.001,858,42	1.626,876	.051,002,78	$.0^4$85,194	.271,839,6
18	.544,024,6	.019,667,98	6.456,569	.544,024,6	.010,030,28	4.326,096	.322,628,0	.002,472,45	1.930,381	.060,541,15	$.0^3$113,370	.322,652,9
19	.639,690,7	.025,747,95	7.582,365	.639,690,7	.013,135,37	5.083,343	.379,394,5	.003,238,91	2.269,310	.071,199,88	$.0^3$148,548	.379,427,1
20	.745,937,6	.033,241,50	8.829,960	.745,937,6	.016,964,25	5.923,349	.442,448,7	.004,184,43	2.645,574	.083,041,06	$.0^3$191,960	.442,490,8
21	.863,313,9	.042,379,78	10.205,09	.863,313,9	.021,635,91	6.850,195	.512,118,6	.005,338,65	3.061,078	.096,126,80	$.0^3$244,979	.512,172,3
22	.992,366,9	.053,417,65	11.713,37	.992,366,9	.027,281,75	7.867,915	.588,731,8	.006,734,20	3.517,716	.110,519,1	$.0^3$309,09	.588,799,6
23	1.133,643	.066,635,2	13.360,35	1.133,643	.034,045,7	8.980,501	.672,615,4	.008,407,17	4.017,372	.126,280,1	$.0^3$385,99	.672,700,1
24	1.287,685	.082,333,7	15.151,43	1.287,685	.042,085,2	10.191,90	.764,096,0	.010,396,54	4.561,920	.143,471,8	$.0^3$477,47	.764,200,8
25	1.455,039	.100,846,9	17.091,90	1.455,039	.051,571,5	11.506,01	.863,499,7	.012,745,33	5.153,223	.162,156,1	$.0^3$585,52	.863,628,2
26	1.636,244	.122,531,2	19.186,95	1.636,244	.062,689,7	12.926,68	.971,152,1	.015,499,88	5.793,134	.182,395,1	$.0^3$712,30	.971,308,2
27	1.831,842	.147,772,6	21.441,63	1.831,842	.075,640,4	14.457,70	1.087,378	.018,710,4	6.483,493	.204,250,5	$.0^3$860,13	1.087,567
28	2.042,370	.176,985,5	23.860,87	2.042,370	.090,639,2	16.102,82	1.212,502	.022,431,2	7.226,127	.227,784,5	.001,031,52	1.212,729
29	2.268,365	.210,613,6	26.449,45	2.268,365	.107,917,4	17.865,73	1.346,848	.026,720,1	8.022,854	.253,058,7	.001,229,20	1.347,118
30	2.510,362	.249,131,7	29.212,04	2.510,362	.127,722,8	19.750,05	1.490,739	.031,640,0	8.875,476	.280,135,2	.001,456,07	1.491,059
31	2.768,895	.293,045,1	32.153,15	2.768,895	.150,319,7	21.759,37	1.644,498	.037,257,36	9.785,783	.309,075,5	.001,715,208	1.644,875
32	3.044,494	.342,891,2	35.277,15	3.044,494	.175,990,2	23.897,18	1.808,446	.043,643,47	10.755,55	.339,941,7	.002,010,049	1.808,888
33	3.337,688	.399,239,9	38.588,28	3.337,688	.205,033,5	26.166,93	1.982,905	.050,874,24	11.786,54	.372,795,2	.002,344,044	1.983,420
34	3.649,006	.462,693,6	42.090,59	3.649,006	.237,766,7	28.572,03	2.168,195	.059,030,13	12.880,50	.407,698,2	.002,721,162	2.168,792
35	3.978,972	.533,888,4	45.788,02	3.978,972	.274,525,4	31.115,77	2.364,636	.068,196,70	14.039,16	.444,711,7	.003,144,89	2.365,327
36	4.328,110	.613,495,8	49.684,31	4.328,110	.315,664,9	33.801,43	2.572,547	.078,464,30	15.264,24	.483,897,8	.003,619,98	2.573,342
37	4.696,940	.702,220,4	53.783,07	4.696,940	.361,558,9	36.632,17	2.792,246	.089,928,47	16.557,43	.525,318,0	.004,150,82	2.793,158
38	5.085,980	.800,802,8	58.087,74	5.085,980	.412,600,8	39.611,11	3.024,051	.102,690,0	17.920,43	.569,033,8	.004,742,10	3.025,092
39	5.495,748	.910,018,9	62.601,57	5.495,748	.469,204,0	42.741,29	3.268,278	.116,855,0	19.354,90	.615,106,7	.005,398,86	3.269,463
40	5.926,757	1.030,680	67.327,66	5.926,757	.531,802,4	46.025,69	3.525,242	.132,535,0	20.862,50	.663,598,1	.006,126,36	3.526,587
41	6.379,519	1.163,636	72.268,95	6.379,519	.600,850,5	49.467,20	3.795,260	.149,847,4	22.444,84	.714,569,5	.006,930,17	3.796,782
42	6.854,543	1.309,770	77.428,16	6.854,543	.676,823,7	53.068,63	4.078,644	.168,915,0	24.103,56	.768,082,2	.007,816,12	4.080,360
43	7.352,335	1.470,004	82.807,88	7.352,335	.760,218,5	56.832,73	4.375,708	.189,866,6	25.840,25	.824,197,7	.008,790,35	4.377,639
44	7.873,400	1.645,297	88.410,49	7.873,400	.851,552,8	60.762,15	4.686,765	.212,836,8	27.656,48	.882,977,0	.009,859,15	4.688,930
45	8.418,239	1.836,644	94.238,18	8.418,239	.951,366,8	64.859,48	5.012,124	.237,966,2	29.553,82	.944,481,6	.011,029,46	5.014,546
46	8.987,350	2.045,079	100.293,0	8.987,350	1.060,222	69.127,22	5.352,098	.265,401,8	31.533,80	1.008,772	.012,308,13	5.354,800
47	9.581,230	2.271,673	106.576,7	9.581,230	1.178,703	73.567,79	5.706,994	.295,296,9	33.597,96	1.075,910	.013,702,58	5.710,003
48	10.200,37	2.517,532	113.091,1	10.200,37	1.307,415	78.183,72	6.077,121	.327,810,6	35.747,78	1.145,958	.015,220,50	6.080,464
49	10.845,26	2.783,803	119.837,4	10.845,26	1.446,989	82.976,66	6.462,788	.363,108,9	37.984,75	1.218,974	.016,869,85	6.466,493
50	11.516,40	3.071,668	126.817,1	11.516,40	1.598,076	87.949,39	6.864,298	.401,364,5	40.310,34	1.295,021	.018,658,81	6.868,396
51	12.214,25	3.382,348	134.031,1	12.214,25	1.761,351	93.103,76	7.281,958	.442,756,1	42.725,97	1.374,160	.020,596,21	7.286,482
52	12.939,31	3.717,098	141.480,3	12.939,31	1.937,513	98.441,78	7.716,073	.487,470,2	45.233,06	1.456,450	.022,690,99	7.721,055
53	13.692,05	4.077,212	149.165,4	13.692,05	2.127,281	103.965,4	8.166,943	.535,698,9	47.833,02	1.541,954	.024,952,6	8.172,424
54	14.472,95	4.464,022	157.086,8	14.472,95	2.331,402	109.676,3	8.634,872	.587,642,0	50.527,22	1.630,732	.027,390,6	8.640,888
55	15.282,48	4.878,895	165.244,9	15.282,48	2.550,644	115.576,3	9.120,161	.643,506,0	53.317,00	1.722,843	.030,015,2	9.126,753
56	16.121,11	5.323,233	173.639,7	16.121,11	2.785,797	121.667,0	9.623,108	.703,505,1	56.203,70	1.818,351	.032,836,8	9.630,320
57	16.989,30	5.798,470	182.271,1	16.989,30	3.037,676	127.950,0	10.144,01	.767,859,5	59.188,61	1.917,314	.035,866,2	10.151,89
58	17.887,52	6.306,088	191.138,8	17.887,52	3.307,124	134.426,6	10.683,17	.836,797	62.273,04	2.019,793	.039,114,8	10.691,76
59	18.816,21	6.847,587	200.242,1	18.816,21	3.594,997	141.098,4	11.240,88	.910,554	65.458,23	2.125,848	.042,593,9	11.250,23
60	19.775,85	7.424,515	209.579,4	19.775,85	3.902,186	147.966,4	11.817,43	.989,374	68.745,41	2.235,541	.046,315,6	11.827,60
61	20.766,90	8.038,460	219.152,8	20.766,90	4.229,599	155.031,8	12.413,12	1.073,504	72.135,86	2.348,931	.050,292,8	12.424,17
62	21.789,75	8.690,985	228.958,2	21.789,75	4.578,169	162.295,9	13.028,24	1.163,205	75.630,60	2.466,078	.054,537,5	13.040,22
63	22.844,90	9.383,777	238.995,1	22.844,90	4.948,851	169.759,4	13.663,08	1.258,740	79.230,96	2.587,043	.059,063,4	13.676,05
64	23.932,77	10.118,49	249.262,1	23.932,77	5.342,629	177.423,3	14.317,93	1.360,384	82.938,00	2.711,885	.063,884,2	14.331,96
65	25.053,81	10.896,84	259.757,3	25.053,81	5.760,502	185.288,4	14.993,08	1.468,417	86.752,86	2.840,666	.069,013,8	15.008,24
66	26.208,44	11.720,55	270.479,0	26.208,44	6.203,500	193.355,3	15.688,81	1.583,126	90.676,61	2.973,444	.074,466,7	15.705,16
67	27.397,09	12.591,38	281.424,8	27.397,09	6.672,670	201.624,6	16.405,41	1.704,810	94.710,32	3.110,279	.080,257,9	16.423,04
68	28.620,19	13.511,12	292.592,6	28.620,19	7.169,085	210.096,9	17.143,16	1.833,771	98.855,02	3.251,232	.086,402,8	17.162,14
69	29.878,16	14.481,60	303.979,7	29.878,16	7.693,842	218.772,5	17.902,35	1.970,321	103.111,7	3.396,361	.092,917,4	17.922,77
70	31.171,42	15.504,66	315.583,5	31.171,42	8.248,057	227.651,7	18.683,26	2.114,782	107.481,4	3.545,728	.099,817,7	18.705,19
71	32.500,39	16.582,16	327.400,9	32.500,39	8.832,871	236.734,7	19.486,16	2.267,479	111.965,0	3.699,391	.107,120,6	19.509,70
72	33.865,47	17.716,00	339.428,9	33.865,47	9.449,447	246.021,7	20.311,34	2.428,751	116.563,5	3.857,409	.114,843,4	20.336,58
73	35.267,06	18.908,10	351.664,2	35.267,06	10.098,96	255.512,4	21.159,07	2.598,939	121.277,8	4.019,843	.123,003,5	21.186,10
74	36.705,58	20.160,40	364.103,3	36.705,58	10.782,64	265.207,1	22.029,62	2.778,397	126.108,7	4.186,751	.131,619,5	22.058,54
75	38.181,41	21.474,86	376.742,4	38.181,41	11.501,68	275.105,3	22.923,28	2.967,485	131.057,1	4.358,192	.140,709,6	22.954,20
76	39.694,95	22.853,45	389.577,7	39.694,95	12.257,36	285.206,8	23.840,30	3.166,571	136.123,8	4.534,227	.150,293,4	23.873,33
77	41.246,60	24.298,18	402.605,1	41.246,60	13.050,93	295.511,2	24.780,97	3.376,032	141.309,7	4.714,914	.160,390,2	24.81 6,21
78	42.836,72	25.811,06	415.820,4	42.836,72	13.883,68	306.018,1	25.745,55	3.596,252	146.615,4	4.900,311	.171,020,2	25.783,13
79	44.465,71	27.394,12	429.219,2	44.465,71	14.756,93	316.726,8	26.734,30	3.827,624	152.041,7	5.090,479	.182,204,2	26.774,34
80	46.133,94	29.049,41	442.796,8	46.133,94	15.671,99	327.636,7	27.747,50	4.070,550	157.589,4	5.285,475	.193,963,3	27.790,12
81	47.841,79	30.778,98	456.548,5	47.841,79	16.630,23	338.747,1	28.785,40	4.325,438	163.259,0	5.485,358	.206,319,0	28.830,74
82	49.589,62	32.584,92	470.469,1	49.589,62	17.633,00	350.056,7	29.848,27	4.592,706	169.051,3	5.690,188	.219,293,5	29.896,47
83	51.377,78	34.469,29	484.553,8	51.377,78	18.681,70	361.564,9	30.936,38	4.872,781	174.966,9	5.900,022	.232,909,8	30.987,57
84	53.206,65	36.434,18	498.797,2	53.206,65	19.777,72	373.270,7	32.049,97	5.166,095	181.006,3	6.114,918	.247,191,0	32.104,30
85	55.076,57	38.481,70	513.193,8	55.076,57	20.922,49	385.172,7	33.189,30	5.473,090	187.170,2	6.334,939	.262,160,7	33.246,92
86	56.987,90	40.613,94	527.738,0	56.987,90	22.117,44	397.269,8	34.354,64	5.794,219	193.459,1	6.560,136	.277,843,5	34.415,71
87	58.940,97	42.833,01	542.424,0	58.940,97	23.364,02	409.560,6	35.546,23	6.129,939	199.873,5	6.790,572	.294,264,1	35.610,91
88	60.936,13	45.141,02	557.246,0	60.936,13	24.663,72	422.043,6	36.764,32	6.480,715	206.413,8	7.026,303	.311,447,8	36.832,78
89	62.973,71	47.540,07	572.197,7	62.973,71	26.018,01	434.717,3	38.009,17	6.847,027	213.080,5	7.267,389	.329,420,8	38.081,59
90	65.054,05	50.032,28	587.273,0	65.054,05	27.428,40	447.580,1	39.281,03	7.229,352	219.874,1	7.513,887	.348,209,5	39.357,58

TABLE 17—Continued

UNIT RADIAL LOAD NO. 5
TWIST AND TANGENTIAL DEFLECTIONS

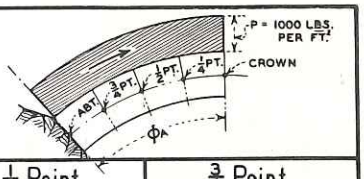


φ	Crown			¼ Point.			½ Point			¾ Point		
	D_1	D_3		D_1	D_3		D_1	D_3		D_1	D_3	
°		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.221,299,2	.002,242,50	1.764,561	.163,427,0	$.0^3$879,07	1.025,292	.069,201,66	$.0^3$161,529	.387.189,4	.011,248,23	$.0^5$6,480	.058,823,66
11	.294,486,5	.003,609,25	2.346,503	.217,487,8	.001,415,19	1.363,904	.092,100,54	$.0^3$260,095	.515,215,7	.014,971,09	$.0^4$10,433	.078,289,07
12	.382,234,3	.005,572,56	3.043,376	.282,310,6	.002,185,57	1.769,641	.119,561,8	$.0^3$401,79	.668,700,5	.019,436,14	$.0^4$16,119	.101,633,1
13	.485,853,9	.008,308,77	3.865,212	.358,867,2	.003,259,67	2.248,454	.151,998,8	$.0^3$599,39	.849,931,7	.024,710,77	$.0^4$24,051	.129,207,3
14	.606,653,5	.012,025,49	4.821,934	.448,127,7	.004,719,28	2.806,258	.189,824,6	$.0^3$868,01	1.061,191	.030,862,38	$.0^4$34,836	.161,363,0
15	.745,937,6	.016,964,25	5.923,349	.551,060,0	.006,659,66	3.448,925	.233,451,9	.001,225,26	1.304,752	.037,958,36	$.0^4$49,183	.198,451,1
16	.905,007,1	.023,402,75	7.179,140	.668,630,6	.009,190,56	4.182,284	.283,292,9	.001,691,44	1.582,881	.046,066,06	$.0^4$67,910	.240,822,5
17	1.085,159	.031,657,4	8.598,854	.801,803,2	.012,436,97	5.012,114	.339,759,4	.002,289,70	1.897,834	.055,252,84	$.0^4$91,948	.288,827,4
18	1.287,685	.042,085,2	10.191,90	.951,539,8	.016,540,41	5.944,144	.403,262,9	.003,046,21	2.251,862	.065,586,02	$.0^3$122,356	.342,815,8
19	1.513,875	.055,086,6	11.967,54	1.118,799	.021,659,4	6.984,048	.474,214,1	.003,990,51	2.647,201	.077,132,90	$.0^3$160,322	.403,137,5
20	1.765,010	.071,106,9	13.934,87	1.304,539	.027,971,0	8.137,445	.553,023,4	.005,155,40	3.086,081	.089,960,76	$.0^3$207,175	.470,141,5
21	2.042,370	.090,639,2	16.102,82	1.509,712	.035,671,3	9.409,892	.640,100,4	.006,577,38	3.570,719	.104,136,9	$.0^3$264,40	.544,176,8
22	2.347,226	.114,225,7	18.480,17	1.735,271	.044,976,1	10.806,88	.735,854,4	.008,296,67	4.103,323	.119,728,4	$.0^3$333,59	.625,591,6
23	2.680,847	.142,460,3	21.075,49	1.982,162	.056,122,6	12.333,85	.840,693,9	.010,357,63	4.686,086	.136,802,7	$.0^3$416,59	.714,733,9
24	3.044,494	.175,990,2	23.897,18	2.251,332	.069,369,6	13.996,15	.955,026,8	.012,808,54	5.321,192	.155,426,8	$.0^3$515,31	.811,951,0
25	3.439,421	.215,518,3	26.953,43	2.543,721	.084,998,2	15.799,08	1.079,260	.015,702,0	6.010,809	.175,667,9	$.0^3$631,93	.917,589,8
26	3.866,879	.261,803,8	30.252,25	2.860,269	.103,313,6	17.747,85	1.213,801	.019,095,4	6.757,095	.197,593,2	$.0^3$768,75	1.031,996
27	4.328,110	.315,664,9	33.801,43	3.201,910	.124,644,8	19.847,59	1.359,055	.023,050,5	7.562,190	.221,269,7	$.0^3$928,30	1.155,518
28	4.824,348	.377,980,6	37.608,53	3.569,576	.149,346,1	22.103,38	1.515,427	.027,633,9	8.428,224	.246,764,4	.001,113,28	1.288,498
29	5.356,824	.449,692,1	41.680,91	3.964,192	.177,797,4	24.520,17	1.683,321	.032,917,4	9.357,308	.274,144,5	.001,326,62	1.431,281
30	5.926,757	.531,802,4	46.025,69	4.386,684	.210,405,3	27.10288	1.863,140	.038,977,8	10.351,54	.303,476,8	.001,571,47	1.584,213
31	6.535,362	.625,381,4	50.649,77	4.837,970	.247,604,1	29.856,29	2.055,288	.045,897,4	11.413,00	.334,828,3	.001,851,14	1.747,635
32	7.183,843	.731,564,2	55.559,79	5.318,964	.289,855,4	32.785,13	2.260,166	.053,763,8	12.543,76	.368,266,1	.002,169,35	1.921,891
33	7.873,400	.851,552,8	60.762,15	5.830,579	.337,650,6	35.894,02	2.478,174	.062,670,5	13.745,85	.403,856,8	.002,529,81	2.107,323
34	8.605,221	.986,618,8	66.263,00	6.373,720	.391,509,0	39.187,48	2.709,713	.072,716,5	15.021,32	.441,667,3	.002,936,62	2.304,271
35	9.380,488	1.138,103	72.068,25	6.949,289	.451,980,8	42.669,95	2.955,182	.084,007,2	16.372,17	.481,764,6	.003,394,12	2.513,078
36	10.200,37	1.307,415	78.183,52	7.558,182	.519,647,7	46.345,77	3.214,978	.096,653,8	17.80041	.524,215,3	.003,906,86	2.734,082
37	11.066,04	1.496,039	84.614,18	8.201,291	.595,121,2	50.219,16	3.489,499	.110,774,0	19.307,99	.569,086,1	.004,479,76	2.967,623
38	11.978,63	1.705,530	91.365,33	8.879,504	.679,044,8	54.294,24	3.779,139	.126,491,7	20.896,88	.616,443,6	.005,117,90	3.214,038
39	12.939,31	1.937,513	98.441,78	9.593,702	.772,095,0	58.575,04	4.084,295	.143,937,6	22.569,02	.666,354,6	.005,826,70	3.473,665
40	13.949,20	2.193,688	105.848,1	10.34476	.874,980,3	63.06548	4.405,359	.163,249,2	24.326,31	.718,885,4	.006,611,85	3.746,841
41	15.009,43	2.475,834	113.588,5	11.133,55	.988,443,9	67.769,36	4.742,724	.184,570,5	26.170,66	.774,102,8	.007,479,35	4.033,901
42	16.121,11	2.785,797	121.667,0	11.960,94	1.113,261	72.690,37	5.096,782	.208,053,0	28.103,93	.832,073,1	.008,435,49	4.335,181
43	17.285,35	3.125,498	130.087,3	12.827,79	1.250,243	77.832,10	5.467,922	.233,855,1	30.127,98	.892,862,9	.009,486,93	4.651,014
44	18.503,23	3.496,937	138.852,7	13.734,95	1.400,233	83.198,00	5.856,534	.262,142,3	32.244,63	.956,538,3	.010,640,4	4.981,733
45	19.775,85	3.902,186	147.965,4	14.683,28	1.564,113	88.791,42	6.263,006	.293,087,8	34.455,70	1.023,166	.011,903,4	5.327,671
46	21.104,28	4.343,392	157.431,1	15.673,60	1.742,795	94.615,59	6.687,723	.326,872,6	36.762,96	1.092,812	.013,283,4	5.689,158
47	22.489,57	4.822,773	167.249,3	16.706,77	1.937,236	100.673,6	7.131,072	.363,684,9	39.168,17	1.165,542	.014,788,3	6.066,524
48	23.932,77	5.342,629	177.423,3	17.783,60	2.148,417	106.968,5	7.593,435	.403,720,7	41.673,08	1.241,423	.016,426,5	6.460,100
49	25.434,93	5.905,325	187.954,9	18.904,92	2.377,363	113.503,1	8.075,197	.447,184,2	44.279,39	1.320,521	.018,206,5	6.870,214
50	26.997,07	6.513,305	198.845,7	20.071,56	2.625,133	120.280,2	8.576,736	.494,267,7	46.988,79	1.402,902	.020,137,2	7.297,189
51	28.620,19	7.169,085	210.096,9	21.284,31	2.892,823	127.302,3	9.098,434	.545,250,8	49.802,93	1.488,631	.022,228,1	7.741,355
52	30.305,31	7.875,252	221.709,6	22.543,99	3.181,567	134.572,0	9.640,669	.600,303,3	52.723,47	1.577,775	.024,488,8	8.203,036
53	32.053,41	8.634,462	233.684,4	23.851,38	3.492,533	142.091,6	10.203,82	.659,681,1	55.752,00	1.670,399	.026,929,5	8.682,556
54	33.865,47	9.449,447	246.021,7	25.207,29	3.826,928	149.863,3	10.788,25	.723,631	58.890,12	1.766,570	.029,560,6	9.180,237
55	35.742,44	10.323,00	258.721,4	26.612,49	4.185,999	157.889,3	11.394,35	.792,405	62.139,36	1.866,353	.032,393,2	9.696,401
56	37.685,29	11.257,99	271.783,3	28.067,75	4.571,021	166.171,5	12.02249	.866,267	65.501,28	1.969,813	.035,438,2	10.231,37
57	39.694,95	12.257,36	285.206,8	29.573,85	4.983,314	174.711,7	12.673,03	.945,489	68.977,37	2.077,017	.038,707,5	10.785,46
58	41.772,34	13.324,09	298.991,1	31.131,55	5.424,235	183.511,8	13.346,34	1.030,350	72.569,10	2.188,029	.042,213,3	11.358,99
59	43.918,37	14.461,26	313.134,8	32.741,59	5.895,174	192.573,1	14.042,80	1.121,140	76.277,93	2.302,915	.045,968,1	11.952,28
60	46.133,94	15.671,99	327.636,7	34.404,74	6.397,561	201.897,2	14.762,77	1.218,158	80.105,27	2.421,741	.049,984,5	12.565,63
61	48.419,94	16.959,47	342.494,7	36.121,71	6.932,860	211.484,1	15.506,60	1.321,711	84.052,51	2.544,572	.054,276,5	13.199,38
62	50.777,22	18.326,95	357.706,8	37.893,24	7.502,574	221.338,2	16.274,68	1.432,116	88.121,01	2.671,473	.058,857,5	13.853,83
63	53.206,65	19.777,72	373.270,7	39.720,07	8.108,244	231.458,3	17.067,35	1.549,698	92.312,12	2.802,509	.063,741,8	14.529,29
64	55.709,07	21.315,16	389.183,4	41.602,88	8.751,442	241.845,0	17.884,97	1.674,794	96.627,11	2.937,745	.068,944,3	15.226,06
65	58.285,29	22.942,67	405.442,2	43.54241	9.433,781	252.499,7	18.727,91	1.807,748	101.067,3	3.077,247	.074,480,2	15.944,47
66	60.936,13	24.663,72	422.043,6	45.539,33	10.15691	263.4229	19.596,51	1.948,913	105.633,8	3.221,078	.080,364,8	16.684,82
67	63.662,39	26.481,84	438.984,0	47.594,35	10.922,50	274.615,1	20.491,13	2.098,656	110.328,0	3.369,305	.086,614,6	17.447,41
68	66.464,85	28.400,57	456.259,6	49.708,13	11.732,28	286.076,7	21.412,13	2.257,347	115.151,0	3.521,991	.093,246,0	18.232,56
69	69.344,27	30.423,53	473.866,1	51.881,36	12.588,00	297.807,9	22.359,85	2.425,371	120.1040	3.679,201	.100,276,3	19.040,55
70	72.301,40	32.554,38	491.799,2	54.114,70	13.49143	309.808,7	23.334,64	2.603,121	125.188,0	3.841,001	.107,723,0	19.871,70
71	75.336,98	34.796,79	510.054,1	56.408,80	14.44442	322.079,0	24.336,85	2.790,999	130.404,2	4.007,454	.115,604,0	20.726,30
72	78.451,73	37.154,51	528.625,7	58.764,30	15.448,79	334.618,8	25.366,83	2.989,418	135.753,5	4.178,625	.123,938,1	21.604,66
73	81.646,34	39.631,29	547.508,8	61.181,85	16.506,45	347.427,7	26.424,90	3.198,799	141.237,2	4.354,578	.132,744,2	22.507,07
74	84.921,52	42.230,92	566.697,9	63.662,07	17.619,31	360.505,1	27.511,42	3.419,576	146.856,0	4.535,377	.142,042,2	23.433,82
75	88.277,93	44.957,24	586.187,1	66.205,58	18.789,32	373.850,5	28.626,72	3.652,189	152.611,0	4.721,087	.151,851,8	24.385,22
76	91.716,22	47.814,09	605.970,5	68.812,99	20.018,45	387.463,1	29.771,14	3.897,092	158.503,1	4.911,771	.162,194,1	25.361,55
77	95.237,05	50.805,35	626.041,5	71.484,90	21.308,74	401.342,1	30.945,01	4.154,745	164.533,2	5.107,494	.173,090,1	26.363,11
78	98.841,01	53.934,90	646.393,8	74.221,90	22.662,19	415.486,5	32.148,66	4.425,622	170.702,2	5.308,320	.184,561,5	27.390,19
79	102.528,7	57.206,67	667.020,5	77.024,58	24.080,90	429.895,1	33.382,43	4.710,203	177.010,9	5.514,311	.196,630,7	28.443,06
80	106.300,8	60.624,57	687.914,6	79.893,50	25.566,96	444.566,7	34.646,65	5.008,979	183.460,0	5.725,533	.209,320,2	29.522,03
81	110.157,8	64.192,55	709.068,8	82.829,22	27.122,50	459.497,8	35.941,63	5.322,456	190.050,4	5.942,048	.222,653,8	30.627,38
82	114.100,3	67.914,54	730.475,7	85.832,31	28.749,67	474.693,1	37.267,70	5.651,142	196.782,7	6.163,920	.236,655,1	31.759,38
83	118.128,7	71.794,49	752.1274	88.903,29	30.450,64	490.144,7	38.625,18	5.995,561	203.657,7	6.391,212	.251,348,8	32.918,32
84	122.243,8	75.836,36	774.016,2	92.042,72	32.227,63	505.853,0	40.014,39	6.356,243	210.676,0	6.623,988	.266,760,0	34.104,48
85	126.445,8	80.044,07	796.134,0	95.251,10	34.082,85	521.815,9	41.435,66	6.733,732	217.838,2	6.862,310	.282,914,1	35.318,14
86	130.735,4	84.421,60	818.472,4	98.528,95	36.018,57	538.031,5	42.889,28	7.128,579	225.145,0	7.106,243	.299,837,7	36.559,58
87	135.113,0	88.972,85	841.022,8	101.876,8	38.037,06	554.497,5	44.375,58	7.541,346	232.596,9	7.355,847	.317,557,4	37.829,06
88	139.579,1	93.701,74	863.776,8	105.295,1	40.140,60	571.211,9	45.894,87	7.972,605	240.194,3	7.611,188	.336,100,6	39.126,86
89	144.134,1	98.612,18	886.725,3	108.784,3	42.331,54	588.172,0	47.447,44	8.422,938	247.937,9	7.872,327	.355,495,4	40.453,26
90	148.778,4	103.708,1	909.859,3	112.345,0	44.612,18	605.375,5	49.033,62	8.892,935	255.828,0	8.139,327	.375,770,3	41.808,52

TABLE 19—CONSTANTS FOR TANGENTIAL DEFLECTIONS, TANGENTIAL LOADS

UNIT TANGENTIAL LOAD NO. 1

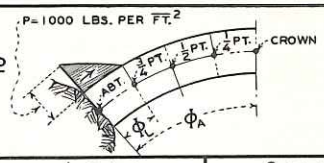
TANGENTIAL DEFLECTIONS



φ °	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_3		D_3		D_3	
		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.038,624,00	.005,373,006	.868,610,9	.03391,390	15.423,05	.0469,751	8.628,329	.056,129,5	3.819,782	.0795,83	.952,684,3
11	.056,537,34	.008,644,710	1.151,254	.03692,926	18.710,38	.03123,525	10.455,71	.0410,856,9	4.624,997	.06169,75	1.152,940
12	.080,054,96	.013,342,03	1.487,730	.001,167,12	22.329,93	.03208,119	12.463,24	.0418,296,3	5.508,114	.06286,11	1.372,344
13	.110,236,8	.019,884,74	1.881,984	.001,885,20	26.286,83	.03336,281	14.652,57	.0429,570,2	6.469,465	.06462,47	1.610,916
14	.148,233,4	.028,766,68	2.337,715	.002,938,38	30.586,62	.03524,336	17.025,49	.0446,118,1	7.509,408	.06721,45	1.868,681
15	.195,286,0	.040,561,22	2.858,360	.004,441,23	35.235,22	.03792,816	19.583,93	.0469,751	8.628,329	.051,091,27	2.145,661
16	.252,725,7	.055,926,64	3.447,075	.006,535,28	40.238,96	.001,167,12	22.329,93	.03102,713	9.826,642	.051,607,24	2.441,884
17	.321,973,1	.075,611,22	4.106,716	.009,392,94	45.604,52	.001,678,19	25.265,68	.03147,738	11.104,79	.052,312,2	2.757,376
18	.404,538,3	.100,458,3	4.839,826	.013,221,47	51.338,95	.002,363,33	28.393,50	.03208,119	12.463,24	.053,257,9	3.092,169
19	.502,019,9	.131,411,0	5.648,610	.018.267,41	57.449,65	.003,266,89	31.715,83	.03287,792	13.902,48	.054,506,0	3.446,293
20	.616,104,7	.169,516,8	6.534,924	.024,820,99	63.944,36	.004,441,23	35.235,22	.03391,390	15.423,05	.056,129,5	3.819,782
21	.748,567,6	.215,931,9	7.500,260	.033,220,98	70.831,13	.005,947,48	38.954,37	.03524,336	17.025,49	.058,213,4	4.212,671
22	.901,270,2	.271,925,6	8.545,724	.043,859,52	78.118,34	.007,856,58	42.876,08	.03692,926	18.710,38	.0410,856,9	4.624,997
23	1.076,161	.338,883,7	9.672,028	.057,187,2	85.814,62	.010,250,13	47.003,26	.03904,418	20.478,31	.0414,174,3	5.056,798
24	1.275,275	.418,312,8	10.879,47	.073,718,6	93.928,93	.013,221,47	51.338,95	.001,167,12	22.329,93	.0418,296,3	5.508,114
25	1.500,731	.511,843,0	12.167,94	.094,037,5	102.470,4	.016,876,71	55.886,28	.001,490,47	24.265,87	.0423,372,0	5.978,989
26	1.754,734	.621,231,9	13.536,85	.118,802,8	111.448,6	.021,335,77	60.648,51	.001,885,20	26.286,83	.0429,570,2	6.469,465
27	2.039,574	.748,366,7	14.985,21	.148,753,6	120.873,1	.026,733,52	65.628,98	.002,363,33	28.393,50	.0437,081,1	6.979,589
28	2.357,621	.895,267,4	16.511,52	.184,716,2	130.753,7	.033,220,98	70.831,13	.002,938,38	30.586,62	.0446,118,1	7.509,408
29	2.711,332	1.064,089	18.113,85	.227,609,6	141.100,5	.040,966,41	76.258,53	.003,625,41	32.866,93	.0456,919	8.058,971
30	3.103,243	1.257,122	19.789,75	.278,451,1	151.923,8	.050,156,62	81.914,79	.004,441,23	35.235,22	.0469,751	8.628,329
31	3.535,971	1.476,798	21.536,29	.338,363,9	163.233,9	.060,998,1	87.803,66	.005,404,35	37.692,29	.0484,908	9.217,534
32	4.012,215	1.725,688	23.350,05	.408,582,5	175.041,3	.073,718,6	93.928,93	.006,535,28	40.238,96	.03102,713	9.826,642
33	4.534,753	2.006,502	25.227,07	.490,459,2	187.356,6	.088,568,1	100.294,5	.007,856,58	42.876,08	.03123,525	10.455,71
34	5.106,441	2.322,095	27.162,89	.585,471,4	200.190,6	.105,820,2	106.904,3	.009,392,94	45.604,52	.03147,738	11.104,79
35	5.730,214	2.675,464	29.152,52	.695,227,4	213.553,9	.125,773,5	113.762,5	.011,171,41	48.425,17	.03175,779	11.773,94
36	6.409,082	3.069,747	31.190,44	.821,473,3	227.457,5	.148,753,6	120.873,1	.013,221,47	51.338,95	.03208,119	12.463,24
37	7.146,134	3.508,226	33.270,58	.966,100,1	241.912,2	.175,113,4	128.240,2	.015,575,21	54.346,79	.03245,273	13.172,73
38	7.944,531	3.994,326	35.386,33	1.131,150	256.928,7	.205,235,4	135.868,2	.018,267,41	57.449,65	.03287,792	13.902,48
39	8.807,509	4.531,611	37.530,56	1.318,823	272.518,0	.239,532,8	143.761,3	.021,335,77	60.648,51	.03336,281	14.652,57
40	9.738,379	5.123,788	39.695,55	1.531,484	288.690,8	.278,451,1	151.923,8	.024,820,99	63.944,36	.03391,390	15.423,05
41	10.740,52	5.774,700	41.873,07	1.771,670	305.457,8	.322,469,7	160.360,1	.028,766,99	67.338,22	.03453,821	16.214,00
42	11.817,39	6.488,328	44.054,30	2.042,096	322.829,8	.372,103,2	169.074,8	.033,220,98	70.831,13	.03524,336	17.025,49
43	12.972,51	7.268,790	46.229,89	2.345,660	340.817,1	.427,903,2	178.072,1	.038,233,69	74.424,15	.03603,744	17.857,59
44	14.209,46	8.120,332	48.389,95	2.685,455	359.430,4	.490,459,2	187.356,6	.043,859,52	78.118,34	.03692,926	18.710,38
45	15.531,92	9.047,332	50.524,02	3.064,770	378.679,7	.560,401,3	196.932,9	.050,156,62	81.914,79	.03792,816	19.583,93
46	16.943,60	10.054,30	52.621,08	3.487,098	398.575,1	.638,400,6	206.805,4	.057,187,2	85.814,62	.03904,418	20.478,31
47	18.448,30	11.145,85	54.669,59	3.956,144	419.126,7	.725,171,6	216.978,8	.065,017,7	89.818,96	.001,028,80	21.393,62
48	20.049,87	12.326,73	56.657,44	4.475,830	440.344,0	.821,473,3	227.457,5	.073,718,6	93.928,93	.001,167,12	22.329,93
49	21.752,24	13.601,81	58.572,04	5.050,301	462.236,4	.928,111,2	238.246,2	.083,365,3	98.145,70	.001,320,57	23.287,32
50	23.559,38	14.976,06	60.400,20	5.683,932	484.813,1	1.045,939	249.349,5	.094,037,5	102.470,4	.001,490,47	24.265,87
51	25.475,35	16.454,55	62.128,25	6.381,331	508.083,0	1.175,858	260.771,9	.105,820,2	106.904,3	.001,678,19	25.265,68
52	27.504,23	18.042,45	63.741,98	7.147,352	532.054,6	1.318,823	272.518,0	.118,802,8	111.448,6	.001,885,20	26.286,83
53	29.650,19	19.745,04	65.226,67	7.987,089	556.736,3	1.475,841	284.592,3	.133,080,4	116.104,4	.002,113,02	27.329,41
54	31.917,45	21.567,68	66.567,14	8.905,892	582.135,8	1.647,971	296.999,3	.148,753,6	120.873,1	.002,363,33	28.393,50
55	34.310,28	23.515,80	67.747,63	9.909,370	608.260,6	1.836,332	309.743,7	.165,928,2	125.755,7	.002,637,88	29.479,21
56	36.833,02	25.594,94	68.752,03	11.003,39	635.118,0	2.042,096	322.829,8	.184,716,2	130.758,7	.002,938,38	30.586,62
57	39.490,03	27.810,67	69.563,64	12.194,08	662.714,6	2.266,496	336.262,0	.205,235,4	135.868,2	.003,266,89	31.715,83
58	42.285,76	30.168,64	70.165,33	13.487,86	691.056,6	2.510,829	350.044,9	.227,609,6	141.100,5	.003,625,41	32.866,93
59	45.224,69	32.674,57	70.539,58	14.891,40	720.150,0	2.776,446	364.182,6	.251,969,1	146.452,0	.004,016,11	34.040,03
60	48.311,36	35.334,20	70.668,42	16.411,67	750.000,0	3.064,770	378.679,7	.278,451,1	151.923,8	.004,441,23	35.235,22
61	51.550,33	38.153,34	70.533,38	18.055,92	780.611,5	3.377,285	393.540,2	.307,199,1	157.517,3	.004,903,13	36.452,61
62	54.946,24	41.137,80	70.115,72	19.831,67	811.988,9	3.715,542	408.768,3	.338,363,9	163.233,9	.005,404,35	37.692,29
63	58.503,77	44.293,43	69.396,21	21.746,74	844.135,9	4.081,160	424.368,2	.372,103,2	169.074,8	.005,947,48	38.954,37
64	62.227,62	47.626,10	68.355,33	23.809,26	877.055,8	4.475,830	440.344,0	.408,582,5	175.041,3	.006,535,28	40.238,96
65	66.122,56	51.141,68	66.973,11	26.027,63	910.751,4	4.901,312	456.699,5	.447,974,2	181.134,8	.007,170,64	41.546,16
66	70.193,38	54.846,03	65.229,35	28.410,57	945.224,8	5.359,439	473.438,6	.490,459,2	187.356,6	.007,856,58	42.876,08
67	74.444,93	58.745,01	63.103,45	30.967,08	980.477,4	5.852,118	490.565,2	.536,226,0	193.708,1	.008,596,23	44.228,83
68	78.882,07	62.844,45	60.574,61	33.706,47	1,016.510	6.381,331	508.083,0	.585,471,4	200.190,6	.009,392,94	45.604,52
69	83.509,71	67.150,14	57.621,60	36.638,37	1,053.324	6.949,139	525.995,5	.638,400,6	206.805,4	.010,250,13	47.003,26
70	88.332,82	71.667,85	54.223,10	39.772,68	1,090.917	7.557,677	544.306,2	.695,227,4	213.553,9	.011,171,41	48.425,17
71	93.356,36	76.403,29	50.357,44	43.119,62	1,129.290	8.209,163	563.018,5	.756,174,4	220.437,5	.012,160,55	49.870,36
72	98.585,35	81.362,09	46.002,79	46.689,73	1,168.441	8.905,892	582.135,8	.821,473,3	227.457,5	.013,221,47	51.338,95
73	104.024,8	86.549,84	41.137,10	50.493,84	1,208.366	9.650,246	601.661,0	.891,365,1	234.615,3	.014,358,27	52.831,05
74	109.679,9	91.972,01	35.738,17	54.543,08	1,249.064	10.444,68	621.597,3	.966,100,1	241.912,2	.015,575,21	54.346,79
75	115.555,5	97.634,02	29.783,61	58.848,90	1,290.530	11.291,75	641.947,6	1.045,939	249.349,5	.016,876,71	55.886,28
76	121.657,0	103.541,2	23.250,93	63.423,01	1,332.761	12.194,08	662.714,6	1.131,150	256.928,7	.018,267,41	57.449,65
77	127.989,4	109.698,6	16.117,58	68.277,49	1,375.750	13.154,40	683.900,9	1.222,015	264.651,1	.019,752,09	59.037,02
78	134.557,9	116.111,4	8.360,84	73.424,65	1,419.492	14.175,48	705.509,0	1.318,823	272.518,0	.021,335,77	60.648,51
79	141.367,7	122.784,4	-.041,97	78.877,13	1,463.981	15.260,25	727.541,4	1.421,876	280.530,8	.023,023,60	62.284,25
80	148.423,9	129.722,5	-9.113,60	84.647,87	1,509.209	16.411,67	750.000,0	1.531,484	288.690,8	.024,820,99	63.944,36
81	155.731,9	136.930,2	-18.876,88	90.750,08	1,555.168	17.632,83	772.887,0	1.647,971	296.999,3	.026,733,52	65.628,98
82	163.296,9	144.412,1	-29.354,43	97.197,25	1,601.850	18.926,87	796.204,2	1.771,670	305.457,8	.028,766,99	67.338,22
83	171.124,1	152.172,2	-40.569,05	104.003,2	1,649.244	20.297,06	819.953,3	1.902,927	314.067,5	.030,927,40	69.072,23
84	179.218,7	160.214,8	-52.543,29	111.181,9	1,697.341	21.746,74	844.135,9	2.042,096	322.829,8	.033,220,98	70.831,13
85	187.586,2	168.543,7	-65.299,73	118.747,8	1,746.129	23.279,35	868.753,2	2.189,547	331.745,9	.035,654,18	72.615,06
86	196.231,7	177.162,5	-78.860,70	126.715,4	1,795.597	24.898,43	893.806,5	2.345,660	340.817,1	.038,233,69	74.424,15
87	205.160,6	186.074,6	-93.248,54	135.099,6	1,845.731	26.607,59	919.296,8	2.510,829	350.044,9	.040,966,41	76.258,53
88	214.378,2	195.283,3	-108.485,2	143.915,6	1,896.519	28.410,57	945.224,8	2.685,455	359.430,4	.043,859,52	78.118,34
89	223.889,7	204.791,5	-124.592,7	153.178,6	1,947.947	30.311,18	971.591,1	2.869,960	368.974,9	.046,920,34	80.003,71
90	233.700,6	214.601,8	-141.592,6	162.904,2	2,000.000	32.313,32	998.396,3	3.064,770	378.679,7	.050,156,62	81.914,79

TABLE 19—Continued

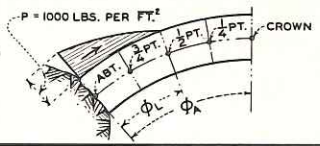
UNIT TANGENTIAL LOAD NO. 2
TANGENTIAL DEFLECTIONS



ϕ	Crown			¼ Point		½ Point		¾ Point	
	D_1	D_2	D_3	D_3		D_3		D_3	
°		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.0⁴30,204,29	.0⁵5,028,372	-041,440,86	.0⁶422,078	.314,744,3	.0⁶231,089	.316,2524	.0⁷97,157	.317,158,4
11	.0⁴44,221,67	.0⁵8,090,191	-055,131,26	.0⁶747,362	.380,189,4	.0⁶409,275	.382,396,6	.0⁶172,298	.383,723,1
12	.0⁴62,630,36	.0⁴12,486,14	-071,537,67	.0⁵1,258,992	.451,608,2	.0⁶689,625	.454,733,3	.0⁶290,034	.456,612,0
13	.0⁴86,263,76	.0⁴18,609,04	-090,901,73	.0⁵2,033,928	.528,930,3	.0⁵1,114,928	.533,233,1	.0⁶468,766	.535,820,6
14	.0³116,027,7	.0⁴26,921,01	-.113,463,8	.0⁵3,170,754	.612,078,1	.0⁵1,737,767	.617,864,2	.0⁶731,127	.621,344,5
15	.0³152,900,5	.0⁴37,958,56	-.139,462,6	.0⁵4,793,374	.700,972,1	.0⁵2,627,875	.708,592,6	.0⁵1,105,87	.713,178,7
16	.0³197,932,8	.0⁴52,337,63	-.169,135,5	.0⁵7,054,889	.795,520,6	.0⁵3,858,964	.805,381,1	.0⁵1,628,51	.811,317,8
17	.0³252,247,6	.0⁴70,758,38	-.202,717,9	.0⁴10,141,92	.895,630,0	.0⁵5,563,872	.908,190,4	.0⁵2,342,49	.915,756,0
18	.0³317,040,3	.0⁴94,009,81	-.240,443,6	.0⁴14,279,00	1.001,200	.0⁵7,836,392	1.016,978	.0⁵3,300,12	1.026,487
19	.0³393,578,7	.0³122,974,3	-.282,544,3	.0⁴19,733,27	1.112,123	.0⁴10,833,96	1.131,701	.0⁵4,563,73	1.143,505
20	.0³483,202,9	.0³158,631,7	-.329,249,8	.0⁴26,819,57	1.228,288	.0⁴14,730,58	1.252,310	.0⁵6,206,95	1.266,802
21	.0³587,325,1	.0³202,063,6	-.380,787,5	.0⁴35,905,46	1.349,576	.0⁴19,729,56	1.378,757	.0⁵8,315,86	1.396,370
22	.0³707,431,1	.0³254,457,4	-437,382,9	.0⁴47,416,94	1.475,863	.0⁴26,066,85	1.510,990	.0⁴10,990,5	1.532,204
23	.0³845,075,3	.0³317,108,5	-499,258,7	.0⁴61,843,66	1.607,019	.0⁴34,014,00	1.648,953	.0⁴14,346,0	1.674,294
24	.001,001,890	.0³391,426,4	-.566,635,5	.0⁴79,745,51	1.742,907	.0⁴43,881,89	1.792,589	.0⁴18,514,5	1.822,630
25	.001,179,575	.0³478,935,3	-639,730,8	.0³101,757,8	1.883,386	.0⁴56,023,81	1.941,838	.0⁴23,645,8	1.977,206
26	.001,379,903	.0³581,277,7	-718,759,9	.0³128,598,7	2.028,310	.0⁴70,839,79	2.096,638	.0⁴29,910,5	2.138,010
27	.001,604,722	.0³700,218,1	-803,934,9	.0³161,074,4	2.177,524	.0⁴88,779,35	2.256,923	.0⁴37,499,8	2.305,034
28	.001,855,949	.0³837,644,0	-895,465,1	.0³200,086,9	2.330,870	.0³110,346,3	2.422,626	.0⁴46,628,6	2.478,266
29	.002,135,574	.0³995,568,7	-993,556,6	.0³246,640,2	2.488,184	.0³136,102,6	2.593,677	.0⁴57,536,7	2.657,697
30	.002,445,660	.001,176,133	-1.098,412	.0³301,847,9	2.649,296	.0³166,672,3	2.770,003	.0⁴70,490,9	2.843,314
31	.002,788,341	.001,381,606	-1.210,232	.0³366,939,1	2.814,030	.0³202,745,6	2.951,528	.0⁴85,786,1	3.035,106
32	.003,165,823	.001,614,388	-1.329,212	.0³443,267,6	2.982,206	.0³245,084,0	3.138,174	.0³103,749,1	3.233,061
33	.003,580,384	.001,877,011	-1.455,546	.0³532,318,1	3.153,636	.0³294,524,1	3.329,861	.0³124,738,3	3.437,166
34	.004,034,376	.002,172,136	-1.589,422	.0³635,713,3	3.328,129	.0³351,982,5	3.526,504	.0³149,147,6	3.647,408
35	.004,530,220	.002,502,557	-1.731,026	.0³755,222,8	3.505,490	.0³418,460,5	3.728,020	.0³177,407,6	3.863,774
36	.005,070,410	.002,871,202	-1.880,539	.0³892,770,4	3.685,512	.0³495,049,1	3.934,319	.0³209,988,5	4.086,249
37	.005,657,514	.003,281,127	-2.038,138	.001,050,442	3.867,992	.0³582,933,6	4.145,309	.0³247,401,4	4.314,819
38	.006,294,169	.003,735,523	-2.203,999	.001,230,492	4.052,714	.0³683,398,0	4.360,898	.0³290,200,8	4.549,468
39	.006,983,084	.004,237,706	-2.378,289	.001,435,355	4.239,462	.0³797,832,5	4.580,989	.0³338,988,9	4.790,182
40	.007,727,042	.004,791,123	-2.561,173	.001,667,650	4.428,012	.0³927,735,3	4.805,483	.0³394,414,9	5.036,943
41	.008,528,895	.005,399,365	-2.752,814	.001,930,190	4.618,135	.001,074,710	5.034,279	.0³457,179,9	5.289,735
42	.009,391,569	.006,066,080	-2.953,366	.002,225,990	4.809,601	.001,240,519	5.267,272	.0³528,037,6	5.548,541
43	.010,318,06	.006,795,130	-3.162,981	.002,558,278	5.002,169	.001,427,004	5.504,355	.0³607,794,1	5.813,342
44	.011,311,44	.007,590,447	-3.381,806	.002,930,496	5.195,599	.001,636,152	5.745,420	.0³697,332,5	6.084,122
45	.012,374,84	.008,456,082	-3.609,982	.003,346,316	5.389,642	.001,870,097	5.990,354	.0³797,569,2	6.360,860
46	.013,511,49	.009,396,201	-3.847,647	.003,809,642	5.584,046	.002,131,109	6.239,042	.0³909,504,1	6.643,537
47	.014,724,65	.010,415,08	-4.094,933	.004,324,625	5.778,555	.002,421,608	6.491,368	.001,034,199	6.932,133
48	.016,017,68	.011,517,10	-4.351,966	.004,895,661	5.972,908	.002,744,165	6.747,210	.001,172,784	7.226,627
49	.017,394,02	.012,706,73	-4.618,868	.005,527,407	6.166,839	.003,101,512	7.006,447	.001,326,465	7.526,998
50	.018,857,16	.013,988,56	-4.895,755	.006,224,790	6.360,078	.003,496,548	7.268,954	.001,496,521	7.833,224
51	.020,410,66	.015,367,25	-5.182,737	.006,993,006	6.552,352	.003,932,342	7.534,602	.001,684,312	8.145,282
52	.022,058,17	.016,847,54	-5.479,919	.007,837,536	6.743,380	.004,412,139	7.803,262	.001,891,275	8.463,149
53	.023,803,39	.018,434,25	-5.787,401	.008,764,150	6.932,883	.004,939,368	8.074,799	.002,118,935	8.786,801
54	.025,650,11	.020,132,28	-6.105,276	.009,778,913	7.120,572	.005,517,648	8.349,078	.002,368,906	9.116,213
55	.027,602,19	.021,946,60	-6.433,632	.010,888,197	7.306,156	.006,150,789	8.625,961	.002,642,887	9.451,361
56	.029,663,55	.023,882,20	-6.772,551	.012,098,688	7.489,344	.006,842,817	8.905,307	.002,942,688	9.792,218
57	.031,838,18	.025,944,17	-7.122,106	.013,417,380	7.669,835	.007,597,928	9.186,971	.003,270,177	10.138,76
58	.034,130,14	.028,137,60	-7.482,369	.014,851,598	7.847,329	.008,420,572	9.470,808	.003,627,368	10.490,95
59	.036,543,58	.030,467,63	-7.853,402	.016,409,018	8.021,519	.009,315,414	9.756,669	.004,016,368	10.848,78
60	.039,082,71	.032,939,44	-8.235,260	.018,097,61	8.192,099	.010,287,302	10.044,40	.004,439,356	11.212,20
61	.041,751,80	.035,558,19	-8.627,997	.019,925,72	8.358,755	.011,341,36	10.333,85	.004,898,663	11.581,19
62	.044,555,20	.038,329,08	-9.031,653	.021,902,04	8.521,171	.012,482,92	10.624,86	.005,396,713	11.955,71
63	.047,497,33	.041,257,28	-9.446,265	.024,035,61	8.679,032	.013,717,58	10.917,28	.005,936,058	12.335,75
64	.050,582,69	.044,347,97	-9.871,864	.026,335,85	8.832,013	.015,051,18	11.210,93	.006,519,366	12.721,26
65	.053,815,83	.047,606,29	-10.308,47	.028,812,53	8.979,792	.016,489,80	11.505,66	.007,149,424	13.112,22
66	.057,201,39	.051,037,34	-10.756,11	.031,475,78	9.122,041	.018,039,78	11.801,29	.007,829,149	13.508,58
67	.060,744,07	.054,646,21	-11.214,77	.034,336,12	9.258,430	.019,707,74	12.097,66	.008,561,595	13.910,32
68	.064,448,64	.058,437,90	-11.684,48	.037,404,46	9.388,627	.021,500,57	12.394,60	.009,349,941	14.317,40
69	.068,319,95	.062,417,34	-12.165,22	.040,692,07	9.512,298	.023,425,44	12.691,93	.010,197,52	14.729,79
70	.072,362,90	.066,589,41	-12.656,97	.044,210,61	9.629,105	.025,489,76	12.989,47	.011,107,78	15.147,44
71	.076,582,48	.070,958,88	-13.159,72	.047,972,16	9.738,709	.027,701,30	13.287,04	.012,084,35	15.570,32
72	.080,983,75	.075,530,42	-13.673,44	.051,989,16	9.840,768	.030,068,07	13.584,45	.013,130,97	15.998,40
73	.085,571,84	.080,308,58	-14.198,10	.056,274,46	9.934,940	.032,598,40	13.881,54	.014,251,57	16.431,62
74	.090,351,91	.085,297,78	-14.733,65	.060,841,27	10.020,88	.035,300,89	14.178,09	.015,450,18	16.869,96
75	.095,329,25	.090,502,31	-15.280,04	.065,703,27	10.098,24	.038,184,51	14.473,94	.016,731,07	17.313,37
76	.100,509,2	.095,926,29	-15.837,21	.070,874,49	10.166,67	.041,258,49	14.768,87	.018,098,61	17.761,81
77	.105,897,1	.101,573,7	-16.405,10	.076,369,39	10.225,83	.044,532,43	15.062,70	.019,557,38	18.215,23
78	.111,498,6	.107,448,3	-16.983,63	.082,202,79	10.275,35	.048,016,18	15.355,22	.021,112,08	18.673,59
79	.117,319,0	.113,553,6	-17.572,72	.088,389,92	10.314,90	.051,719,98	15.646,25	.022,767,63	19.136,85
80	.123,364,0	.119,893,1	-18.172,28	.094,946,48	10.344,11	.055,654,41	15.935,57	.024,529,12	19.604,96
81	.129,639,4	.126,469,9	-18.782,21	.101,888,5	10.362,63	059,830,33	16.222,97	.026,401,81	20.077,88
82	.136,150,8	.133,286,9	-19.402,40	.109,232,3	10.370,12	064,258,97	16.508,26	.028,391,14	20.555,54
83	.142,904,1	.140,346,7	-20.032,74	.116,994,9	10.366,20	068,951,96	16.791,22	.030,502,76	21.037,86
84	.149,905,2	.147,651,8	-20.673,10	.125,193,5	10.350,54	073,920,82	17.071,63	.032,742,50	21.524,95
85	.157,159,9	.155,204,2	-21.323,35	.133,845,6	10.322,76	079,178,90	17.349,29	.035,116,40	22.016,58
86	.164,674,5	.163,005,8	-21.983,35	.142,969,3	10.282,53	084,737,82	17.623,97	.037,630,70	22.512,77
87	.172,454,9	.171,058,0	-22.652,95	.152,583,0	10.229,47	090,610,91	17.895,45	.040,291,81	23.013,47
88	.180,507,3	.179,362,1	-23.331,99	.162,705,5	10.163,25	096,811,58	18.163,51	.043,106,43	23.518,59
89	.188,838,0	.187,918,8	-24.020,31	.173,355,9	10.083,49	103,353,45	18.427,93	.046,081,33	24.028,11
90	.197,453,2	.196,728,7	-24.717,73	.184,553,7	9.989,85	110,250,77	18.688,49	.049,223,68	24.541,96

TABLE 19—Continued

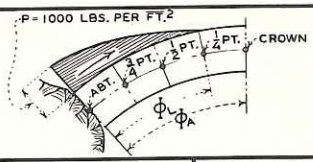
UNIT TANGENTIAL LOAD NO. 3
TANGENTIAL DEFLECTIONS



ϕ °	Crown D_1	Crown D_2 1ST Term	Crown D_2 2ND Term	Crown D_3 1ST Term	Crown D_3 2ND Term	¼ Point D_3 1ST Term	¼ Point D_3 2ND Term	½ Point D_3 1ST Term	½ Point D_3 2ND Term	¾ Point D_3 1ST Term	¾ Point D_3 2ND Term
10	.$0^3$483,202,9	.$0^4$76,971,58	-.110,957,7	.$0^5$6,206,95	1.266,802	.$0^5$3,303,49	1.270,436	.$0^5$1,313,55	1.271,652	.$0^7$58,182	.635,072,5
11	.$0^3$707,431,1	.$0^3$123,845,7	-.147,739,0	.$0^4$10,990,5	1.532,204	.$0^5$5,850,64	1.537,528	.$0^5$2,326,69	1.539,310	.$0^6$103,064	.768,552,8
12	.001,001,890	.$0^3$191,148,2	-.191,882,4	.$0^4$18,514,5	1.822,630	.$0^5$9,858,196	1.830,175	.$0^5$3,921,04	1.832,703	.$0^6$173,711	.914,791,3
13	.001,379,903	.$0^3$284,897,3	-.244,067,7	.$0^4$29,910,5	2.138,010	.$0^4$15,930,13	2.148,409	.$0^5$6,337,18	2.151,896	.$0^6$280,789	1.073,800
14	.001,855,949	.$0^3$412,173,0	-.304,977,3	.$0^4$46,628,6	2.478,266	.$0^4$24,840,80	2.492,262	.$0^5$9,883,82	2.496,961	.$0^6$438,020	1.245,593
15	.002,445,660	.$0^3$581,197,7	-.375,296,6	.$0^4$70,490,9	2.843,314	.$0^4$37,564,14	2.861,772	.$0^4$14,949,2	2.867,975	.$0^6$662,565	1.430,186
16	.003,165,823	.$0^3$801,412,3	-.455,714,0	.$0^3$103,749,1	3.233,061	.$0^4$55,304,52	3.256,974	.$0^4$22,014,0	3.265,020	.$0^6$975,830	1.627,593
17	.004,034,376	.001,083,551	-.546,921,0	.$0^3$149,147,6	3.647,408	.$0^4$79,530,88	3.677,910	.$0^4$31,664,4	3.688,185	.$0^5$1,403,85	1.837,831
18	.005,070,410	.001,439,714	-.649,612,5	.$0^3$209,988,5	4.086,249	.$0^3$112,012,9	4.124,620	.$0^4$44,607,5	4.137,562	.$0^5$1,978,04	2.060,918
19	.006,294,169	.001,883,435	-.764,487,0	.$0^3$290,200,8	4.549,468	.$0^3$154,857,6	4.597,149	.$0^4$61,685,2	4.613,252	.$0^5$2,735,82	2.296,874
20	.007,727,042	.002,429,749	-.892,246,7	.$0^3$394,414,9	5.036,943	.$0^3$210,551,1	5.095,540	.$0^4$83,892,5	5.115,357	.$0^5$3,721,5	2.545,717
21	.009,391,569	.003,095,258	-1.033,598	.$0^3$528,037,6	5.548,541	.$0^3$282,005,3	5.619,840	.$0^3$112,393	5.643,988	.$0^5$4,986,8	2.807,470
22	.011,311,44	.003,898,183	-1.189,250	.$0^3$697,332,5	6.084,122	.$0^3$372,590	6.170,096	.$0^3$148,553	6.199,259	.$0^5$6,591,8	3.082,153
23	.013,511,49	.004,858,436	-1.359,918	.$0^3$909,503,9	6.643,537	.$0^3$486,156	6.746,358	.$0^3$193,876	6.781,290	.$0^5$8,606,0	3.369,792
24	.016,017,68	.005,997,658	-1.546,319	.001,172,784	7.226,627	.$0^3$627,189	7.348,673	.$0^3$250,198	7.390,205	.$0^4$11,108,7	3.670,409
25	.018,857,16	.007,339,277	-1.749,176	.001,496,521	7.833,224	.$0^3$800,707	7.977,093	.$0^3$319,528	8.026,136	.$0^4$14,190,4	3.984,030
26	.022,058,17	.008,908,552	-1.969,215	.001,891,275	8.463,149	.001,012,427	8.631,668	.$0^3$404,162	8.689,216	.$0^4$17,953,7	4.310,681
27	.025,650,11	.010,732,62	-2.207,169	.002,368,906	9.116,213	.001,268,796	9.312,448	.$0^3$506,686	9.379,585	.$0^4$22,514,0	4.650,390
28	.029,663,55	.012,840,53	-2463,773	.002,942,688	9.792,218	.001,576,995	10.019,487	.$0^3$630,007	10.097,39	.$0^4$28,001,0	5.003,185
29	.034,130,14	.015,263,28	-2.739,767	.003,627,368	10.490,95	.001,945,030	10.752,833	.$0^3$777,334	10.842,77	.$0^4$34,558,9	5.369,095
30	.039,082,71	.018,033,84	-3.035,896	.004,439,356	11.212,20	.002,381,846	11.512,54	.$0^3$952,298	11.615,90	.$0^4$42,350,1	5.748,152
31	.044,555,20	.021,187,20	-3.352,910	.005,396,713	11.955,71	.002,897,277	12.298,65	.001,158,857	12.416,92	.$0^4$51,552,8	6.140,386
32	.050,582,69	.024,760,34	-3.691,562	.006,519,366	12.721,26	.003,502,214	13.111,23	.001,401,430	13.246,00	.$0^4$62,363,4	6.545,831
33	.057,201,39	.028,792,34	-4.052,610	.007,829,149	13.508,58	.004,208,594	13.950,31	.001,684,843	14.103,30	.$0^4$75,000	6.964,519
34	.064,448,64	.033,324,30	-4.436,819	.009,349,941	14.317,40	.005,029,507	14.815,94	.002,014,411	14.989,00	.$0^4$89,701	7.396,486
35	.072,362,90	.038,399,36	-4.844,955	.011,107,78	15.147,44	.005,979,252	15.708,18	.002,395,940	15.903,28	.$0^3$106,727	7.841,767
36	.080,983,75	.044,062,80	-5.277,790	.013,130,97	15.998,40	.007,073,407	16.627,06	.002,835,766	16.846,31	.$0^3$126,363	8.300,399
37	.090,351,91	.050,361,89	-5.736,101	.015,450,18	16.869,96	.008,328,871	17.572,63	.003,340,762	17.818,28	.$0^3$148,922	8.772,420
38	.100,509,2	.057,346,00	-6.220,668	.018,098,61	17.761,81	.009,764,00	18.544,92	.003,918,43	18.819,37	.$0^3$174,739	9.257,867
39	.111,498,6	.065,066,56	-6.732,275	.021,112,08	18.673,59	.011,398,63	19.543,98	.004,576,85	19.849,78	.$0^3$204,180	9.756,782
40	.123,364,0	.073,576,99	-7.271,711	.024,529,12	19.604,96	.013,254,14	20.569,84	.005,324,78	20.909,71	.$0^3$237,641	10.269,21
41	.136,150,8	.082,932,74	-7.839,768	.028,391,14	20.555,54	.015,353,56	21.622,53	.006,171,67	21.999,35	.$0^3$275,548	10.795,18
42	.149,905,2	.093,191,26	-8.437,243	.032,742,50	21.524,95	.017,721,75	22.702,08	.007,127,64	23.118,91	.$0^3$318,367	11.334,74
43	.164,674,5	.104,411,9	-9.064,934	.037,630,70	22.512,77	.020,384,93	23.808,51	.008,203,61	24.268,59	.$0^3$366,579	11.887,94
44	.180,507,3	.116,656,1	-9.723,645	.043,106,43	23.518,59	.023,371,84	24.941,84	.009,411,29	25.448,60	.$0^3$420,737	12.454,82
45	.197,453,2	.129,986,8	-10.414,18	.049,223,68	24.541,96	.026,712,70	26.102,10	.010,763,18	26.659,15	.$0^3$481,384	13.035,42
46	.215,562,8	.144,469,1	-11.137,35	.056,039,98	25.582,44	.030,439,93	27.289,29	.012,272,67	27.900,47	.$0^3$549,146	13.629,80
47	.234,888,0	.160,169,9	-11.893,97	.063,616,40	26.639,53	.034,588,03	28.503,42	.013,954,05	29.172,76	.$0^3$624,674	14.238,00
48	.255,481,5	.177,157,5	-12.684,84	.072,017,65	27.712,76	.039,193,66	29.744,50	.015,822,49	30.476,26	.$0^3$708,658	14.860,07
49	.277,397,5	.195,502,1	-13.510,79	.081,312,37	28.801,60	.044,295,83	31.012,51	.017,894,22	31.811,18	.$0^3$801,836	15.496,05
50	.300,691,0	.215,275,4	-14.372,63	.091,572,99	29.905,53	.049,935,80	32.307,45	.020,186,41	33.177,75	.$0^3$905,002	16.146,00
51	.325,418,0	.236,550,6	-15.271,18	.102,876,1	31.023,98	.056,157,35	33.629,31	.022,717,29	34.576,20	.001,018,985	16.809,98
52	.351,635,9	.259,402,3	-16.207,25	.115,302,4	32.156,39	.063,006,75	34.978,06	.025,506,23	36.006,77	.001,144,678	17.488,02
53	.379,402,9	.283,906,6	-17.181,68	.128,936,8	33.302,16	.070,532,87	36.353,68	.028,573,66	37.469,68	.001,283,016	18.180,19
54	.408,778,5	.310,140,6	-18.195,27	.143,868,7	34.460,67	.078,787,30	37.756,13	.031,941,27	38.965,17	.001,435,007	18.886,54
55	.439,823,1	.338,182,7	-19.248,86	.160,191,9	35.631,30	.087,824,32	39.185,36	.035,631,83	40.493,48	.001,601,714	19.607,13
56	.472,598,3	.368,112,4	-20.343,25	.178,005,1	36.813,36	.097,701,33	40.641,33	.039,669,65	42.054,85	.001,784,194	20.342,00
57	.507,166,4	.400,010,0	-21.479,27	.197,410,9	38.006,20	.108,478,0	42.123,98	.044,079,66	43.649,52	.001,983,669	21.091,23
58	.543,591,4	.433,956,8	-22.657,73	.218,517,6	39.209,10	.120,217,9	43.633,24	.048,889,03	45.277,73	.002,201,37	21.854,85
59	.581,937,8	.470,034,6	-23.879,45	.241,438,0	40.421,32	.132,987,1	45.169,03	.054,125,75	46.939,73	.002,438,62	22.632,95
60	.622,271,5	.508,326,0	-25.145,24	.266,289,9	41.642,13	.146,855,2	46.731,28	.059,819,38	48.635,77	.002,696,76	23.425,56
61	.664,659,3	.548,914,1	-26.455,91	.293,196,2	42.870,75	.161,894,9	48.319,88	.066,000,89	50.366,08	.002,977,24	24.232,76
62	.709,169,0	.591,882,0	-27.812,27	.322,284,9	44.106,37	.178,182,1	49.934,74	.072,702,83	52.130,91	.003,281,60	25.054,60
63	.755,869,6	.637,313,5	-29.215,11	.353,689,5	45.348,16	.195,796,6	51.575,74	.079,959,26	53.930,52	.003,611,41	25.891,16
64	.804,831,0	.685,292,1	-30.665,24	.387,548,5	46.595,29	.214,821,2	53.242,76	.087,805,90	55.765,15	.003,968,35	26.742,48
65	.856,124,2	.735,901,3	-32.163,44	.424,006,2	47.846,88	.235,342,8	54.935,65	.096,280,08	57.635,05	.004,354,17	27.608,63
66	.909,821,2	.789,224,3	-33.710,50	.463,211,9	49.102,02	.257,451,6	56.654,29	.105,420,7	59.540,47	.004,770,72	28.489,69
67	.965,995,1	.845,344,2	-35.307,19	.505,320,9	50.359,79	.281,241,7	58.398,50	.115,268,7	61.481,66	.005,219,86	29.385,71
68	1.024,720	.904,343,2	-36.954,30	.550,494,0	51.619,23	.306,811,1	60.168,11	.125,866,6	63.458,87	.005,703,67	30.296,76
69	1.086,071	.966,303,0	-38.652,59	.598,897,6	52.879,38	.334,261,7	61.962,96	.137,258,6	65.472,35	.006,224,22	31.222,91
70	1.150,124	1.031,304	-40.402,80	.650,703,9	54.139,22	.363,699,4	63.782,85	.149,491,2	67.522,35	.006,783,67	32.164,22
71	1.216,957	1.099,427	-42.205,69	.706,090,7	55.397,71	.395,234,1	65.627,56	.162,612,3	69.609,11	.007,384,35	33.120,77
72	1.286,646	1.170,749	-44.061,99	.765,242,0	56.653,81	.428,980,0	67.496,88	.176,672,3	71.732,88	.008,028,62	34.092,63
73	1.359,273	1.245,348	-45.972,43	.828,347,4	57.906,42	.465,055,4	69.390,57	.191,723,2	73.893,92	.008,718,98	35.079,86
74	1.434,916	1.323,300	-47.937,71	.895,602,4	59.154,43	.503,583,0	71.308,40	.207,819,3	76.092,46	.009,457,99	36.082,53
75	1.513,657	1.404,677	-49.958,55	.967,208,4	60.396,69	.544,689,7	73.250,09	.225,016,9	78.328,75	.010,248,36	37.100,73
76	1.595,579	1.489,553	-52.035,63	1.043,373	61.632,04	.588,507,1	75.215,39	.243,374,4	80.603,05	.011,092,92	38.134,51
77	1.680,763	1.577,996	-54.169,63	1.124,310	62.859,28	.635,171,4	77.203,97	.262,952,9	82.915,57	.011,994,55	39.183,96
78	1.769,295	1.670,073	-56.361,20	1.210,238	64.077,19	.684,822,6	79.215,57	.283,814,6	85.266,58	.012,956,31	40.249,15
79	1.861,259	1.765,851	-58.611,00	1.301,383	65.284,50	.737,606,5	81.249,85	.306,025,6	87.656,31	.013,981,33	41.330,15
80	1.956,741	1.865,389	-60.919,64	1.397,976	66.479,95	.793,672,7	83.306,46	.329,653,0	90.084,99	.015,072,89	42.427,04
81	2.055,829	1.968,748	-63.287,75	1.500,254	67.662,21	.853,175,9	85.385,07	.354,766,7	92.552,85	.016,234,38	43.539,89
82	2.158,610	2.075,983	-65.715,92	1.608,461	68.829,95	.916,275,7	87.485,30	.381,439,4	95.060,14	.017,469,32	44.668,79
83	2.265,174	2.187,148	-68.204,72	1.722,845	69.981,81	.983,136,7	89.606,78	.409,746,0	97.607,09	.018,781,38	45.315,80
84	2.375,609	2.302,290	-70.754,70	1.843,663	71.116,39	1.053,928	91.749,09	.439,763,7	100.193,9	.020,174,36	46.975,02
85	2.490,008	2.421,455	-73.327,99	1.971,175	72.232,26	1.128,824	93.911,82	.471,572,8	102.820,8	.021,652,04	48.152,52
86	2.608,462	2.544,684	-76.040,36	2.105,647	73.327,99	1.208,004	96.094,53	.505,256,0	105.488,1	.023,218,65	49.346,37
87	2.731,064	2.672,015	-78.777,04	2.247,353	74.402,08	1.291,653	98.296,78	.540,898,5	108.195,9	.024,878,32	50.556,66
88	2.857,906	2.803,479	-81.576,92	2.396,570	75.453,03	1.379,961	100.518,1	.578,588,4	110.944,4	.026,635,41	51.783,47
89	2.989,085	2.939,105	-84.440,44	2.553,583	76.479,31	1.473,122	102.758,0	.618,416,7	113.734,0	.028,494,37	53.026,88
90	3.124,695	3.078,916	-87.368,02	2.718,680	77.479,35	1.571,336	105.015,9	.660,476,8	116.564,7	.030,459,90	54.286,97

TABLE 19—Continued

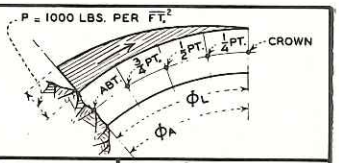
UNIT TANGENTIAL LOAD NO. 4
TANGENTIAL DEFLECTIONS



φ	Crown					¼ Point		½ Point		¾ Point	
	D_1	D_2		D_3		D_3		D_3		D_3	
°		1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term	1ST Term	2ND Term
10	.002,445,660	$.0^3$371,976,5	-.126,057,5	$.0^4$28,874,4	2.865,203	$.0^4$14,949,2	2.867,975	$.0^5$2,918,8	2.121,028	$.0^7$70,73	.740,943,1
11	.003,580,384	$.0^3$598,508,7	-.168,186,1	$.0^4$51,125,4	3.469,258	$.0^4$26,474,7	3.473,332	$.0^5$5,170,0	2.567,872	$.0^6$125,30	.896,681,7
12	.005,070,410	$.0^3$923,774,2	-.218,924,5	$.0^4$86,123,3	4.131,767	$.0^4$44,607,5	4.137,562	$.0^5$8,712,8	3.057,840	$.0^6$211,17	1.067,309
13	.006,983,084	.001,376,860	-.279,134,1	$.0^3$139,130,0	4.852,976	$.0^4$72,079,3	4.860,996	$.0^4$14,081,6	3.591,086	$.0^6$341,35	1.252,839
14	.009,391,569	.001,991,993	-.349,696,5	$.0^3$216,888,1	5.633,147	$.0^3$112,393	5.643,988	$.0^4$21,961,9	4.167,777	$.0^6$532,50	1.453,289
15	.012,374,84	.002,808,912	-.431,515,4	$.0^3$327,870	6.472,558	$.0^3$169,950	6.486,922	$.0^4$33,216,5	4.788,093	$.0^6$805,47	1.668,678
16	.016,017,68	.003,873,279	-.525,517,7	$.0^3$482,541	7.371,504	$.0^3$250,198	7.390,205	$.0^4$48,913,6	5.452,227	$.0^5$1,186,30	1.899,025
17	.020,410.66	.005,236,977	-.632,655,1	$.0^3$693,664	8.330,295	$.0^3$359,778	8.354,273	$.0^4$70,355,6	6.160,386	$.0^5$1,706,63	2.144,346
18	.025,650,11	.006,958,505	-.753,905,5	$.0^3$976,585	9.349,255	$.0^3$506,686	9.379,585	$.0^4$99,111,0	6.912,787	$.0^5$2,404,61	2.404,668
19	.031,838,18	.009,103,312	-.890,274,5	.001,349,564	10.428,72	$.0^3$700,448	10.466,62	$.0^3$137,054	7.709,662	$.0^5$3,325,8	2.680,013
20	.039,082,71	.011,744,11	- 1.042,798	.001,834,123	11.569,04	$.0^3$952,298	11.615,90	$.0^3$186,391	8.551,255	$.0^5$4,524,2	2.970,405
21	.047,497,33	.014,961,18	- 1.212,537	.002,455,367	12.770,56	.001,275,350	12.827,94	$.0^3$249,707	9.437,822	$.0^5$6,062,3	3.275,870
22	.057,201,39	.018,842,68	- 1.400,594	.003,242,418	14.033,67	.001,684,843	14.103,30	$.0^3$329,998	10.369,63	$.0^5$8,013,5	3.596,434
23	.068,319,95	.023,484,90	- 1.608,094	.004,228,734	15.358,72	.002,198,301	15.442,56	$.0^3$430,723	11.346,96	$.0^4$10,462,1	3.932,127
24	.080,983,75	.028,992,53	- 1.836,204	.005,452,542	16.746,11	.002,835,766	16.846,31	$.0^3$555,837	12.370,11	$.0^4$13,504,6	4.282,977
25	.095,329,25	.035,478,95	- 2.086,121	.006,957,250	18.196,21	.003,620,02	18.315,17	$.0^3$709,843	13.439,38	$.0^4$17,250,9	4.649,016
26	.111,498,6	.043,066,38	- 2.359,082	.008,791,891	19.709,42	.004,576,85	19.849,78	$.0^3$897,841	14.555,09	$.0^4$21,825,8	5.030,276
27	.129,639,4	.051,886,15	- 2.656,360	.011,011,53	21.286,13	.005,735,24	21.450,80	.001,125,57	15.717,56	$.0^4$27,369,7	5.426,790
28	.149,905,2	.062,078,86	- 2.979,265	.013,677,73	22.926,72	.007,127,64	23.118,91	.001,399,47	16.927,13	$.0^4$34,040,0	5.838,592
29	.172,454,9	.073,794,59	- 3.329,147	.016,859,04	24.631,58	.008,790,23	24.854,79	.001,726,69	18.184,16	$.0^4$42,012,3	6.265,721
30	.197,453,2	.087,192,98	- 3.707,399	.020,631,41	26.401,10	.010,763,18	26.659,15	.002,115,28	19.489,01	$.0^4$51,484,1	6.708,211
31	.225.070,2	.102,443,4	- 4.115,452	.025,078,80	28.235,66	.013,090,95	28.532,73	.002,574,02	20.842,04	$.0^4$62,671	7.166,102
32	.255,481,5	.119,725,2	- 4.554,779	.030,293,51	30.135,63	.015,822,49	30.476,26	.003,112,71	22.243,65	$.0^4$75,814	7.639,435
33	.288,868,6	.139,227,5	- 5.026,897	.036,376,82	32.101,37	.019,011,64	32.490,49	.003,742,09	23.694,23	$.0^4$91,174	8.128,248
34	.325,418,0	.161,149,5	- 5.533,363	.043,439,42	34,133,24	.022,717,29	34.576,20	.004,473,92	25.194,17	$.0^3$109,047	8.632,587
35	.365,322,1	.185,700,4	- 6.075,782	.051,601,98	36.231,57	.027,003,82	36.734,16	.005,321,10	26.743,91	$.0^3$129,745	9.152,493
36	.408,778,5	.213,099,6	- 6.655,800	.060,995,60	38.396,70	.031,941,27	38.965,17	.006,297,67	28.343,85	$.0^3$153,616	9.688,012
37	.455,990,4	.243,576,4	- 7.275,107	.071,762,45	40.628,92	.037,605,73	41.270,02	.007,418,91	29.994,45	$.0^3$181,039	10.239,19
38	.507,166,4	.277,370,3	- 7.935,439	.084,056,27	42.928,53	.044,079,66	43.649,52	.008,701,42	31.696,13	$.0^3$212,423	10.806,07
39	.562,520,3	.314,730,8	- 8.638,579	.098,042,78	45.295,81	.051,452,16	46.104,49	.010,163,2	33.449,36	$.0^3$248,213	11.388,71
40	.622.271,5	.355,917,3	- 9.386,350	.113,900,5	47.730,98	.059,819,38	48.635,77	.011,823,5	35.254,59	$.0^3$288,891	11.987,15
41	.686,644,6	.401,199,0	- 10.180,63	.131,820,9	50.234,29	.069,284,72	51.244,16	.013,703,4	37.112,31	$.0^3$334,973	12.601,45
42	.755,869,6	.450,855,0	- 11.023,33	.152,009,4	52.805,92	.079,959,26	53.930,52	.015,825,4	39.022,98	$.0^3$387,020	13.231,66
43	.830,181,6	.505,173,7	- 11.916,41	.174,685,5	55.446,03	.091,962,15	56.695,68	.018,213,6	40.987,11	$.0^3$445,634	13.877,82
44	.909,821,2	.564,453,2	- 12.861,89	.200.083,6	58.154,78	.105,420,7	59.540,47	.020.894,0	43.005,18	$.0^3$511,463	14.540,00
45	.995,034,0	.629,000,8	- 13.861,81	.228,453,5	60.932,24	.120,471,2	62.465,75	.023.894,3	45.077,70	$.0^3$585,194	15.218,26
46	1.086,071	.699,132,8	- 14.918,28	.260,060,9	63.778,50	.137,258,6	65.472,35	.027,244,2	47.205,19	$.0^3$667,569	15.912,64
47	1.183,188	.775,174,1	- 16.033,43	.295,187,7	66.693,57	.155,937,5	68.561,11	.030,975,3	49.388,16	$.0^3$759,384	16.623.21
48	1.286,646	.857,458,3	- 17.209,45	.334,133,2	69.677,45	.176,672,3	71.732,88	.035,121,2	51.627,15	$.0^3$861,477	17.350,02
49	1.396,712	.946,327,4	- 18.448,58	.377,213,8	72.730,09	.199,637,2	74.988,48	.039,717,9	53.922,68	$.0^3$974,746	18.093,14
50	1.513,657	1.042,131	- 19.753,08	.424,764,1	75.851,39	.225.016,9	78.328,75	.044,803,4	56.275,31	.001,100,159	18.852,63
51	1.637,758	1.145,226	- 21.125,28	.477,137,5	79.041,19	.253,007,2	81.754,51	.050,418,2	58.685,58	.001,238,719	19.628,54
52	1.769,295	1.255,978	- 22.567,52	.534,705,8	82.299,32	.283,814,6	85.266,58	.056,605,1	61.154,04	.001,391,533	20.420,96
53	1.908,555	1.374,757	- 24.082,22	.597,861,1	85.625,52	.317,657,9	88.865,76	.063,409,2	63.681,26	.001,559,678	21.229,93
54	2.055,829	1.501,941	- 25.671,80	.667,014,9	89.019,51	.354,766,7	92.552,85	.070.878,7	66.267,80	.001,744,449	22.055,53
55	2.211,413	1.637,915	- 27.338,75	.742,599,9	92.480,92	.395,383,6	96.328,65	.079,064,0	68.914,22	.001,947,10	22.897,82
56	2.375,609	1.783,068	- 29.085,57	.825.069,1	96.009,37	.439,763,7	100.193,9	.088,018,5	71.621,12	.002,168,92	23.756,87
57	2.548,722	1.937,793	- 30.914,83	.914,897,6	99.604,36	.488,174,9	104.149,4	.097,798,2	74.389,07	.002,411,41	24.632,76
58	2.731,064	2.102,489	- 32.829,11	1.012,582	103.265,4	.540,898,5	108.195,9	.108,462,5	77.218,66	.002,676,05	25.525,55
59	2.922,948	2.277,560	- 34.831,03	1.118,641	106.991,8	.598,229,4	112.334,1	.120,073,5	80.110,47	.002,964,45	26.435,31
60	3.124,695	2.463,412	- 36.923,25	1.233,618	110.783,1	.660,476,8	116.564,7	.132,696,6	83.065,11	.003,278,25	27.362,11
61	3.336,631	2.660,454	- 39.108,45	1.358,076	114.638,4	.727,963,6	120.888,4	.146,400,3	86.083,16	.003,619,20	28.306,04
62	3.559,084	2.869,097	- 41.389,35	1.492,604	118.557,0	.801,028,1	125.305,9	.161,256,5	89.165,24	.003,989,18	29.267,17
63	3.792,388	3.089,755	- 43.768,71	1.637,815	122.537,9	.880,023,6	129.817,9	.177,340,5	92.311,93	.004,390,11	30.245,56
64	4.036,882	3.322,842	- 46.249,27	1.794,346	126.580,4	.965,318,6	134.424,9	.194,731,4	95.523,86	.004,824,00	31.241,31
65	4.292,909	3.568,771	- 48.833,86	1.962,857	130.683,3	1.057,297	139.127,6	.213,511,4	98.801,63	.005,293,00	32.254,48
66	4.560,817	3.827,959	- 51.525,28	2.144,035	134.845,5	1.156,360	143.926,5	.233,766,9	102.145,9	.005,799,33	33.285,15
67	4.840,956	4.100,816	- 54.326,38	2.338,592	139.066,0	1.262,924	148.822,2	.255,587,8	105.557,1	.006,345,32	34.333,42
68	5.133,684	4.387,756	- 57.240,02	2.547,264	143.343,3	1.377,422	153.815,3	.279,068,2	109.036,1	.006,933,42	35.399,35
69	5.439,361	4.689,187	- 60.269,07	2.770,813	147.676,4	1.500,305	158.906,2	.304,306,0	112.583,4	.007,566,18	36.483,03
70	5.758,352	5.005,514	- 63.416,44	3.010,028	152.063,6	1.632,042	164.095,3	.331,403,2	116.199,5	.008,246,26	37.584,54
71	6.091,027	5.337,140	- 66.685,02	3.265,724	156.503,5	1.773,117	169.383,2	.360,466,4	119.885,2	.008,976,42	38.703,97
72	6.437,757	5.684,461	- 70.077,73	3.538,742	160.994,6	1.924,035	174.770,2	.391,606,0	123.641,1	.009,759,57	39.841,40
73	6.798,922	6.047,869	- 73.597,52	3.829,948	165.535,1	2.085,317	180.256,6	.424,937,2	127.467,7	.010,598,74	40.996,92
74	7.174,903	6.427,749	- 77.247,29	4.140,238	170.123,3	2.257,503	185.842,9	.460,579,6	131.365,7	.011,497,06	42.170,62
75	7.566,085	6.824,478	- 81.029,99	4.470,530	174.757,4	2.441,154	191.529,3	.498,657,4	135.335,7	.012,457,81	43.362,58
76	7.972,858	7.238,425	- 84.948,55	4.821,773	179.435,3	2.636,848	197.316,0	.539,299,6	139.378,3	.013,484,41	44.572,89
77	8.395,616	7.669,957	- 89.005,93	5.194,943	184.155,1	2.845,183	203.203,3	.582,640,2	143.494,1	.014,580,40	45.801,64
78	8.834,756	8.119,409	- 93.205,03	5.591,034	188.914,7	3.066,775	209.191,3	.628,817,4	147.683,7	.015,749,46	47.048,93
79	9.290,681	8.587,137	- 97.548,80	6.011,081	193.711,7	3.302,263	215.280,2	.677,975,6	151.947,8	.016,995,42	48.314,85
80	9.763,795	9.073,462	- 102.040,1	6.456,135	198.543,9	3.552,305	221.470,0	.730,263,4	156.286,9	.018,322,25	49.599,48
81	10.254,51	9.578,703	- 106.682,0	6.927,278	203.408,9	3.817,577	227.760,9	.785,834,8	160.701,7	.019,734,09	50.902,92
82	10.763,23	10.103,16	- 111.477,2	7.425,617	208.304,1	4.098,779	234.152,8	.844,849,5	165.192,7	.021,235,21	52.225,27
83	11.290,38	10.647,12	- 116.428,6	7.952,285	213.226,9	4.396,630	240.645,6	.907,472,9	169.760,6	.022,830,05	53.566,61
84	11.836,38	11.210,86	- 121.539,1	8.508,445	218.174,7	4.711,871	247.239,3	.973,874,5	174.405,9	.024,523,20	54.927,06
85	12.401,65	11.794,64	- 126.811,6	9.095,281	223.144,5	5.045,262	253.933,8	1.044,231	179.129,2	.026,319,42	56.306,70
86	12.986,62	12.398,69	- 132.248,8	9.714,006	228.133,5	5.397,588	260.728,8	1.118,724	183.931,1	.028,223,66	57.705,63
87	13.591,72	13.023,23	- 137.853,4	10.365,86	233.138,6	5.769,653	267.624,0	1.197,542	188.812,2	.030,241,02	59.123,95
88	14.217,38	13.668,47	- 143.628,4	11.052,10	238.156,8	6.162,281	274.619,3	1.280,877	193.773,1	032,376,78	60.561,76
89	14.864,05	14.334,60	- 149.576,2	11.774,02	243.184,8	6.576,322	281.714,2	1.368,931	198.814,3	034,636,35	62.019,16
90	15.532,16	15.021,84	- 155.699,7	12.532,96	248.219,2	7.012,646	288.908,4	1.461,908	203.936,4	037,025,47	63.496,25

TABLE 19—Continued

UNIT TANGENTIAL LOAD NO. 5
TANGENTIAL DEFLECTIONS

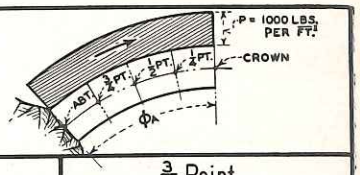


Φ °	Crown D_1	Crown D_2 1ST Term	Crown D_2 2ND Term	Crown D_3 1ST Term	Crown D_3 2ND Term	¼ Point D_3 1ST Term	¼ Point D_3 2ND Term	½ Point D_3 1ST Term	½ Point D_3 2ND Term	¾ Point D_3 1ST Term	¾ Point D_3 2ND Term
10	.007,727,042	.001,119,538	-.004,035,59	.04 83,892,5	5.11 5,357	.04 28,649,7	4.308,063	.05 3,721,5	2.545,717	.07 77,01	.793,878,4
11	.011,311,44	.001,801,296	-.006,494,89	.03 148,553	6.199,259	.04 50,737,3	5.218,925	.05 6,591,8	3.082,153	.06 1,136,41	.960,746,3
12	.016,017,68	.002,780,168	-.010,027,5	.03 .250,198	7.390,205	.04 85,485	6.218,981	.04 11,108,7	3.670,409	.06 ,229,91	1.143,567
13	.022,058,17	.004,143,664	-.014,950,3	.03 ,404,162	8.689,216	.03 ,138,130	7.308,889	.04 17,953,7	4.31 0,681	.06 ,371,63	1.342,358
14	.029,663,55	.005,994,747	-.021,636,6	.03 ,630,007	10.097,39	.03 ,215,371	8.489,364	.04 28,001,0	5.003,185	.06 ,579,73	1.557,137
15	.039,082,71	.008,452,995	-.030,520,7	.03 ,952,298	11.61 5,90	.03 ,325,668	9.761,173	.04 42,350,1	5.748,152	.06 ,876,92	1.787,924
16	.050,582,69	.011,655,67	-.042,101,4	.001,401,430	13.246,00	.03 ,479,433	11.125,14	.04 62,363,4	6.545,831	.05 1,291,54	2.034,738
17	.064,448,64	.015,758,88	-.056,947,4	.002,014,411	14.989,00	.03 ,689,384	12.582,13	.04 89,701,2	7.396,486	.05 1,858,03	2.297,604
18	.080,983,75	.020,938,55	-.075,699,8	.002,835,766	16.846,31	.03 ,970,847	14.133,06	.03 1,126,363	8.300,399	.05 2,61 8,0	2.576,544
19	.100,509,2	.027,391,45	-.099,077,7	.003,918,43	18.819,37	.001,342,058	15.778,92	.03 1,174,739	9.257,867	.05 3,620,9	2.871,583
20	.123,364,0	.035,336,22	-.127,881	.005,324,78	20.909,71	.001,824,531	17.520,73	.03 ,237,641	10.269,21	.05 4,925,5	3.182,749
21	.149,905,2	.045,014,18	-.162,993	.007,127,64	23.118,91	.002,443,382	19.359,55	.03 ,318,367	11.334,74	.05 6,600,1	3.510,070
22	.180,507,3	.056,690,37	-.205,389	.009,411,29	25.448,60	.003,227,78	21.296,49	.03 ,420,739	12.454,82	.05 8,724,3	3.853,575
23	.215,562,8	.070,654,03	-.256,132	.012,272,67	27.900,47	.004,211,26	23.332,73	.03 ,549,149	13.629,80	.04 11,390,1	4.213,295
24	.255,481,5	.087,219,96	-.316,383	.015,822,49	30.476,26	.005,432,19	25.469,47	.03 ,708,661	14.860,07	.04 14,702,4	4.589,261
25	.300,691,0	.106,728,7	-.387,403	.020,186,41	33.177,75	.006,934,20	27.707,95	.03 ,905,003	16.146,00	.04 18,781,2	4.981,509
26	.351,635,9	.129,547,5	-.470,548	.025,506,23	36.006,77	.008,766,58	30.049,46	.001,144,678	17.488,02	.04 23,761,9	5.390,073
27	.408,778,5	.156,070,7	-.567,288	.031,941,27	38.965,17	.010,984,81	32.495,35	.001,435,007	18.886,54	.04 29,797,6	5.814,990
28	.472,598,3	.186,720,8	-.679,195	.039,669,65	42.054,85	.013,650,97	35.046,96	.001,784,194	20.342,00	.04 37,059,5	6.256,296
29	.543,591,4	.221,948,2	-.807,952	.048,889,03	45.277,73	.016,834,27	37.705,72	.002,201,37	21.854,85	.04 45,738,9	6.714,033
30	.622,271,5	.262,232,4	-.955,352	.059,819,38	48.635,77	.020,611,54	40.473,06	.002,696,76	23.425,56	.04 56,050,6	7.188,240
31	.709,169,0	.308,081,9	-1.123,310	.072,702,83	52.130,91	.025,067,73	43.350,46	.003,281,60	25.054,60	.04 68,230	7.678,960
32	.804,831,0	.360,034,6	-1.313,852	.087,805,90	55.765,15	.030,296,52	46.339,42	.003,968,35	26.742,48	.04 82,538	8.186,236
33	.909,821,2	.418,658,3	-1.529,123	.105,420,7	59.540,47	.036,400,75	49.441,49	.004,770,72	28.489,69	.04 99,262	8.710,113
34	1.024,720	.484,550,7	-1.771,394	.125,866,6	63.458,87	.043,493,01	52.658,24	.005,703,67	30.296,76	.03 ,118,720	9.250,637
35	1.150,124	.558,339,4	-2.043,057	.149,491,2	67.522,35	.051,696,24	55.991,24	.006,783,67	32.164,22	.03 ,141,253	9.807,855
36	1.286,646	.640,682,2	-2.346,623	.176,672,3	71.732,88	.061,144,36	59.442,14	.008,028,62	34.092,63	.03 ,167,248	10.381,82
37	1.434,916	.732,267,1	-2.684,737	.207,819,3	76.092,46	.071,982,64	63.012,56	.009,457,99	36.082,53	.03 ,197,098	10.972,57
38	1.595,579	.833,812,1	-3.060,165	.243,374,4	80.603,05	.084,368,6	66.704,18	.011,092,92	38.134,51	.03 ,231,265	11.580,18
39	1.769,295	.946,065,0	-3.475,801	.283,814,6	85.266,58	.098,472,4	70.518,69	.012,956,31	40.249,15	.03 ,270,231	12.204,68
40	1.956,741	1.069,803	-3.934,669	.329,653,0	90.084,99	.114,477,3	74.457,77	.015,072,89	42.427,04	.03 ,314,516	12.846,13
41	2.158,610	1.205,834	-4.439,920	.381,439,4	95.060,14	.132,581,0	78.523,16	.017,469,32	44.668,79	.03 ,364,685	13.504,59
42	2.375,609	1.354,993	-4.994,838	.439,763,7	100.193,9	.152,995,2	82.716,58	.020,174,36	46.975,02	.03 ,421,347	14.180,11
43	2.608,462	1.518,145	-5.602,833	.505,256,0	105.488,1	.175,947,4	87.039,78	.023,218,65	49.346,37	.03 ,485,161	14.872,76
44	2.857,906	1.696,183	-6.267,446	.578,588,4	110.944,4	.201,680,4	91.494,51	.026,635,41	51.783,47	.03 ,556,829	15.582,60
45	3.124,695	1.890,027	-6.992,345	.660,476,8	116.564,7	.230,453,7	96.082,53	.030,459,90	54.286,97	.03 ,637,099	16.309,67
46	3.409,597	2.100,623	-7.781,327	.751,681,5	122.350,5	.262,544,4	100.805,6	.034,730,10	56.857,55	.03 ,726,781	17.054,06
47	3.713,394	2.328,946	-8.638,319	.853,010,4	128.303,4	.298,246,0	105.665,5	.039,485,87	59.495,86	.03 ,826,739	17.815,81
48	4.036,882	2.575,994	-9.567,376	.965,318,6	134.424,9	.337,872,7	110.664,0	.044,770,55	62.202,59	.03 ,937,887	18.595,00
49	4.380,874	2.842,790	-10.572,68	1.089,511	140.716,5	.381,755,6	115.802,9	.050,629,76	64.978,44	.001,061,206	19.391,68
50	4.746,195	3.130,381	-11.658,53	1.226,543	147.179,5	.430,247,4	121.083,9	.057,111,95	67.824,10	.001,197,738	20.205,94
51	5.133,684	3.439,836	-12.829,36	1.377,422	153.815,3	.483,719,8	126.508,9	.064,268,74	70.740,27	.001,348,590	21.037,83
52	5.544,194	3.772,247	-14.089,72	1.543,210	160.624,9	.542,566,7	132.079,4	.072,154,5	73.727,68	.001,514,936	21.887,42
53	5.978,592	4.128,724	-15.444,28	1.725,024	167.609,6	.607,203,5	137.797,4	.080,827,0	76.787,05	.001,698,01	22.754,80
54	6.437,757	4.510,399	-16.897,83	1.924,035	174.770,2	.678,067,8	143.664,5	.090,347,4	79.919,11	.001,899,16	23.640,02
55	6.922,584	4.918,420	-18.455,27	2.141,473	182.107,6	.755,620,7	149.682,4	.100,780,0	83.124,60	.002,119,80	24.543,17
56	7.433,978	5.353,951	-20.121,62	2.378,628	189.622,7	.840,346,7	155.852,9	.112,193,0	86.404,26	.002,361,29	25.464,31
57	7.972,858	5.818,173	-21.902,01	2.636,848	197.316,0	.932,755,2	162.177,6	.124,657,5	89.758,85	.002,625,28	26.403,53
58	8.540,190	6.312,279	-23.801,69	2.917,541	205.188,1	1.033,381	168.658,1	.138,249,3	93.189,12	.002,913,39	27.360,89
59	9.136,816	6.837,476	-25.825,99	3.222,183	213.239,3	1.142,784	175.296,2	.153,047,5	96.695,84	.003,227,36	28.336,49
60	9.763,795	7.394,979	-27.980,35	3.552,305	221.470,0	1.261,550	182.093,4	.169,135,2	100.279,8	.003,569,00	29.330,39
61	10.422,06	7.986,015	-30.270,32	3.909,506	229.880,3	1.390,294	189.051,4	.186,600,0	103.941,7	.003,940,19	30.342,68
62	11.112,59	8.611,815	-32.701,54	4.295,451	238.470,1	1.529,657	196.171,5	.205,533,4	107.682,4	.004,34298	31.373,45
63	11.836,38	9.273,617	-35.279,74	4.711,871	247.239,3	1.680,308	203.455,6	.226,031,3	111.502,6	.004,779,45	32.422,76
64	12.594,43	9.972,662	-38.010,73	5.160,562	256.187,6	1.842,946	210.904,7	.248,194,3	115.403,2	.005,251,82	33.490,72
65	13.387,76	10.710,19	-40.900,41	5.643,387	265.314,5	2.018,301	218.520,6	.272,127,1	119.384,9	.005,762,41	34.577,40
66	14.217,38	11.487,45	-43.954.78	6.162,281	274.619,3	2.207,130	226.304,5	.297,940,0	123.448,5	.006,313,64	35.682,88
67	15.084,35	12.305,68	-47.179,85	6.719,244	284.101,2	2.410,222	234.258,0	.325,747,4	127.594,9	.006,908,05	36.807,27
68	15.989,69	13.166,11	-50.581,79	7.316,348	293.759,4	2.628,399	242.382,0	.355,669,0	131.824,7	.007,548,30	37.950,64
69	16.934,48	14.069,97	-54.166,76	7.955,734	303.592,6	2.862,514	250.678,5	.387,829,6	136.138,9	.008,237,17	39.113,09
70	17.919,77	15.018,49	-57.941,02	8.639,612	313.599,5	3.113,451	259.148,0	.422,358,8	140.538,1	.008,977,57	40.294,70
71	18.946,64	16.012,86	-61.910,87	9.370,266	323.778,6	3.382,129	267.792,0	.459,393,4	145.023,3	.009,772,45	41.495,57
72	20.016,18	17.054,30	-66.082,66	10.150,05	334.128,3	3.669,499	276.611,6	.499,072,8	149.595,2	.010,625,04	42.715,79
73	21.129,48	18.143,98	-70.462,79	10.981,39	344.646,6	3.976,548	285.607,7	.541,544,2	154.254,6	.011,538,63	43.955,46
74	22.287,65	19.283,05	-75.057,71	11.866,77	355.331,5	4.304,298	294.781,5	.586,959,7	159.002,3	.012,516,59	45.214,66
75	23.491,81	20.472,68	-79.873,88	12.808,77	366.180,7	4.653,804	304.133,8	.635,477,7	163.839,1	.013,562,54	46.493,50
76	24.743,06	21.713,97	-84.917,81	13.810,03	377.191,8	5.026,157	313.665,6	.687,262,3	168.765,9	.014,680,16	47.792,08
77	26.042,55	23.008,02	-90.195,98	14.873,25	388.362,1	5.422,486	323.377,7	.742,483,8	173.783,3	.015,873,32	49.110,49
78	27.391,41	24.355,90	-95.714,99	16.001,22	399.688,8	5.843,951	333.270,7	.801,318,8	178.892,3	.017,146,04	50.448,83
79	29.790,79	25.758,66	-101.481,3	17.196,79	411.168,8	6.291,759	343.345,5	.863,950,6	184.093,5	.018,502,46	51.807,20
80	30.241,85	27.217,29	-107.501,6	18.462,88	422.798,8	6.767,147	353.602,5	.930,568,6	189.387,9	.019,946,93	53.185,70
81	31.745,73	28.732,77	-113.782,3	19.802,49	434.575,1	7.271,389	364.042,4	1.001,369	194.776,1	.021,483,95	54.584,43
82	33.303,62	30.306,03	-120.330,0	21.218,67	446.494,8	7.805,802	374.665,7	1.076,555	200.259,0	.023,118,16	56.003,50
83	34.916,69	31.937,98	-127.151,2	22.714,56	458.553,1	8.371,737	385.472,6	1.156,336	205.837,3	.024,854,39	57.443,02
84	36.586,12	33.629,46	-134.252,3	24.293,36	470.746,3	8.970,588	396.463,5	1.240,930	211.511,8	.026,697,64	58.903,08
85	38.313,11	35.381,28	-141.640,0	25.958,33	483.070,0	9.603,785	407.638,6	1.330,560	217.283,4	.028,653,12	60.383,79
86	40.098,83	37.194,20	-149.320,6	27.712,80	495.519,6	10.272,80	418.998,2	1.425,458	223.152,6	.030,726,17	61.885,26
87	41.944,50	39.068,93	-157.300,4	29.560,18	508.090,4	10.979,14	430.542,2	1.525,864	229.120,4	.032,922,37	63.407,59
88	43.851,33	41.006,14	-165.585,9	31.503,92	520.777,5	11.724,35	442.270,7	1.632,022	235.187,4	.035,247,47	64.950,90
89	45.820,52	43.006,43	-174.183,3	33.547,56	533.575,5	12.51 0,04	454.183,4	1.744,188	241.354,4	.037,707,35	66.515,30
90	47.853,28	45.070,34	-183.098,9	35.694,66	546.479,1	13.337,82	466.280,3	1.862,623	247.622,2	.040,308,26	68.100,88

TABLE 20—CONSTANTS FOR RADIAL AND TWIST DEFLECTIONS, TANGENTIAL LOADS

UNIT TANGENTIAL LOAD NO. 1

RADIAL AND TWIST DEFLECTIONS

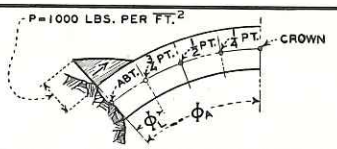


φ °	Crown			1/4 Point			1/2 Point			3/4 Point		
	D_1	D_2		D_1	D_2		D_1	D_2		D_1	D_2	
		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.038,624,00	.005,373,006	.868,610,9	.012,226,31	.001,277,674	.369,666,3	.002,415,839	$0^3$168,501,5	.110,214,2	$0^3$151,018,7	$0^5$5,270,328	.013,828,12
11	.056,537,34	.008,644,710	1.151,254	.017,898,39	.002,056,560	.490,867,1	.003,536,841	$0^3$271,306,4	.146,542,2	$0^3$221,103,5	$0^5$8,487,391	.018,400,44
12	.080,054,96	.013,342,03	1.487,730	.025,346,10	.003,175,552	.635,632,5	.005,008,925	$0^3$419,068,2	.190,034,2	$0^3$313,143,6	$0^4$13,112,56	.023,881,98
13	.110,236,8	.019,884,74	1.881,984	.034,905,79	.004,735,236	.805,877,8	.006,898,670	$0^3$625,125,5	.241,311,4	$0^3$431,305,6	$0^4$19,564,38	.030,354,43
14	.148,233,4	.028,766,68	2.337,715	.046,942,83	.006,854,153	1.003,459	.009,278,426	$0^3$905,216,3	.300,987,2	$0^3$580,118,1	$0^4$28,337,07	.037,899,26
15	.195,286,0	.040,561,22	2.858,360	.061,851,50	.009,670,204	1.230,168	.012,226,31	.001,277,674	.369,666,3	$0^3$764,471,5	$0^4$40,006,82	.046,597,68
16	.252,725,7	.055,926,64	3.447,075	.080,054,96	.013,342,03	1.487,730	.015,826,18	.001,763,619	.447,944,7	$0^3$989,618,1	$0^4$55,237,98	.056,530,59
17	.321,973,1	.075,611,22	4.106,716	.102,005,2	.018,050,37	1.777,797	.020,167,65	.002,387,155	.536,408,2	.001,261,172	$0^4$74,789,45	.067,778,60
18	.404,538,3	.100,458,3	4.839,826	.128,182,8	.023,999,40	2.101,944	.025,346,10	.003,175,552	.635,632,5	.001,585,109	$0^4$99,520,73	.080,421,98
19	.502,019,9	.131,411,0	5.648,610	.159,097,1	.031,418,03	2.461,662	.031,462,61	.004,159,443	.746,182,4	.001,967,765	$0^3$130,398,2	.094,540,67
20	.616,104,7	.169,516,8	6.534,924	.195,286,0	.040,561,22	2.858,360	.038,624,00	.005,373,006	.868,610,9	.002,415,839	$0^3$168,501,5	.110,214,2
21	.748,567,6	.215,931,9	7.500,260	.237,315,8	.051,711,18	3.293,353	.046,942,83	.006,854,153	1.003,459	.002,936,391	$0^3$215,029,3	.127,521,8
22	.901,270,2	.271,925,6	8.545,724	.285,781,1	.065,178,61	3.767,864	.056,537,34	.008,644,710	1.151,254	.003,536,841	$0^3$271,306,4	.146,542,2
23	1.076,161	.338,883,7	9.672,028	.341,305,0	.081,303,91	4.283,014	.067,531,50	.010,790,60	1.312,511	.004,224,972	$0^3$338,787,8	.167,353,6
24	1.275,275	.418,312,8	10.879,47	.404,538,3	.100,458,3	4.839,826	.080,054,96	.013,342,03	1.487,730	.005,008,925	$0^3$419,068,2	.190,034,2
25	1.500,731	.511,843,0	12.167,94	.476,160,3	.123,044,9	5.439,212	.094,243,04	.016,353,65	1.677,398	.005,897,203	$0^3$513,885,0	.214,661,1
26	1.754,734	.621,231,9	13.536,85	.556,878,0	.149,500,1	6.081,974	.110,236,8	.019,884,74	1.881,984	.006,898,670	$0^3$625,125,5	.241,311,4
27	2.039,574	.748,366,7	14.985,21	.647,426,3	.180,293,9	6.768,800	.128,182,8	.023,999,40	2.101,944	.008,022,549	$0^3$754,8339	.270,061,4
28	2.357,621	.895,267,4	16.511,52	.748,567,6	.215,931,9	7.500,260	.148,233,4	.028,766,68	2.337,715	.009,278,426	$0^3$905,216,3	.300,987,2
29	2.711,332	1.064,089	18.113,85	.861,092,0	.256,955,4	8.276,800	.170,546,8	.034,260,78	2.589,718	.010,676,24	.001,078,647	.334,163,8
30	3.103,243	1.257,122	19.789,75	.985,816,9	.303,942,8	9.098,743	.195,286,0	.040,561,22	2.858,360	.012,226,31	.001,277,674	.369,666,3
31	3.535,971	1.476,798	21.536,29	1.123,587	.357,510,3	9.966,281	.222,620,8	.047,752,97	3.144,023	.013,939,28	.001,505,026	.407,568,7
32	4.012,215	1.725,688	23.350,05	1.275,275	.418,312,8	10.879,47	.252,725,7	.055,926,64	3.447,075	.015,826,18	.001,763,619	.447,944,7
33	4.534,753	2.006,502	25.227,07	1.441,778	.487,044,6	11.838,25	.285,781,1	.065,178,61	3.767,864	.017,898,39	.002,056,560	.490,867,1
34	5.106,441	2.322,095	27.162,89	1.624,023	.564,440,3	12.842,39	.321,973,1	.075,611,22	4.106,716	.020,167,65	.002,387,155	.536,408,2
35	5.730,214	2.675,464	29.152,52	1.822,962	.651,275,3	13.891,54	.361,493,2	.087,332,90	4.463,942	.022,646,07	.002,758,912	.584,639,7
36	6.409,082	3.069,747	31.190,44	2.039,574	.748,366,7	14.985,21	.404,538,3	.100,458,3	4.839,826	.025,346,10	.003,175,552	.635,632,5
37	7.146,134	3.508,226	33.270,58	2.274,863	.856,573,6	16.122,73	.451,311,2	.115,108,5	5.234,634	.028,280,55	.003,641,010	.689,457,0
38	7.944,531	3.994,326	35.386,33	2.529,862	.976,797,9	17.303,32	.502,019,9	.131,411,0	5.648,610	.031,462,61	.004,159,443	.746,182,4
39	8.807,509	4.531,611	37.530,56	2.805,628	1.109,985	18.526,03	.556,878,0	.149,500,1	6.081,974	.034,905,79	.004,735,236	.805,877,8
40	9,738,379	5.123,788	39.695,55	3.103,243	1.257,122	19.789,75	.616,104,7	.169,516,8	6.534,924	.038,624,00	.005,373,006	.868,610,9
41	10.740,52	5.774,700	41.873,07	3.423,816	1.419,243	21.093,21	.679,924,6	.191,609,0	7.007,636	.042,631,48	.006,077,612	.934,449,1
42	11.817,39	6.488,328	44.054,30	3.768,482	1.597,425	22.435,00	.748,567,6	.215,931,9	7.500,260	.046,942,83	.006,854,153	1.003,459
43	12.972,51	7.268,790	46.229,89	4.138,399	1.792,789	23.813,54	.822,269,1	.242,647,6	8.012,921	.051,573,01	.007,707,983	1.075,705
44	14.209,46	8.120,332	48.389,95	4.534,753	2.006,502	25.227,07	.901,270,2	.271,925,6	8.545,724	.056,537,34	.008,644,710	1.151,254
45	15.531,92	9.047,332	50.524,02	4.958,752	2.239,775	26.673,69	.985,816,9	.303,942,8	9.098,743	.061,851,50	.009,670,204	1.230,168
46	16.943,60	10.054,30	52.621,08	5.411,631	2.493,865	28.151,31	1.076,161	.338,883,7	9.672,028	.067,531,50	.010,790,60	1.312,511
47	18.448,30	11.145,85	54.669,59	5.894,647	2.770,073	29.657,70	1.172,559	.376,940,4	10.265,61	.073,593,74	.012,012,31	1.398,345
48	20.049,87	12.326,73	56.657,44	6.409,082	3.069,747	31.190,44	1.275,275	.418,312,8	10.879,47	.080,054,96	.013,342,03	1.487,730
49	21.752,24	13.601,81	58.572,04	6.956,245	3.394,278	32.746,94	1.384,574	.463,208,4	11.513,60	.086,932,24	.014,786,72	1.580,728
50	23.559,38	14.976,06	60.400,20	7.537,464	3.745,104	34.324,44	1.500,731	.511,843,0	12.167,94	.094,243,04	.016,353,65	1.677,398
51	25.475,35	16.454,55	62.128,25	8.154,093	4.123,706	35.920,01	1.624,023	.564,440,3	12.842,39	.102,005,2	.018,050,37	1.777,797
52	27.504,23	18.042,45	63.741,98	8.807,509	4.531,611	37.530,56	1.754,734	.621,231,9	13.536,85	.110,236,8	.019,884,74	1.881,984
53	29.650,19	19.745,04	65.226,67	9.499,113	4.970,390	39.152,80	1.893,153	.682,458,0	14.251,18	.118,956,3	.021,864,92	1.990,014
54	31.917,45	21.567,68	66.567,14	10.230,33	5.441,656	40.783,28	2.039,574	.748,366,7	14.985,21	.128,182,8	.023,999,40	2.101,944
55	34.310,28	23.515,80	67.747,63	11.002,60	5.947,069	42.418,39	2.194,295	.819,214,8	15.738,73	.137,935,3	.026,296,94	2.217,826
56	36.833,02	25.594,94	68.752,03	11.817,39	6.488,328	44.054,30	2.357,621	.895,267,4	16.511,52	.148,233,4	.028,766,68	2.337,715
57	39.490,03	27.810,67	69.563,64	12.676,20	7.067,179	45.687,06	2.529,862	.976,797,9	17.303,32	.159,097,1	.031,418,03	2.461,662
58	42.285,76	30.168,64	70.165,33	13.580,53	7.685,405	47.312,51	2.711,332	1.064,089	18.113,85	.170,546,8	.034,260,78	2.589,718
59	45.224,69	32.674,57	70.539,58	14.531,92	8.344,834	48.926,33	2.902,351	1.157,430	18.942,77	.182,602,6	.037,305,03	2.721,935
60	48.311,36	35.334,20	70.668,42	15.531,92	9.047,332	50.524,02	3.103,243	1.257,122	19.789,75	.195,286,0	.040,561,22	2.858,360
61	51.550,33	38.153,34	70.553,38	16.582,11	9.794,807	52.100,90	3.314,338	1.363,473	20.654,39	.208,618,2	.044,040,15	2.999,040
62	54.946,24	41.137,80	70.115,72	17.684,08	10.589,20	53.652,14	3.535,971	1.476,798	21.536,29	.222,620,8	.047,752,97	3.144,023
63	58.503,77	44.293,43	69.396,21	18.839,46	11.432,51	55.172,72	3.768,482	1.597,425	22.435,00	.237,315,8	.051,711,18	3.293,353
64	62.227,62	47.626,10	68.355,33	20.049,87	12.326,73	56.657,44	4.012,215	1.725,688	23.350,05	.252,725,7	.055,926,64	3.447,075
65	66.122,56	51.141,68	66.973,11	21.316,98	13.273,94	58.100,97	4.267,521	1.861,929	24.280,92	.268,873,1	.060,411,58	3.605,231
66	70.193,38	54.846,03	65.229,35	22.642,47	14.276,23	59.497,77	4.534,753	2.006,502	25.227,07	.285,781,1	.065,178,61	3.767,864
67	74.444,93	58.745,01	63.103,45	24.028,01	15.335,71	60.842,14	4.814,272	2.159,767	26.187,93	.303,473,2	.070,240,71	3.935,012
68	78.882,07	62.844,45	60.574,61	25.475,35	16.454,55	62.128,25	5.106,441	2.322,095	27.162,89	.321,973,1	.075,611,22	4.106,716
69	83.509,71	67.150,14	57.621,60	26.986,19	17.634,93	63.350,06	5.411,631	2.493,865	28.151,31	.341,305,0	.081,303,91	4.283,014
70	88.332,82	71.667,85	54.223,10	28.562,31	18.879,08	64.501,40	5.730,214	2.675,464	29.152,52	.361,493,2	.087,332,90	4.463,942
71	93.356,36	76.403,29	50.357,44	30.205,47	20.189,23	65.575,92	6.062,570	2.867,289	30.165,81	.382,562,6	.093,712,72	4.649,535
72	98.585,35	81.362,09	46.002,79	31.917,45	21.567,68	66.567,14	6.409,082	3.069,747	31.190,44	.404,538,3	.100,458,3	4.839,826
73	104.024,8	86.549,84	41.137,10	33.700,07	23.016,71	67.468,39	6.770,139	3.283,251	32.225,63	.427,445,9	.107,585,0	5.034,849
74	109.679,9	91.972,01	35.738,17	35.555,14	24.538,65	68.272,87	7.146,134	3.508,226	33.270,58	.451,311,2	.115,108,5	5.234,634
75	115.555,5	97.634,02	29.783,61	37.484,51	26.135,84	68.973,67	7.537,464	3.745,104	34.324,44	.476,160,3	.123,044,9	5.439,212
76	121.657,0	103.541,2	23.250,93	39.490,03	27.810,67	69.563,64	7.944,531	3.994,326	35.386,33	.502,019,9	.131,411,0	5.648,610
77	127.989,4	109.698,6	16.117,58	41.573,58	29.565,50	70.035,54	8.367,742	4.256,342	36.455,35	.528,916,7	.140,223,5	5.862,856
78	134.557,9	116.111,4	8.360,84	43.737,05	31.402,75	70.382,01	8.807,509	4.531,611	37.530,56	.556,878,0	.149,500,1	6.081,974
79	141.367,7	122.784,4	-.041,97	45.982,33	33.324,84	70.595,51	9.264,248	4.820,601	38.610,96	.585,931,4	.159,258,4	6.305,989
80	148.423,9	129.722,5	-9.113,60	48.311,36	35.334,20	70.668,42	9.738,379	5.123,788	39.695,55	.616,104,7	.169,516,8	6.534,924
81	155.731,9	136.930,2	-18.876,88	50.726,06	37.433,28	70.592,91	10.230,33	5.441,656	40.783,28	.647,426,3	.180,293,9	6.768,800
82	163.296,9	144.412,1	-29.354,43	53.228,38	39.624,54	70.361,09	10.740,52	5.774,700	41.873,07	.679,924,6	.191,609,0	7.007,636
83	171.124,1	152.172,2	-40.569,05	55.820,29	41.910,43	69.964,90	11.269,40	6.123,420	42.963,79	.713,628,6	.203,481,7	7.251,450
84	179.218,7	160.214,8	-52.543,29	58.503,77	44.293,43	69.396,21	11.817,39	6.488,328	44.054,30	.748,567,6	.215,931,9	7.500,260
85	187.586,2	168.543,7	-65.299,73	61.280,81	46.776,02	68.646,77	12.384,94	6.869,942	45.143,41	.784,771,1	.228,980,2	7.754,079
86	196.231,7	177.162,5	-78.860,70	64.153,41	49.360,66	67.708,14	12.972,51	7.268,790	46.229,89	.822,269,1	.242,647,6	8.012,921
87	205.160,6	186.074,6	-93.248,54	67.123,59	52.049,84	66.571,85	13.580,53	7.685,405	47.312,51	.861,092,0	.256,955,4	8.276,800
88	214.378,2	195.283,3	-108.485,2	70.193,36	54.846,03	65.229,35	14.209,46	8.120,332	48.389,95	.901,270,2	.271,925,6	8.545,724
89	223.889,7	204.791,5	-124.592,7	73.364,83	57.751,70	63.671,89	14.859,77	8.574,121	49.460,92	.942,834,7	.287,580,4	8.819,703
90	233.700,8	214.601,8	-141.592,6	76.639,99	60.769,31	61.890,70	15.531,92	9.047,332	50.524,02	.985,816,9	.303,942,8	9.098,743

TABLE 20—Continued

UNIT TANGENTIAL LOAD NO. 2

RADIAL AND TWIST DEFLECTIONS

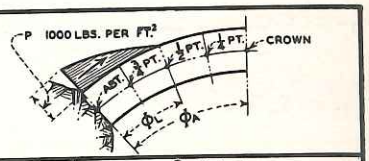


Φ	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_2		D_1	D_2		D_1	D_2		D_1	D_2	
o		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.0⁴30,204,29	.0⁵5,028,372	-.04 1,440,86	.0⁴30,204,29	.0⁵3,724,504	-.027,672,47	.0⁴30,204,29	.0⁵2,413,317	-.013,851,39	.0⁴30,204,29	.0⁵1,097,995	-.0⁵3,954
11	.0⁴44,221,67	.0⁵8,090,191	-.055,131,26	.0⁴44,221,67	.0⁵5,995,058	-.036,827,00	.0⁴44,221,67	.0⁵3,885,931	-.018,437,92	.0⁴44,221,67	.0⁵1,768,226	-.0⁵6,366
12	.0⁴62,630,36	.0⁴12,486,14	-.071,537,67	.0⁴62,630,36	.0⁵9,257,101	-.047,804,28	.0⁴62,630,36	.0⁵6,002,687	-.023,939,87	.0⁴62,630,36	.0⁵2,731,820	-.0⁵9,836
13	.0⁴86,263,76	.0⁴18,609,04	-.090,901,73	.0⁴86,263,76	.0⁴13,803,89	-.060,769,00	.0⁴86,263,76	.0⁵8,954,334	-.030,440,80	.0⁴86,263,76	.0⁵4,075,975	-.0⁴14,676
14	.0³1 16,027,7	.0⁴26,921,01	-.113,463,8	.0³1 16,027,7	.0⁴19,981,04	-.075,885,61	.0³1 16,027,7	.0⁴12,966,54	-.038,024,36	.0³1 16,027,7	.0⁵5,903,662	-.0⁴21,259
15	.0³1 52,900,5	.0⁴37,958,56	-.139,462,6	.0³1 52,900,5	.0⁴28,190,62	-.093,318,25	.0³1 52,900,5	.0⁴18,301,96	-.046,774,28	.0³1 52,900,5	.0⁵8,334,923	-.0⁴30,012
16	.0³1 97,932,8	.0⁴52,337,63	-.169,135,5	.0³1 97,932,8	.0⁴38,895,17	-.113,230,8	.0³1 97,932,8	.0⁴25,263,22	-.056,774,39	.0³1 97,932,8	.0⁴11,508,18	-.0⁴41,426
17	.0³2 52,247,6	.0⁴70,758,38	-.202,717,9	.0³2 52,247,6	.0⁴52,621,72	-.135,786,7	.0³2 52,247,6	.0⁴34,195,66	-.068,108,68	.0³2 52,247,6	.0⁴15,581,54	-.0⁴56,108
18	.0³3 17,040,3	.0⁴94,009,81	-.240,443,6	.0³3 17,040,3	.0⁴69,965,64	-.161,149,1	.0³3 17,040,3	.0⁴45,490,09	-.080,861,18	.0³3 17,040,3	.0⁴20,734,09	-.0⁴74,658
19	.0³3 93,578,7	.0³1 22,974,3	-.282,544,3	.0³3 93,578,7	.0⁴91,594,45	-.189,481,0	.0³3 93,578,7	.0⁴59,585,46	-.095,116,07	.0³3 93,578,7	.0⁴27,167,18	-.0⁴97,827
20	.0³4 83,202,9	.0³1 58,631,7	-.329,249,8	.0³4 83,202,9	.0³1 18,251,6	-.220,944,5	.0³4 83,202,9	.0⁴76,971,58	-.110,957,7	.0³4 83,202,9	.0⁴35,105,75	-.0³1 26,42
21	.0³5 87,325,1	.0³2 02,063,6	-.380,787,5	.0³5 87,325,1	.0³1 50,760,1	-.255,701,7	.0³5 87,325,1	.0⁴98,191,77	-.128,470,5	.0³5 87,325,1	.0⁴44,799,56	-.0³1 61,33
22	.0³7 07,431,1	.0³2 54,457,4	-.437,382,9	.0³7 07,431,1	.0³1 90,026,4	-.293,914,1	.0³7 07,431,1	.0³1 23,845,7	-.147,739,0	.0³7 07,431,1	.0⁴56,524,65	-.0³2 03,57
23	.0³8 45,075,3	.0³3 17,108,5	-.499,258,7	.0³8 45,075,3	.0³2 37,042,5	-.335,742,6	.0³8 45,075,3	.0³1 54,591,1	-.168,847,9	.0³8 45,075,3	.0⁴70,584,15	-.0³2 54,21
24	.001,001,890	.0³3 91,426,4	-.566,635,5	.001,001,890	.0³2 92,891,8	-.381,348,0	.001,001,890	.0³1 91,148,2	-.191,882,4	.001,001,890	.0⁴87,310,42	-.0³3 14,47
25	.001,179,575	.0³4 78,935,3	-.639,730,8	.001,179,575	.0³3 58,750,1	-.430,890,1	.001,179,575	.0³2 34,300,4	-.216,927,2	.001,179,575	.0³1,107,065,4	-.0³3 85,64
26	.001,379,903	.0³5 81,277,7	-.718,759,9	.001,379,903	.0³4 35,889,4	-.484,528,4	.001,379,903	.0³2 84,897,3	-.244,067,7	.001,379,903	.0³1,130,242,4	-.0³4 69,13
27	.001,604,722	.0³7 00,218,1	-.803,934,9	.001,604,722	.0³5 25,681,9	-.542,421,9	.001,604,722	.0³3 43,858,1	-.273,389,2	.001,604,722	.0³1,157,267,4	-.0³5 66,50
28	.001,855,949	.0³8 37,644,0	-.895,465,1	.001,855,949	.0³6 29,601,5	-.604,728,8	.001,855,949	.0³4 12,173,0	-.304,977,3	.001,855,949	.0³1,188,600,1	-.0³6 79,39
29	.002,135,574	.0³9 95,568,7	-.993,556,6	.002,135,574	.0³7 49,227,5	-.671,606,7	.002,135,574	.0³4 90,906,2	-.338,917,8	.002,135,574	.0³1,224,735,2	-.0³7 93,59
30	.002,445,660	.001,176,133	-1.098,412	.002,445,660	.0³8 86,247,4	-.743,212,9	.002,445,660	.0³5 81,197,7	-.375,296,6	.002,445,660	.0³1,266,203,7	-.0³9 59,031
31	.002,788,341	.001,381,606	-1.210,232	.002,788,341	.001,042,458	-.819,703,5	.002,788,341	.0³6 84,266,6	-.414,200,0	.002,788,341	.0³1,313,574,4	-.001,129,76
32	.003,165,823	.001,614,388	-1.329,212	.003,165,823	.001,219,771	-.901,234,2	.003,165,823	.0³8 01,412,3	-.455,714,0	.003,165,823	.0³1,367,454,7	-.001,323,94
33	.003,580,384	.001,877,011	-1.455,546	.003,580,384	.001,420,212	-.987,959,9	.003,580,384	.0³9 34,017,7	-.499,925,4	.003,580,384	.0³1,428,492,3	-.001,543,94
34	.004,034,376	.002,172,136	-1.589,422	.004,034,376	.001,645,922	-1.080,035	.004,034,376	.001,083,551	-.546,921,0	.004,034,376	.0³1,497,375,9	-.001,792,26
35	.004,530,220	.002,502,557	-1.731,026	.004,530,220	.001,899,166	-1.177,612	.004,530,220	.001,251,568	-.596,787,6	.004,530,220	.0³1,574,837,0	-.002,071,48
36	.005,070,410	.002,871,202	-1.880,539	.005,070,410	.002,182,326	-1.280,845	.005,070,410	.001,439,714	-.649,612,5	.005,070,410	.0³1,661,650,8	-.002,384,46
37	.005,657,514	.003,281,127	-2.038,138	.005,657,514	.002,497,908	-1.389,884	.005,657,514	.001,649,725	-.705,483,0	.005,657,514	.0³1,758,637,5	-.002,734,15
38	.006,294,169	.003,735,523	-2.203,999	.006,294,169	.002,848,545	-1.504,881	.006,294,169	.001,883,435	-.764,487,0	.006,294,169	.0³1,866,663,9	-.003,123,67
39	.006,983,084	.004,237,706	-2.378,289	.006,983,084	.003,236,991	-1.625,986	.006,983,084	.002,142,767	-.826,712,2	.006,983,084	.0³1,986,643,2	-.003,556,35
40	.007,727,042	.004,791,123	-2.561,173	.007,727,042	.003,666,132	-1.753,347	.007,727,042	.002,429,749	-.892,246,7	.007,727,042	.001,119,538	-.004,035,59
41	.008,528,895	.005,399,365	-2.752,814	.008,528,895	.004,138,992	-1.887,113	.008,528,895	.002,746,509	-.961,179,1	.008,528,895	.001,266,361	-.004,565,19
42	.009,391,569	.006,066,080	-2.953,366	.009,391,569	.004,658,679	-2.027,431	.009,391,569	.003,095,258	-1.033,598	.009,391,569	.001,428,176	-.005,148,86
43	.010,318,06	.006,795,130	-3.162,981	.010,318,06	.005,228,493	-2.174,447	.010,318,06	.003,478,339	-1.109,592	.010,318,06	.001,606,098	-.005,790,67
44	.011,311,44	.007,590,447	-3.381,806	.011,311,44	.005,851,830	-2.328,305	.011,311,44	.003,898,183	-1.189,250	.011,311,44	.001,801,296	-.006,494,89
45	.012,374,84	.008,456,082	-3.609,982	.012,374,84	.006,532,222	-2.489,150	.012,374,84	.004,357,333	-1.272,662	.012,374,84	.002,014,994	-.007,266,02
46	.013,511,49	.009,396,201	-3.847,647	.013,511,49	.007,273,333	-2.657,125	.013,511,49	.004,858,436	-1.359,918	.013,511,49	.002,248,471	-.008,108,5
47	.014,724,65	.010,415,08	-4.094,933	.014,724,65	.008,078,957	-2.832,371	.014,724,65	.005,404,254	-1.451,106	.014,724,65	.002,503,064	-.009,027,3
48	.016,017,68	.011,517,10	-4.351,966	.016,017,68	.008,953,022	-3.015,028	.016,017,68	.005,997,658	-1.546,319	.016,017,68	.002,780,168	-.010,027,5
49	.017,394,02	.012,706,73	-4.618,868	.017,394,02	.009,899,586	-3.205,236	.017,394,02	.006,641,632	-1.645,645	.017,394,02	.003,081,233	-.011,114,3
50	.018,857,16	.013,988,56	-4.895,755	.018,857,16	.010,922,84	-3.403,133	.018,857,16	.007,339,277	-1.749,176	.018,857,16	.003,407,779	-.012,293,2
51	.020,410,66	.015,367,25	-5.182,737	.020,410,66	.012,027,09	-3.608,855	.020,410,66	.008,093,806	-1.857,002	.020,410,66	.003,761,375	-.013,569,7
52	.022,058,17	.016,847,54	-5.479,919	.022,058,17	.013,216,79	-3.822,539	.022,058,17	.008,908,552	-1.969,215	.022,058,17	.004,143,664	-.014,950,3
53	.023,803,39	.018,434,25	-5.787,401	.023,803,39	.014,496,52	-4.044,317	.023,803,39	.009,786,967	-2.085,907	.023,803,39	.004,556,346	-.016,440,5
54	.025,650,11	.020,132,28	-6.105,276	.025,650,11	.015,870,97	-4.274,323	.025,650,11	.010,732,62	-2.207,169	.025,650,11	.005,001,188	-.018,047,4
55	.027,602,19	.021,946,60	-6.433,632	.027,602,19	.017,344,97	-4.512,688	.027,602,19	.011,749,21	-2.333,094	.027,602,19	.005,480,023	-.019,777,4
56	.029,663,55	.023,882,20	-6.772,551	.029,663,55	.018,923,47	-4.759,540	.029,663,55	.012,840,53	-2.463,773	.029,663,55	.005,994,747	-.021,636,6
57	.031,838,18	.025,944,17	-7.122,106	.031,838,18	.020,611,55	-5.015,010	.031,838,18	.014,010,54	-2.599,300	.031,838,18	.006,547,340	-.023,633,3
58	.034,130,14	.028,137,60	-7.482,369	.034,130,14	.022,414,39	-5.279,224	.034,130,14	.015,263,28	-2.739,767	.034,130,14	.007,139,821	-.025,774,2
59	.036,543,58	.030,467,63	-7.853,402	.036,543,58	.024,337,30	-5.552,306	.036,543,58	.016,602,95	-2.885,268	.036,543,58	.007,774,319	-.028,067,4
60	.039,082,71	.032,939,44	-8.235,260	.039,082,71	.026,385,71	-5.834,380	.039,082,71	.018,033,84	-3.035,896	.039,082,71	.008,452,995	-.030,520,7
61	.041,751,80	.035,558,19	-8.627,997	.041,751,80	.028,565,15	-6.125,569	.041,751,80	.019,560,40	-3.191,745	.041,751,80	.009,178,104	-.033,142,0
62	.044,555,20	.038,329,08	-9.031,653	.044,555,20	.030,881,28	-6.425,991	.044,555,20	.021,187,20	-3.352,910	.044,555,20	.009,951,977	-.035,940.1
63	.047,497,33	.041,257,28	-9.446,265	.047,497 33	.033,339,83	-6.735,767	.047,497,33	.022,918,90	-3.519,483	.047,497,33	.010,777,01	-.038,923,5
64	.050,582,69	.044,347,97	-9.871,864	.050,582,69	.035,946,67	-7.055,012	.050,582,69	.024,760,34	-3.691,562	.050,582,69	.011,655,67	-.042,101,4
65	.053,815,83	.047,606,29	-10.308,47	.053,815,83	.038,707,76	-7.383,841	.053,815,83	.026,716,47	-3.869,239	.053,815,83	.012,590,53	-.045,483,2
66	.057,201,39	.051,037,34	-10.756,11	.057,201,39	.041,629,14	-7.722,368	.057,201,39	.028,792,34	-4.052,610	.057,201,39	.013,584,19	-.049,078,0
67	.060,744,07	.054,646,21	-11.214,77	.060,744,07	.044,716,97	-8.070,702	.060,744,07	.030,993,18	-4.241,772	.060,744,07	.014,639,39	-.052,895,8
68	.064,448,64	.058,437,90	-11.684,48	.064,448,64	.047,977,48	-8.428,954	.064,448,64	.033,324,30	-4.436,819	.064,448,64	.015,758,88	-.056,947,4
69	.068,319,95	.062,417,34	-12.165,22	.068,319,95	.051,416,99	-8.797,231	.068,319,95	.035,791,16	-4.637,848	.068,319,95	.016,945,56	-.061,242,5
70	.072,362,90	.066,589,41	-12.656,97	.072,362,90	.055,041,89	-9.175,639	.072,362,90	.038,399,36	-4.844,955	.072,362,90	.018,202,36	-.065,792,2
71	.076,582,48	.070,958,88	-13.159,72	.076,582,48	.058,858,67	-9.564,279	.076,582,48	.041,154,63	-5.058,237	.076,582,48	.019,532,31	-.070,607,5
72	.080,983,75	.075,530,42	-13.673,44	.080,983,75	.062,873,87	-9.963,253	.080,983,75	.044,062,80	-5.277,790	.080,983,75	.020,938,55	-.075,699,8
73	.085,571,84	.080,308,58	-14.198,10	.085,571,84	.067,094,11	-10.372,66	.085,571,84	.047,129,85	-5.503,713	.085,571,84	.022,424,25	-.081,081,0
74	.090,351,91	.085,297,78	-14.733,65	.090,351,91	.071,526,04	-10.792,60	.090,351,91	.050,361,89	-5.736,101	.090,351,91	.023,992,70	-.086,762,6
75	.095,329,25	.090,502,31	-15.280,04	.095,329,25	.076,176,40	-11.223,16	.095,329,25	.053,765,16	-5.975,053	.095,329,25	.025,647,29	-.092,757,3
76	.100,509,2	.095,926,29	-15.837,21	.100,509,2	.081,051,97	-11.664,44	.100,509,2	.057,346,00	-6.220,668	.100,509,2	.027,391,45	-.099,077,7
77	.105,897,1	.101,573,7	-16.405.10	.105,897,1	.086,159,58	-12.116,52	.105,897,1	.061,110,94	-6.473,042	.105,897,1	.029,228,76	-.105,736,7
78	.111,498,6	.107,448,3	-16.983,63	.111,498,6	.091,506,06	-12.579,49	.111,498,6	.065,066,56	-6.732,275	.111,498,6	.031,162,82	-.112,747,8
79	.117,319,0	.113,553,6	-17.572,72	.117,319,0	.097,098,33	-13.053,45	.117,319,0	.069,219,62	-6.998,465	.117,319,0	.033,197,36	-.120,124,3
80	.123,364,0	.119,893,1	-18.172.28	.123,364,0	.102,943,3	-13.538,47	.123,364,0	.073,576,99	-7.271,711	.123,364,0	.035,336,22	-.127,880,6
81	.129,639,4	.126,469,9	-18.782,21	.129,639,4	.109,047,9	-14.034,62	.129,639,4	.078,145,65	-7.552,111	.129,639,4	.037,583,28	-.136,030,8
82	.136,150,8	.133,286,9	-19.402,40	.136,150,8	.115,419,0	-14.542,00	.136,150,8	.082,932,74	-7.839,768	.136,150,8	.039,942,55	-.144,589,5
83	.142,904,1	.140,346,7	-20.032,74	.142,904,1	.122,063,7	-15.060,66	.142,904,1	.087,945,50	-8.134,778	.142,904,1	.042,418,13	-.153,571,8
84	.149,905,2	.147,651,8	-20.673,10	.149,905,2	.128,988,9	-15.590,70	.149,905,2	.093,191,26	-8.437.243	.149,905,2	.045,014,18	-.162,993.3
85	.157,159,9	.155,204,2	-21.323,35	.157,159,9	.136,201,5	-16.132,16	.157,159,9	.098,677,55	-8.747,262	.157,159,9	.047,735,01	-.172,869,2
86	.164,674,5	.163,005,8	-21.983,35	.164,674,5	.143,708,4	-16.685,13	.164,674,5	.104,411,9	-9.064,934	.164,674,5	.050,584,98	-.183,216.1
87	.172,454,9	.171,058,0	-22.652,95	.172,454,9	.151,516,6	-17.249,67	.172,454,9	.110,402,2	-9.390,363	.172,454,9	.053,568,56	-.194,050,3
88	.180,507,3	.179,362,1	-23.331,99	.180,507,3	.159,632,9	-17.825,83	.180,507,3	.116,656,1	-9.723,645	.180,507,3	.056,690,37	-.205,388,7
89	.188,838,0	.187,918,8	-24.020,31	.188,838,0	.168,064,2	-18.413,67	.188,838,0	.123,181,6	-10.064,89	.188,838,0	.059,954,93	-.217,248,1
90	.197,453,2	.196,728,7	-24.717,73	.197,453,2	.176,817,1	-19.013,25	.197,453,2	.129,986,8	-10.414,18	.197,453,2	.063,367,16	-.229,646,5

TABLE 20—Continued

UNIT TANGENTIAL LOAD NO. 3

RADIAL AND TWIST DEFLECTIONS

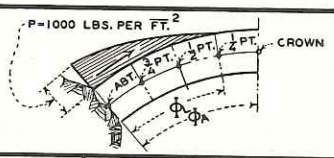


ϕ °	Crown D_1	Crown D_2 1ST Term	Crown D_2 2ND Term	¼ Point D_1	¼ Point D_2 1ST Term	¼ Point D_2 2ND Term	½ Point D_1	½ Point D_2 1ST Term	½ Point D_2 2ND Term	¾ Point D_1	¾ Point D_2 1ST Term	¾ Point D_2 2ND Term
10	.03,483,2029	.0476,971,58	-1 10,957,7	.03,483,2029	.0456092,05	-055,594,95	.03,483,2029	.0435 10,575	-03,126,424	.0490,611,49	.05 3,183,161	.006912,084
11	.03,707,431,1	.03,123,8457	-147,739,0	.03,707,431,1	.0490,289,13	-074,056,56	.03,707,431,1	.0456,524,65	-03,203,57	.03,132,6626	.05 5,127,808	.009,197,036
12	.001,001,890	.03,191,148,2	-1 91,882,4	.001,001,890	.03,139,420,4	-096,230,30	.001,001,890	.0487,310,42	-03,314,47	.03,187,887,0	.05 7922,192	.011,936,07
13	.001,379,903	.03,284,897,3	-244,067,7	.001,379,903	.03,207,902,2	-1 22,465,4	.001,379,903	.03,130,2424	-03,469,13	.03,258,784,7	.0411,820,18	.015,169,88
14	.001,855,949	.03,412,1730	-304,977,3	.001,855,949	.03,300,947,9	-153,113,9	.001,855,949	.03,188,800,1	-03,679,39	.03,348,0729	.0417,120,37	.018,939,00
15	.002,445,660	.03,581,197,7	-375,296,6	.002,445,660	.03,424,6099	-188,531,5	.002,445,660	.03,266,203,7	-03,959,031	.03,593,7754	.0433,373,08	.028,244,58
16	.003,165,823	.03,801,412,3	-455,714,0	.003,165,823	.03,585,860,7	-229,077,0	.003,165,823	.03,367,454,7	-001,323,94	.03,793,775,4	.0460,127,41	.040,173,66
17	.004,034,376	.001,083,551	-546,921,0	.004,034,376	.03,792,643,1	-275,113,1	.004,034,376	.03,497,3759	-001,792,26	.03,756,709,7	.0445,185,50	.033,861,25
18	.005,070,410	.001,439,714	-649,612,5	.005,070,410	.001,053,931	-327,006,5	.005,070,410	.03,661,6508	-002,384,46	.03,951,0744	.0460,127,41	.040,173,66
19	.006,294,169	.001,883,435	-764,487,0	.006,294,169	.001,379,788	-385,128,1	.006,294,169	.03,866,6639	-003,123,67	.001,180,672	.0478,782,70	.047,221,42
20	.007,727,042	.002,429,749	-892,246,7	.007,727,042	.001,781,422	-449,853,0	.007,727,042	.001,119,538	-004,035,59	.001,449,521	.03,101,803,6	.055,043,89
21	.009,391,569	.003,095,258	-1.033,598	.009,391,569	.002,271,245	-521,561,3	.009,391,569	.001,428,176	-005,148,86	.001,761,858	.03,129,914,5	.063,680,22
22	.011,311,44	.003,898,183	-1.189,250	.011,311,44	.002,862,920	-600,637,7	.011,311,44	.001,801,296	-006,494,89	.002,122,136	.03,163,915,5	.073,169,30
23	.013,511,49	.004,858,436	-1.359,918	.013,511,49	.003,571,422	-687,471,9	.013,511,49	.002,248,671	-008,108,46	.002,535,024	.03,204,886,0	.083,549,72
24	.016,017,68	.005,997,658	-1.546,319	.016,017,68	.004,413,088	-.782,459,3	.016,017,68	.002,780,168	-.010,027,5	.003,005,407	.03,253,189,3	.094,859,85
25	.018,857,16	.007,339,277	-1.749,176	.018,857,16	.005,405,657	-.886,000,4	.018,857,16	.003,407,779	-.012,293,2	.003,538,389	.03,310,475,2	.107,137,7
26	.022,058,17	.008,908,552	-1.969,215	.022,058,17	.006,568,329	-.998,501,2	.022,058,17	.004,143,664	-.014,950,3	.004,139,286	.03,377,683,9	.120,421,1
27	.025,650,11	.010,732,62	-2.207,169	.025,650,11	.007,921,818	-1.120,374	.025,650,11	.005,001,188	-.018,047,4	.004,813,636	.03,456,050,7	.134,747,5
28	.029,663,55	.012,840,53	-2.463,773	.029,663,55	.009,488,361	-1.252,037	.029,663,55	.005,994,747	-.021,636,8	.005,567,188	.03,546,908,2	.150,153,9
29	.034,130,14	.015,263,28	-2.739,767	.034,130,14	.011,291,83	-1.393,915	.034,130,14	.007,139,821	-.025,774,2	.006,405,909	.03,651,8909	.166,677,1
30	.039,082,71	.018,033,84	-3.035,896	.039,082,71	.013,357,70	-1.546,438	.039,082,71	.008,452,995	-.030,520,7	.007,335,982	.03,771,938,8	.184,353,6
31	.044,555,20	.021,187,20	-3.352,910	.044,555,20	.015,713,11	-1.710,045	.044,555,20	.009,951,977	-.035,940,1	.008,383,808	.03,909,3004	.203,219,5
32	.050,582,69	.024,760,34	-3.691,562	.050,582,69	.018,386,95	-1.885,178	.050,582,69	.011,655,67	-.042,101,4	.009,495,999	.001,065,537	.223,310,4
33	.057,201,39	.028,792,34	-4.052,610	.057,201,39	.021,409,83	-2.072,289	.057,201,39	.013,584,19	-.049,078,0	.010,739,39	.001,242,526	.244,661,5
34	.064,448,64	.033,324,30	-4.436,819	.064,448,64	.024,814,15	-2.271,837	.064,448,64	.015,758,88	-.056,947,4	.012,101,01	.001,442,265	.267,308,0
35	.072,362,90	.038,399,36	-4.844,955	.072,362,90	.028,834,12	-2.484,287	.072,362,90	.018,202,36	-.065,792,2	.013,588,14	.001,666,874	.291,284,1
36	.080,983,75	.044,062,80	-5.277,790	.080,983,75	.032,905,80	-2.710,111	.080,983,75	.020,938,55	-.075,699,8	.015,208,25	.001,918,601	.316,624,0
37	.090,351,91	.050,361,89	-5.736,101	.090,351,91	.037,667,11	-2.949,790	.090,351,91	.023,992,70	-.086,762,6	.016,969,03	.002,199,824	.343,361,4
38	.100,509,2	.057,346,00	-6.220,668	.100,509,2	.042,957,87	-3.203,811	.100,509,2	.027,391,45	-.099,077,7	.018,878,39	.002,51 3,053	.371,529,4
39	.111,498,6	.065,066,56	-6.732,275	.111,498,6	.048,819,84	-3.472,670	.111,498,6	.031,162,82	-.112,748	.020,944,44	.002,860,939	.401,160,7
40	.123,364,0	.073,576,99	-7.271,711	.123,364,0	.055,296,69	-3.756,871	.123,364,0	.035,336,22	-.127,881	.023,175,52	.003,246,272	.432,287,6
41	.136,150,8	.082,932,74	-7.839,768	.136,150,8	.062,434,05	-4.056,925	.136,150,8	.039,942,55	-.144,589	.025,580,19	.003,671,986	.464,941,9
42	.149,905,2	.093,191,26	-8.437,243	.149,905,2	.070,279,56	-4.373,350	.149,905,2	.045,014,18	-.162,993	.028,167,21	.004,141,165	.499,155,0
43	.164,674,5	.104,411,9	-9.064,934	.164,674,5	.078,882,82	-4.706,676	.164,674,5	.050,584,98	-.183,216	.030,945,54	.004,657,041	.534,957,4
44	.180,507,3	.116,656,1	-9.723,645	.180,507,3	.088,295,47	-5.057,436	.180,507,3	.056,690,37	-.205,389	.033,924,40	.005,223,003	.572,379,6
45	.197,453,2	.129,986,8	-10.414,18	.197,453,2	.098,570,99	-5.426,177	.197,453,2	.063,367,16	-.229,646	.037,113,17	.005,842,599	.611,451,1
46	.215,562,8	.144,469,1	-11.137,35	.215,562,8	.109,765,1	-5.813,449	.215,562,8	.070,654,03	-.256,132	.040,521,49	.006,519,536	.652,201,2
47	.234,888,0	.160,169,9	-11.893,97	.234,888,0	.121,935,6	-6.219,813	.234,888,0	.078,591,05	-.284,993	.044,159,19	.007,257,689	.694,658,8
48	.255,481,5	.177,157,5	-12.684,84	.255,481,5	.135,141,9	-6.645,839	.255,481,5	.087,219,96	-.316,383	.048,036,32	.008,061,098	.738,851,4
49	.277,397,5	.195,502,1	-13.510,79	.277,397,5	.149,445,9	-7.092,106	.277,397,5	.096,584,16	-.350,464	.052,163,13	.008,933,976	.784,806,7
50	.300,691,0	.215,275,4	-14.372,63	.300,691,0	.164,911,1	-7.559,198	.300,691,0	.106,728,7	-.387,403	.056,550,10	.009,880,711	.832,552,5
51	.325,418,0	.236,550,6	-15.271,18	.325,418,0	.181,603,4	-8.047,712	.325,418,0	.117,700,3	-.427,371	.061,207,91	.010,905,87	.882,113,8
52	.351,635,9	.259,402,3	-16.207,25	.351,635,9	.199,590,4	-8.558,248	.351,635,9	.129,547,5	-.470,548	.066,147,46	.012,014,20	.933,517,0
53	.379,402,9	.283,906,6	-17.181,68	.379,402,9	.218,941,8	-9.091,422	.379,402,9	.142,320,3	-.517,124	.071,379,86	.013,210,63	.986,786,9
54	.408,778,5	.310,140,6	-18.195,27	.408,778,5	.239,729,4	-9.647,852	.408,778,5	.156,070,7	-.567,288	.076,916,45	.014,500,29	1.041,948
55	.439,823,1	.338,182,7	-19.248,86	.439,823,1	.262,026,7	-10.228,17	.439,823,1	.170,852,4	-.621,243	.082,768,74	.015,888,48	1.099,024
56	.472,598,3	.368,112,4	-20343,25	.472,598,3	.285,909,3	-10.833,01	.472,598,3	.186,720,8	-.679,195	.088,948,48	.017,380,71	1.158,039
57	.507,166,4	.400,010,0	-21.479,27	.507,166,4	.311,454,7	-11.463,02	.507,166,4	.203,733,1	-.741,358	.095,467,64	.018,982,69	1.219,015
58	.543,591,4	.433,956,8	-22657,73	.543,591,4	.338,742,2	-12.118,86	.543,591,4	.221,948,2	-.807,952	.102,338,4	.020,700,30	1.281,972
59	.581,937,8	.470,034,6	-23.879,45	.581,937,8	.367,853,1	-12.801,18	.581,937,8	.241,427,2	-.879,205	.109,573,1	.022,539,67	1.346,934
60	.622,271,5	.508,326,0	-25.145,24	.622,271,5	.398,870,4	-13.510,66	.622,271,5	.262,232,4	-.955,352	.117,1844	.024,507,11	1.413,920
61	.664,659,3	.548,914,1	-26455,91	.664,659,3	.431,878,9	-14.247,98	.664,659,3	.284,428,6	-1.036,637	.125,185,0	.026,609,13	1.482,949
62	.709,169,0	.591,882,0	-27.812,27	.709,169,0	.466,965,3	-15.013,84	.709,169,0	.308,081,9	-1.123,310	.133,588,0	.028,852,47	1.554,041
63	.755,869,6	.637,313,5	-29215,11	.755,869,6	.504,217,8	-15.808,91	.755,869,6	.333,260,6	-1.215,626	.142,406,6	.031,244,09	1.627,215
64	.804,831,0	.685,292,1	-30665,24	.804,831,0	.543,726,4	-16.633,91	.804,831,0	.360,034,6	-1.313,852	.151,654,2	.033,791,15	1.702,487
65	.856,124,2	.735,901,3	-32163,44	.856,124,2	.585,582,8	-17.489,56	.856,124,2	.388,476,0	-1.418,257	.161,344,5	.036,501,06	1.779,874
66	.909,821,2	.789,224,3	-33710,50	.909,821,2	.629,879,9	-18.376,56	.909,821,2	.418,658,3	-1.529,123	.171,491,3	.039,381,40	1.859,393
67	.965,995,1	.845,344,2	-35307,19	.965,995,1	.676,712,8	-19.295,65	.965,995,1	.450,657,4	-1.646,737	.182,108,6	.042,440,05	1.941,058
68	1.024,720	.904,343,2	-36954,30	1.024,720	.726,177,4	-20.247,57	1.024,720	.484,550,7	-1.771,394	.193,210,9	.045,685,05	2.024,885
69	1.086,071	.966,303,0	-38652,59	1.086,071	.778,371,5	-21.233,06	1.086,071	.520,417,6	-1.903,397	.204,812,5	.049,124,73	2.110,886
70	1.150,124	1.031,304	-40402,80	1.150,124	.833,393,8	-22.252,86	1.150,124	.558,339,4	-2.043,057	.216,928,0	.052,767,63	2.199,075
71	1.216,957	1.099,427	-42.205,69	1.216,957	.891,344,8	-23.307,73	1.216,957	.598,399,2	-2.190,689	.229,572,5	.056,622,51	2.289,464
72	1.286,646	1.170,749	-44.061,99	1.286,646	.952,325,8	-24.398,45	1.286,646	.640,682,2	-2.346,623	.242,761,0	.060,698,42	2.382,063
73	1.359,273	1.245,348	-45972,43	1.359,273	1.016,440	-25.525,78	1.359,273	.685,275,2	-2.511,194	.256,508,9	.065,004,61	2.476,884
74	1.434,916	1.323,300	-47.937,71	1.434,916	1.083,790	-26.690,49	1.434,916	.732,267,1	-2.684,737	.270,831,5	.069,550,58	2.573,936
75	1.513,657	1.404,677	-49.958,55	1.513,657	1.154,481	-27.893,38	1.513,657	.781,748,6	-2.867,609	.285,744,8	.074,346,11	2.673,227
76	1.595,579	1.489,553	-52035,63	1.595,579	1.228,620	-29.135,23	1.595,579	.833,812,1	-3.060,165	.301,264,5	.079,401,21	2.774,766
77	1.680,763	1.577,996	-54.169,63	1.680,763	1.306,311	-30.416,83	1.680,763	.888,552,2	-3.262,769	.317,406,9	.084,726,15	2.878,560
78	1.769,295	1.670,073	-56361,20	1.769,295	1.387,664	-31.739,01	1.769,295	.946,065,0	-3.475,801	.334,188,3	.090,331,44	2.984,613
79	1.861,259	1.765,851	-58611,00	1.861,259	1.472,785	-33.102,54	1.861,259	1.006,449	-3.699,637	.351,625,2	.096,227,87	3.092,932
80	1.956,741	1.865,389	-60.919,64	1.956,741	1.561,783	-34.508,26	1.956,741	1.069,803	-3.934,669	.369,734,4	.102,426,5	3.203,522
81	2.055,829	1.968,748	-63.287,75	2.055,829	1.654,768	-35.956,98	2.055,829	1.136,231	-4.181,296	.388,532,8	.108,938,6	3.316,384
82	2.158,610	2.075,983	-65.715,92	2.158,610	1.751,849	-37.449,51	2.158,610	1.205,834	-4.439,920	.408,037,7	.115,775,8	3.431,524
83	2.265,174	2.187,148	-68.204,72	2.265,174	1.853,137	-38.986,70	2.265,174	1.278,719	-4.710,962	.428,266,4	.122,949,9	3.548,939
84	2.375,609	2.302,290	-70754,70	2.375,609	1.958,740	-40.569,37	2.375,609	1.354,993	-4.994,839	.449,236,4	.130,473,1	3.668,634
85	2.490,008	2.421,455	-73.327,99	2.490,008	2.068,770	-42.198,36	2.490,008	1.434,765	-5.291,984	.470,965,5	.138,357,6	3.790,604
86	2.608,462	2.544,684	-76040,36	2.608,462	2.183,337	-43.874,49	2.608,462	1.518,145	-5.602,833	.493,471,8	.146,616,3	3.914,853
87	2.731,064	2.672,015	-78.777,04	2.731,064	2.302,550	-45.598,62	2.731,064	1.605,247	-5.927,836	.516,773,4	.155,262,0	4.041,375
88	2.857,906	2.803,479	-81.576,92	2.857,906	2.426,521	-47.371,60	2.857,906	1.696,183	-6.267.446	.540,888,8	.164,308,0	4.170,167
89	2.989,085	2.939,105	-84440,44	2.989,085	2.555,359	-49.194,26	2.989,085	1.791,070	-6.622,125	.565,836,4	.173,767,7	4.301,228
90	3.124,695	3.078,916	-87.368,02	3.124,695	2.689,173	-51.067,46	3.124,695	1.890,027	-6.992,345	.591,635,1	.183,655,0	4.434,548

TABLE 20—Continued

UNIT TANGENTIAL LOAD NO. 4

RADIAL AND TWIST DEFLECTIONS

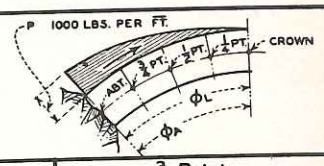


φ	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_2		D_1	D_2		D_1	D_2		D_1	D_2	
°		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term		1ST Term	2ND Term
10	.002,445,660	$0^3$371,976,5	-.126,057,5	.002,445,660	$0^3$266,203,7	$-0^3$959,031	.001,127,415	$0^3$079,571,0	.036,653,80	$0^3$110,747,2	$0^5$3,879,550	.009,217,426
11	.003,580,384	$0^3$598,508,7	-.168,186,1	.003,580,384	$0^3$428,492,3	-.001,543,94	.001,650,568	$0^3$128,118,6	.048,711,63	$0^3$162,142,9	$0^5$6,247,669	.012,264,84
12	.005,070,410	$0^3$923,774,2	-.218,924,5	.005,070,410	$0^3$661,650,8	-.002,384,46	.002,337,568	$0^3$197,896,3	.063,135,07	$0^3$229,639,2	$0^5$9,652,315	.015,918,04
13	.006,983,084	.001,376,860	-.279,134,1	.006,983,084	$0^3$986,643,2	-.003,556,35	.003,219,492	$0^3$295,203,4	.080,124,36	$0^3$316,291,7	$0^4$14,401,58	.020,231,39
14	.009,391,569	.001,991,993	-.349,696,5	.009,391,569	.001,428,176	-.005,148,86	.004,330,108	$0^3$427,472,1	.099,876,11	$0^3$425,421,3	$0^4$20,859,27	.025,259,09
15	.012,374,84	.002,808,912	-.431,515,4	.012,374,84	.002,014,994	-.007,266,02	.005,705,875	$0^3$603,360,4	.122,582,7	$0^3$560,614,5	$0^4$29,449,46	.031,055,12
16	.016,017,68	.003,873,279	-.525,517,7	.016,017,68	.002,780,168	-.010,027,5	.007,385,940	$0^3$832,843,0	.148,432,3	$0^3$725,723,0	$0^4$40,661,38	.037,673,25
17	.020,410,66	.005,236,977	-.632,655,1	.020,410,66	.003,761,375	-.013,569,7	.009,412,135	.001,127,302	.177,607,9	$0^3$924,863,8	$0^4$55,053,48	.045,167,03
18	.025,650,11	.006,958,505	-.753,905,5	.025,650,11	.005,001,188	-.018,047,4	.011,828,97	.001,499,618	.210,287,9	.001,162,419	$0^4$73,258,51	.053,589,77
19	.031,838,18	.009,103,312	-.890,274,5	.031,838,18	.006,547,340	-.023,633,3	.014,683,65	.001,964,257	.246,645,0	.001,443,036	$0^4$95,987,87	.062,994,50
20	.039,082,71	.011,744,11	-1.042,798	.039,082,71	.008,452,995	-.030,520,7	.018,026,03	.002,537,361	.286,846,6	.001,771,627	$0^3$124,036,2	.073,434,00
21	.047,497,33	.014,961,18	-1.212,537	.047,497,33	.010,777,01	-.038,923,5	.021,908,66	.003,236,835	.331,053,7	.002,153,369	$0^3$158,286,1	.084,960,78
22	.057,201,39	.018,842,68	-1.400,594	.057,201,39	.013,584,19	-.049,078,0	.026,386,75	.004,082,434	.379,421,4	.002,593,705	$0^3$199,712,5	.097,626,91
23	.068,319,95	.023,484,90	-1.608,094	.068,319,95	.016,945,56	-.061,242,5	.031,518,16	.005,095,850	.432,098,0	.003,098,340	$0^3$249,386,6	.111,484,5
24	.080,983,75	.028,992,53	-1.836,204	.080,983.75	.020,938,55	-.075,699,8	.037,363,44	.006,300,790	.489,225,1	.003,673,246	$0^3$308,482,3	.126,584,6
25	.095,329,25	.035,478,95	-2.086,121	.095,329,25	.025,647,29	-.092,757,3	.043,985,78	.007,723,068	.550,937,3	.004,324,660	$0^3$378,278,5	.142,978,8
26	.111,498,6	.043,066,38	-2.359,082	.111,498,6	.031,162,82	-.112,748	.051,451,03	.009,390,686	.617,361,2	.005,059,081	$0^3$460,164,4	.160,717,9
27	.129,639,4	.051,886,15	-2.656,360	.129,639,4	.037,583,28	-.136,031	.059,827,67	.011,333,93	.688,616,4	.005,883,274	$0^3$555,645,1	.179,852,1
28	.149,905,2	.062,078,86	-2.979,265	.149,905,2	.045,014,18	-.162,993	.069,186,84	.013,585,39	.764,813,9	.006,804,267	$0^3$666,344,2	.200,431,6
29	.172,454,9	.073,794,59	-3.329,147	.172,454,9	.053,568,56	-.194,050	.079,602,30	.016,180,14	.846,056,9	.007,829,353	$0^3$794,009,5	.222,506,0
30	.197,453,2	.087,192,98	-3.707,399	.197,453,2	.063,367,16	-.229,646	.091,150,48	.019,155,74	.932,439,5	.008,966,090	$0^3$940,517,2	.246,124,5
31	.225,070,2	.102,443,4	-4.115,452	.225,070,2	.074,538,70	-.270,256	.103,910,4	.022,552,31	1.024,047	.010,222,30	.001,107,876	.271,335,9
32	.255,481,5	.119,725,2	-4.554,779	.255,481,5	.087,219,96	-.316,383	.117,963,7	.026,412,66	1.120,957	.011,606,06	.001,298,231	.298,188,5
33	.288,868,6	.139,227,5	-5.026,897	.288,868,6	.101,556,0	-.368,566	.133,394,6	.030,782,33	1.223,236	.013,125,72	.001,513,871	.326,730,1
34	.325,418,0	.161,149,5	-5.533,363	.325,418,0	.117,700,3	-.427,371	.150,290,1	.035,709,66	1.330,940	.014,789,89	.001,757,228	.357,008,0
35	.365,322,1	.185,700,4	-6.075,782	.365,322,1	.135,815,0	-.493,399	.168,739,6	.041,245,90	1.444,119	.016,607,45	.002,030,887	.389,069,3
36	.408,778,5	.213,099,6	-6.655,800	.408,778,5	.156,070,7	-.567,288	.188,835,3	.047,445,13	1.562,809	.018,587,53	.002,337,585	.422,960,2
37	.455,990,4	.243,576,4	-7.275,107	.455,990,4	.178,647,2	-.649,707	.210,671,7	.054,364,62	1.687,036	.020,739,54	.002,680,219	.458,726,6
38	.507,166,4	.277,370,3	-7.935,439	.507,166,4	.203,733,1	-.741,358	.234,346,1	.062,064,62	1.816,818	.023,073,13	.003,061,850	.496,413,7
39	.562,520,3	.314,730,8	-8.638,579	.562,520,3	.231,525,9	-.842,981	.259,958,4	.070,608,57	1.952,159	.025,598,22	.003,485,705	.536,066,4
40	.622,271,5	.355,917,3	-9.386,350	.622,271,5	.262,232,4	-.955,352	.287,610,9	.080,063,07	2.093,054	.028,325,01	.003,955,183	.577,728,7
41	.686,644,6	.401,199,0	-10.180,63	.686,644,6	.296,068,9	-1.079,284	.317,408,7	.090,498,05	2.239,485	.031,263,95	.004,473,862	.621,444,3
42	.755,869,6	.450,855,0	-11.023,33	.755,869,6	.333,260,6	-1.215,626	.349,459,3	.101,986,8	2.391,425	.034,425,74	.005,045,494	.667,256,3
43	.830,181,6	.505,173,7	-11.916,41	.830,181,6	.374,042,3	-1.365,265	.383,872,7	.114,605,8	2.548,829	.037,821,36	.005,674,021	.715,206,7
44	.909,821,2	.564,453,2	-12.861,89	.909,821,2	.418,658,3	-1.529,123	.420,761,6	.128,435,4	2.711,649	.041,462,04	.006,363,572	.765,337,7
45	.995,034,0	.629,000,8	-13.861,81	.995,034,0	.467,362,3	-1.708,167	.460,241,1	.143,559,0	2.879,817	.045,359,28	.007,118,467	.817,690,1
46	1.086,071	.699,132,8	-14.918,28	1.086,071	.520,417,6	-1.903,397	.502,428,8	.160,064,0	3.053,255	.049,524,83	.007,943,225	.872,304,5
47	1.183,188	.775,174,1	-16.033,43	1.183,188	.578,096,8	-2.115,857	.547,445,1	.178,040,8	3.231,873	.053,970,71	.008,842,563	.929,220,8
48	1.286,646	.857,458,3	-17.209,45	1.286,646	.640,682,2	-2.346,623	.595,412,5	.197,584,2	3.415,570	.058,709,20	.009,821,408	.988,477,7
49	1.396,712	.946,327,4	-18.448,58	1.396,712	.708,465,7	-2.596,820	.646,456,3	.218,792,2	3.604,226	.063,752,83	.010,884,89	1.050,114
50	1.513,657	1.042,131	-19.753,08	1.513,657	.781,748,6	-2.867,609	.700,704,2	.241,766,8	3.797,711	.069,114,41	.012,038,36	1.114,168
51	1.637,758	1.145,226	-21.125,28	1.637,758	.860,841,6	-3.160,188	.758,286,3	.266,613,6	3.995,883	.074,806,99	.013,287,37	1.180,675
52	1.769,295	1.255,978	-22.567,52	1.769,295	.946,065,0	-3.475,801	.819,335,3	.293,442,3	4.198,585	.080,843,89	.014,637,71	1.249,673
53	1.908,555	1.374,757	-24.082,22	1.908,555	1.037,748	-3.815,728	.883,986,3	.322,366,2	4.405,645	.087,238,69	.016,095,40	1.321,196
54	2.055,829	1.501,941	-25.671,80	2.055,829	1.136,231	-4.181,296	.952,376,9	.353,502,7	4.616,876	.094,005,23	.017,666,66	1.395,280
55	2.211,413	1.637,915	-27.338,75	2.211,413	1.241,860	-4.573,864	1.024,647	.386,973,2	4.832,081	.101,157,6	.019,357,97	1.471,958
56	2.375,609	1.783,068	-29.085,57	2.375,609	1.354,993	-4.994,839	1.100,939	.422,903,0	5.051,043	.108,710,1	.021,176,03	1.551,264
57	2.548,722	1.937,793	-30.914,83	2.548,722	1.475,997	-5.445,668	1.181,398	.461,421,3	5.273,536	.116,677,5	.023,127,80	1.633,230
58	2.731,064	2.102,489	-32.829,11	2.731,064	1.605,247	-5.927,836	1.266,172	.502,661,7	5.499,315	.125,074,5	.025,220,46	1.717,888
59	2.922,948	2.277,560	-34.831,03	2.922,948	1.743,126	-6.442,874	1.355,409	.546,761,5	5.728,121	.133,916,3	.027,461,46	1.805,268
60	3.124,695	2.463,412	-36.923,25	3.124,695	1.890,027	-6.992,345	1.449,262	.593,862,3	5.959,680	.143,218,2	.029,858,48	1.895,400
61	3.336,631	2.660,454	-39.108,45	3.336,631	2.046,351	-7.577,860	1.547,886	.644,109,9	6.193,706	.152,996,0	.032,419,47	1.988,313
62	3.559,084	2.869,097	-41.389,35	3.559,084	2.212,507	-8.201,070	1.651,436	.697,654,0	6.429,891	.163,265,6	.035,152,64	2.084,035
63	3.792,388	3.089,755	-43.766,71	3.792,388	2.388,913	-8.863,668	1.760,074	.754,648,8	6.667,916	.174,043,0	.038,066,45	2.182,594
64	4.036,882	3.322,842	-46.249,27	4.036,882	2.575,994	-9.567,376	1.873,959	.815,252,3	6.907,448	.185,344,7	.041,169,65	2.284,016
65	4.292,909	3.568,771	-48.833,86	4.292,909	2.774,182	-10.313,97	1.993,256	.879,627,1	7.148,135	.197,187,3	.044,471,23	2.388,327
66	4.560,817	3.827,959	-51.525,28	4.560,817	2.983,919	-11.105,26	2.118,132	.947,939,6	7.389,608	.209,587,9	.047,980,47	2.495,550
67	4.840,956	4.100,816	-54.326,38	4.840,956	3.205,652	-11.943,10	2.248,754	1.020,361	7.631,485	.222,563,5	.051,706,93	2.605,710
68	5.133,684	4.387,756	-57.240,02	5.133,684	3.439,836	-12.829,36	2.385,294	1.097,065	7.873,368	.236,131,6	.055,660,44	2.718,829
69	5.439,361	4.689,187	-60.269,07	5.439,361	3.686,931	-13.765,98	2.527,924	1.178,233	8.114,840	.250,310,0	.059,851,13	2.834,929
70	5.758,352	5.005,514	-63.416,44	5.758,352	3.947,407	-14.754,92	2.676,821	1.264,048	8.355,469	.265,116,4	.064,289,38	2.954,031
71	6.091,027	5.337,140	-66.685,02	6.091,027	4.221,736	-15.798,20	2.832,161	1.354,696	8.594,811	.280,569,2	.068,985,92	3.076,154
72	6.437,757	5.684,461	-70.077,73	6.437,757	4.510,399	-16.897,83	2.994,125	1.450,370	8.832,396	.296,686,8	.073,951,72	3.201,317
73	6.798,922	6.047,869	-73.597,52	6.798,922	4.813,881	-18.055,90	3.162,895	1.551,267	9.067,748	.313,487,9	.079,198,06	3.329,539
74	7.174,903	6.427,749	-77.247,29	7.174,903	5.132,673	-19.274,51	3.338,655	1.657,587	9.300,367	.330,991,4	.084,736,54	3.460,835
75	7.566,085	6.824,478	-81.029,99	7.566,085	5.467,270	-20.555,83	3.521,593	1.769,534	9.529,739	.349,216,6	.090,579,06	3.595,222
76	7.972,858	7.238,425	-84.948,55	7.972,858	5.818,173	-21.902,01	3.711,896	1.887,317	9.755,335	.368,183,0	.096,737,80	3.732,715
77	8.395,616	7.669,957	-89.005,93	8.395,616	6.185,885	-23.315,30	3.909,756	2.011,149	9.976,604	.387,910,2	.103,225,3	3.873,325
78	8.834,756	8.119,409	-93.205,03	8.834,756	6.570,915	-24.797,92	4.115,366	2.141,247	10.192,99	.408,418,2	.110,054,3	4.017,067
79	9.290,681	8.587,137	-97.548,80	9.290,681	6.973,775	-26.352,17	4.328,922	2.277,833	10.403,90	.429,727,3	.117,238,0	4.163,951
80	9.763,795	9.073,462	-102.040,1	9.763,795	7.394,979	-27.980,35	4.550,620	2.421,132	10.608,74	.451,857,8	.124,789,9	4.313,990
81	10.254,51	9.578,703	-106.682,0	10.254,51	7.835,045	-29.684,82	4.780,661	2.571,372	10.806,89	.474,830,7	.132,723,7	4.467,191
82	10.763,23	10.103,16	-111.477,2	10.763,23	8.294,492	-31.467,91	5.019,247	2.728,789	10.997,74	.498,666,7	.141,053,5	4.623,561
83	11.290,38	10.647,12	-116.428,6	11.290,38	8.773,842	-33.332,09	5.266,581	2.893,620	11.180,62	.523,387,1	.149,793,8	4.783,110
84	11.836,38	11.210,86	-121.539,1	11.836,38	9.273,617	-35.279,74	5.522,870	3.066,105	11.354,87	.549,013,4	.158,959,3	4.945,842
85	12.401,65	11.794,64	-126.811,6	12.401,65	9.794,341	-37.313,33	5.788,320	3.246,491	11.519,82	.575,567,4	.168,565,2	5.111,763
86	12.986,62	12.398,69	-132.248,8	12.986,62	10.336,54	-39.435,37	6.063,143	3.435,027	11.674,74	.603,070,9	.178,626,7	5.280,876
87	13.591,72	13.023,23	-137.853,4	13.591,72	10.900,73	-41.648,33	6 347,552	3.631,966	11.818,95	.631,546,3	.189,159,8	5.453,183
88	14.217,38	13.668,47	-143.628,4	14.217,38	11.487,45	-43.954,78	6.641,758	3.837,566	11.951,69	.661,015,9	.200,180,5	5.628,687
89	14.864,05	14.334,60	-149.576,2	14.864,05	12.097,21	-46.357,25	6.945,981	4.052,087	12.072,22	.691,502,5	.211,705,3	5.807,366
90	15.532,16	15.021,84	-155.699,7	15.532,16	12.730,63	-48.858.32	7.260,436	4.275,795	12.179,78	.723,029,0	.223,750,9	5.989,280

TABLE 20—Continued

UNIT TANGENTIAL LOAD NO. 5

RADIAL AND TWIST DEFLECTIONS

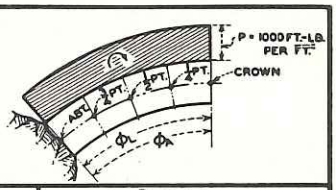


φ °	Crown			¼ Point			½ Point			¾ Point		
	D_1	D_2 1ST Term	D_2 2ND Term	D_1	D_2 1ST Term	D_2 2ND Term	D_1	D_2 1ST Term	D_2 2ND Term	D_1	D_2 1ST Term	D_2 2ND Term
10	.007,727,042	.001,119,538	-.004,035,59	.004,890,821	$.0^3$3,519,071,3	.091,697,30	.001,449,521	$.0^3$1,101,803,6	.055,043,89	$.0^3$1,120,815,1	$.0^5$4,227,445	.010,370,10
11	.011,311,44	.001,801,296	-.006,494,89	.007,159,885	$.0^3$5,835,509,2	.121,558,8	.002,122,136	$.0^3$1,163,915,5	.073,169,30	$.0^3$1,176,883,1	$.0^5$6,807,600	.013,798,74
12	.016,017,68	.002,780,168	-.010,027,5	.010,139,33	.001,290,126	.157,119,8	.003,005,407	$.0^3$2,253,189,3	.094,859,85	$.0^3$1,250,515,3	$.0^4$10,517,38	.017,909,02
13	.022,058,17	.004,143,664	-.014,950,3	.013,963,76	.001,923,791	.198,802,2	.004,139,286	$.0^3$3,377,683,9	.120,421,1	$.0^3$1,345,045,2	$.0^4$15,692,28	.022,762,15
14	.029,663,55	.005,994,747	-.021,636,6	.018,779,38	.002,784,671	.247,003,1	.005,567,188	$.0^3$4,548,908,2	.150,153,9	$.0^3$1,464,095,5	$.0^4$22,728,72	.028,419,13
15	.039,082,71	.008,652,995	-.030,520,7	.024,744,01	.003,928,792	.302,092,5	.007,335,982	$.0^3$7,771,938,8	.184,353,6	$.0^3$1,611,578,7	$.0^4$32,086,84	.034,940,76
16	.050,582,69	.011,655,67	-.042,101,4	.032,027,00	.005,420,633	.364,412,0	.009,495,999	.001,065,537	.223,310,4	$.0^3$1,791,696,8	$.0^4$44,305,53	.042,387,59
17	.064,448,64	.015,758,88	-.056,947,4	.040,809,28	.007,333,627	.434,272,0	.012,101,01	.001,442,265	.267,308,0	.001,008,941	$.0^4$59,987,47	.050,819,92
18	.080,983,75	.020,938,55	-.075,699,8	.051,283,28	.009,750,742	.511,950,4	.015,208,25	.001,918,601	.316,624,0	.001,268,092	$.0^4$79,824,07	.060,297,82
19	.100,509,2	.027,39,1,45	-.099,077,7	.063,652,91	.012,765,01	.597,690,7	.018,878,39	.002,513,053	.371,529,4	.001,574,218	$.0^3$1,04,590,5	.070,88 1,05
20	.123,364,0	.035,336,22	-.127,881	.078,133,54	.016,480,05	.691,699,4	.023,175,52	.003,246,272	.432,287,6	.001,932,680	$.0^3$1,35,152,5	.082,629,06
21	.149,905,2	.045,014,18	-.162,993	.094,951,95	.021,010,55	.794,145,5	.028,167,21	.004,141,165	.499,155,0	.002,349,125	$.0^3$1,72,471,9	.095,600,96
22	.180,507,3	.056,690,37	-.205,389	.114,346,3	.026,482,80	.905,157,4	.033,924,40	.005,223,003	.572,379,6	.002,829,489	$.0^3$2,17,611,0	.109,855,7
23	.215,562,8	.070,654,03	-.256,132	.136,566,2	.033,035,15	1.024,822	.040,521,49	.006,519,536	.652,201,2	.003,379,998	$.0^3$2,71,736,9	.125,451,7
24	.255,481,5	.087,219,96	-.316,383	.161,872,4	.040,818,48	1.153,182	.048,036,32	.008,061,098	.738,851,4	.004,007,166	$.0^3$3,36,128,7	.142,447,0
25	.300,691,0	.106,728,7	-.387,403	.190,537,0	.049,996,70	1.290,235	.056,550,10	.009,880,711	.832,552,5	.004,717,796	$.0^3$4,12,180,1	.160,899,4
26	.351,635,9	.129,547,5	-.470,548	.222,843,4	.060,747,13	1.435,933	.066,147,46	.012,014,20	.933,517,0	.005,518,978	$.0^3$5,01,404,7	.180,866,2
27	.408,778,5	.156,070,7	-.567,288	.259,086,1	.073,260,95	1.590,177	.076,916,45	.014,500,29	1.041,948	.006,418,093	$.0^3$6,05,442,3	.202,404,5
28	.472,598,3	.186,720,8	-.679,195	.299,570,8	.087,743,62	1.752,820	.088,948,48	.017,380,71	1.158,039	.007,422,807	$.0^3$7,26,062,2	.225,570,5
29	.543,591,4	.221,948,2	-.807,952	.344,614,2	.104,415,3	1.923,663	1.02,338,4	.020,700,30	1.281,972	.008,541,076	$.0^3$8,65,168,8	.250,420,5
30	.622,271,5	.262,232,4	-.955,352	.394,544,1	.123,511,1	2.102,451	1.17,184,4	.024,507,11	1.413,920	.009,781,144	.001,024,806	.277,010,0
31	.709,169,0	.308,081,9	-1.123,310	.449,699,4	.145,281,6	2.288,879	1.33,588,0	.028,852,47	1.554,041	.011,151,54	.001,207,163	.305,394,1
32	.804,831,0	.360,034,6	-1.313,852	.510,429,8	.169,993,2	2.482,581	1.51,654,2	.033,791,15	1.702,487	.012,661,09	.001,414,578	.335,627,5
33	.909,821,2	.418,658,3	-1.529,123	.577,095,9	.197,928,2	2.683,138	1.71,491,3	.039,381,40	1.859,393	.014,318,89	.001,649,543	.367,764,3
34	1.024,720	.484,550,7	-1.771,394	.650,069,2	.229,385,3	2.890,071	1.93,210,9	.045,685,05	2.024,885	.016,134,33	.001,914,710	.401,858,1
35	1.150,124	.558,339,4	-2.043,057	.729,732,0	.264,680,1	3.102,836	2.16,928,0	.052,767,63	2.199,075	.018,117,11	.002,212,893	.437,961,9
36	1.286,646	.640,682,2	-2.346,623	.816,477,3	.304,144,7	3.320,834	.242,761,0	.060,698,42	2.382,063	.020,277,17	.002,547,077	.476,128,2
37	1.434,916	.732,267,1	-2.684,737	.910,708,7	.348,128,8	3.543,403	.270,831,5	.069,550,58	2.573,936	.022,624,79	.002,920,417	.516,409,2
38	1.595,579	.833,812,1	-3.060,165	1.012,840	.396,999,3	3.769,812	.301,264,5	.079,401,21	2.774,766	.025,170,50	.003,336,248	.558,855,9
39	1.769,295	.946,065,0	-3.475,801	1.123,297	.451,140,5	3.999,271	.334,188,3	.090,331,44	2.984,613	.027,925,12	.003,798,088	.603,519,2
40	1.956,741	1.069,803	-3.934,669	1.242,514	.510,954,9	4.230,923	.369,734,6	.102,426,5	3.203,522	.030,899,76	.004,309,639	.650,449,3
41	2.158,810	1.205,834	-4.439,920	1.370,938	.576,862,5	4.463,839	.408,037,7	.115,775,8	3.431,524	.034,105,83	.004,874,799	.699,695,5
42	2.375,609	1.354,993	-4.994,838	1.509,023	.649,301,7	4.697,031	.449,236,4	.130,473,1	3.668,634	.037,555,01	.005,497,659	.751,306,8
43	2.608,462	1.518,145	-5.602,833	1.657,236	.728,729,1	4.929,437	.493,471,8	.146,616,3	3.914,853	.041,259,27	.006,182,514	.805,331,3
44	2.857,906	1.696,183	-6.267,446	1.816,054	.815,619,3	5.159,926	.540,888,8	.164,308,0	4.170,167	.045,230,87	.006,933,857	.861,816,9
45	3.124,695	1.890,027	-6.992,345	1.985,964	.910,465,5	5.387,297	.591,635,1	.183,655,0	4.434,548	.049,482,34	.007,756,400	.920,809,5
46	3.409,597	2.100,623	-7.781,327	2.167,461	1.013,779	5.610,281	.645,861,9	.204,768,9	4.707,948	.054,026,50	.008,655,073	.982,356,2
47	3.713,394	2.328,946	-8.638,319	2.361,053	1.126,091	5.827,533	.703,723,7	.227,765,7	4.990,306	.058,876,51	.009,635,001	1.046,502
48	4.036,882	2.575,994	-9.567,376	2.567,255	1.247,948	6.037,641	.765,378,0	.252,766,4	5.281,546	.064,045,64	.010,701,56	1.113,291
49	4.380,874	2.842,790	-10.572,68	2.786,596	1.379,919	6.239,118	.830,985,8	.279,896,3	5.581,570	.069,547,68	.011,860,35	1.182,767
50	4.746,195	3.130,381	-11.658,53	3.019,609	1.522,587	6.430,401	.900,710,8	.309,285,9	5.890,268	.075,396,57	.013,117,18	1.254,975
51	5.133,684	3.439,836	-12.829,36	3.266,841	1.676,558	6.609,862	.974,720,5	.341,070,3	6.207,511	.081,606,53	.014,478,12	1.329,956
52	5.544,194	3.772,247	-14.089,72	3.528,848	1.842,452	6.775,789	1.053,185	.375,389,7	6.533,152	.088,192,11	.015,949,47	1.407,751
53	5.978,592	4.128,724	-15.444,28	3.806,194	2.020,909	6.926,405	1.136,278	.412,389,1	6.867,030	.095,168,1	.017,537,78	1.488,401
54	6.437,757	4.510,399	-16.897,83	4.099,453	2.212,587	7.059,848	1.224,176	.452,218,7	7.208,958	.102,549,6	.019,249,84	1.571,946
55	6.922,584	4.918,420	-18.455,27	4.409,209	2.418,162	7.174,199	1.317,059	.495,033,6	7.558,742	.110,352,0	.021,092,76	1.658,425
56	7.433,978	5.353,951	-20.121,62	4.736,054	2.638,327	7.267,446	1.415,110	.540,994,1	7.916,163	.118,591,0	.023,073,69	1.747,877
57	7.972,858	5.818,173	-21.902,01	5.080,591	2.873,793	7.337,514	1.518,514	.590,265,5	8.280,983	.127,282,4	.025,200,36	1.840,339
58	8.540,190	6.312,279	-23.801,69	5.443,430	3.125,286	7.382,251	1.627,462	.643,018,4	8.652,947	.136,442,5	.027,480,54	1.935,847
59	9.136,816	6.837,476	-25.825,99	5.825,190	3.393,553	7.399,427	1.742,144	.699,428,7	9.031,783	.146,087,9	.029,922,35	2.034,434
60	9.763,795	7.394,979	-27.980,35	6.226,501	3.679,353	7.386,744	1.862,757	.759,677,3	9.417,197	.156,235,2	.032,534,16	2.136,139
61	10.422,06	7.986,015	-30.270,32	6.648,001	3.983,465	7.341,830	1.989,499	.823,950,5	9.808,877	.166,901,6	.035,324,64	2.240,995
62	11.112,59	8.611,815	-32.701,54	7.090,334	4.306,681	7.262,231	2.122,570	.892,440,1	10.206,49	.178,104,4	.038,302,72	2.349,032
63	11.836,38	9.273,617	-35.279,74	7.554,156	4.649,811	7.145,429	2.262,176	.965,342,9	10.609,69	.189,861,2	.041,477,63	2.460,284
64	12.594,43	9.972,662	-38.010,73	8.040,130	5.013,679	6.988,828	2.408,523	1.042,861	11.018,10	.202,189,9	.044,858,90	2.574,781
65	13.387,76	10.710,19	-40.900,41	8.548,928	5.399,123	6.789,763	2.561,822	1.125,203	11.431,33	.215,108,8	.048,456,32	2.692,553
66	14.217,38	11.487,45	-43.954,78	9.081,229	5.806,996	6.545,497	2.722,287	1.212,580	11.848,97	.228,636,2	.052,280,01	2.813,628
67	15.084,35	12.305,68	-47.179,85	9.637,721	6.238,166	6.253,21	2.890,133	1.305,212	12.270,60	.242,790,9	.056,340,38	2.938,035
68	15.989,69	13.166,11	-50.581,79	10.219,10	6.693,513	5.910,05	3.065,581	1.403,320	12.695,75	.257,592,0	.060,648,17	3.065,800
69	16.934,48	14.069,97	-54.166,76	10.826,07	7.173,931	5.513,03	3.248,851	1.507,141	13.123,96	.273,058,7	.065,214,32	3.196,950
70	17.919,77	15.018,49	-57.941,02	11.459,34	7.680,324	5.059,15	3.440,169	1.616,902	13.554,73	.289,210,6	.070,050,26	3.331,508
71	18.946,64	16.012,86	-61.910,87	12.119,64	8.213,610	4.545,33	3.639,763	1.732,844	13.987,56	.306,067,5	.075,167,62	3.469,499
72	20.016,18	17.054,30	-66.082,66	12.807,68	8.774,718	3.968,42	3.847,864	1.855,215	14.421,91	.323,649,7	.080,578,36	3.610,945
73	21.129,48	18.143,98	-70.462,79	13.524,21	9.364,587	3.325,18	4.064,706	1.984,263	14.857,22	.341,977,4	.086,294,78	3.755,867
74	22.287,65	19.283,05	-75.057,71	14.269,96	9.984,166	2.612,33	4.290,525	2.120,247	15.292,92	.361,071,4	.092,329,52	3.904,285
75	23.491,81	20.472,68	-79.873,88	15.045,69	10.634,41	1.826,55	4.525,561	2.263,426	15.728,41	.380,952,6	.098,695,53	4.056,219
76	24.743,06	21.713,97	-84.917,81	15.852,15	11.316,30	.964,40	4.770,055	2.414,069	16.163,08	.401,642,2	.105,406,1	4.211,688
77	26.042,55	23.008,02	-90.195,98	16.690,11	12.030,79	.022,61	5.024,253	2.572,447	16.596,29	.423,161,8	.112,474,8	4.370,707
78	27.391,41	24.355,90	-95.714,99	17.560,33	12.778,88	-1.002,94	5.288,402	2.738,838	17.027,38	.445,533,2	.119,915,8	4.533,293
79	29.790,79	25.758,66	-101.481,3	18.463,59	13.561,54	-2.115,24	5.562,753	2.913,525	17.455,66	.468,778,3	.127,743,9	4.699,461
80	30.241,85	27.217,29	-107.501,6	19.400,69	14.379,79	-3.318,16	5.847,560	3.096,800	17.880,44	.492,919,6	.135,971,6	4.869,223
81	31.745,73	28.732,77	-113.782,3	20.372,39	15.234,61	-4.615,38	6.143,077	3.288,943	18.300,99	.517,979,6	.144,616,3	5.042,592
82	33.303,62	30.306,03	-120.330,0	21.379,52	16.127,00	-6.010,67	6.449,565	3.490,267	18.716,57	.543,981,1	.153,692,4	5.219,579
83	34.916,69	31.937,98	-127.151,2	22.422,86	17.057,99	-7.507,84	6.767,285	3.701,070	19.126,42	.570,947,5	.163,215,8	5.400,195
84	36.586,12	33.629,46	-134.252,3	23.503,23	18.028,57	-9.110,76	7.096,500	3.921,661	19.529,73	.598,902,0	.173,202,5	5.584,447
85	38.313,11	35.381,28	-141.640,0	24.621,44	19.039,76	-10.823,31	7.437,476	4.152,354	19.925,71	.627,868,3	.183,668,9	5.772,342
86	40.098,83	37.194,20	-149.320,6	25.778,32	20.092,57	-12.649,49	7.790,484	4.393,468	20.313,53	.657,870,5	.194,631,9	5.963,887
87	41.944,50	39.068,93	-157.300,4	26.974,69	21.188,01	-14.593,29	8.155,796	4.645,326	20.692,34	.688,932,7	.206,108,7	6.159,088
88	43.851,33	41.006,14	-165.585,9	28.211,38	22.327,10	-16.658,74	8.533,684	4.908,257	21.061,25	.721,079,5	.218,116,8	6.357,946
89	45.820,52	43.006,43	-174.183,3	29.489,25	23.510,83	-18.849,96	8.924,429	5.182,596	21.419,40	.754,335,5	.230,674,1	6.560,465
90	47.853,28	45.070,34	-183.098,9	30.809,12	24.740,30	-21.171,07	9.328,307	5.468,679	21.765,84	.788,726,0	.243,798,9	6.766,645

TABLE 21—CONSTANTS FOR TWIST, RADIAL, AND TANGENTIAL DEFLECTIONS, TWIST LOADS

UNIT TWIST LOAD NO. 1

TWIST, RADIAL AND TANGENTIAL DEFLECTIONS

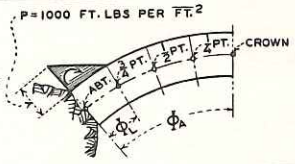


ϕ	Crown			¼ Point			½ Point			¾ Point		
°	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
10	15.230,87	1.766,800	.115,794	8.567,365	.746,363,4	.036,664,9	3.807,718	.221,355,4	.007,246,2	.951,929,4	.027,685,23	$.0^3$453,0
11	18.429,35	2.350,105	.169,473	10.366,51	.993,052,0	.053,670	4.607,339	.294,576,9	.010,608,4	1.151,835	.036,847,57	$.0^3$663,3
12	21.932,45	3.048,936	.239,931	12.337,01	1.288,743	.075,997	5.483,114	.382,373,9	.015,023,1	1.370,778	.047,836,08	$.0^3$939,4
13	25.740,17	3.873,499	.330,332	14.478,85	1.637,822	.104,650	6.435,043	.486,062,2	.020,690,1	1.608,761	.060,816,47	.001,293,8
14	29.852,51	4.833,924	.444,110	16.792,04	2.044,653	.140,723	7.463,127	.606,955,1	.027,826,1	1.865,782	.075,954,40	.001,740,2
15	34.269,46	5.940,255	.584,966	19.276,57	2.513,583	.185,395	8.567,365	.746,363,4	.036,664,9	2.141,841	.093,415,43	.002,293,2
16	38.991,03	7.202,451	.756,864	21.932,45	3.048,936	.239,931	9.747,757	.905,594,8	.047,457,9	2.436,939	.113,365,0	.002,968,5
17	44.017,22	8.630,372	.964,030	24.759,69	3.655,016	.305,679	11.004,30	1.085,954	.060,473	2.751,076	.135,968,6	.003,783,1
18	49.348,02	10.233,78	1.210,954	27.758,26	4.336,099	.384,074	12.337,01	1.288,743	.075,997	3.084,251	.161,391,4	.004,754,7
19	54.983,44	12.022,32	1.502,380	30.928,19	5.096,437	.476,635	13.745,86	1.515,260	.094,330	3.436,465	.189,798,7	.005,902,4
20	60.923,48	14.005,54	1.843,311	34.269,46	5.940,255	.584,966	15.230,87	1.766,800	.115,794	3.807,718	.221,355,4	.007,246,2
21	67.168,14	16.192,85	2.239,001	37.782,08	6.871,749	.710,752	16.792,04	2.044,653	.140,723	4.198,009	.256,226,5	.008,807,5
22	73.717,42	18.593,55	2.694,954	41.466,05	7.895,083	.855,763	18.429,35	2.350,105	.169,473	4.607,339	.294,576,9	.010,608,4
23	80.571,31	21.216,80	3.216,927	45.321,36	9.014,393	1.021,852	20.142,83	2.684,440	.202,413	5.035,707	.336,571,2	.012,672,1
24	87.729,82	24.071,62	3.810,913	49.348,02	10.233,78	1.210,954	21.932,45	3.048,936	.239,931	5.483,114	.382,373,9	.015,023,1
25	95.192,94	27.166,89	4.483,153	53.546,03	11.557,31	1.425,082	23.798,24	3.444,867	.282,430	5.949,559	.432,149,5	.017,687,0
26	102.960,7	30.511,35	5.240,13	57.915,39	12.989,00	1.666,335	25.740,17	3.873,499	.330,332	6.435,043	.486,062,2	.020,690,1
27	111.033,1	34.113,57	6.088,54	62.456,09	14.532,87	1.936,890	27.758,26	4.336,099	.384,074	6.939,566	.544,276,1	.024,060,2
28	119.410,0	37.981,97	7.035,34	67.168,14	16.192,85	2.239,001	29.852,51	4.833,924	.444,110	7.463,127	.606,955,1	.027,826,1
29	128.091,6	42.124,81	8.087,71	72.051,54	17.972,87	2.575,006	32.022,91	5.368,226	.510,912	8.005,727	.674,263,0	.032,017,4
30	137.077,8	46.550,16	9.253,05	77.106,29	19.876,79	2.947,320	34.269,46	5.940,255	.584,966	8.567,365	.746,363,4	.036,664,9
31	146.368,7	51.265,93	10.538,95	82.332,38	21.908,44	3.358,432	36.592,17	6.551,251	.666,777	9.148,042	.823,419,5	.041,800,8
32	155.964,1	56.279,86	11.953,27	87.729,82	24.071,62	3.810,913	38.991,03	7.202,451	.756,864	9.747,757	.905,594,8	.047,457,9
33	165.864,2	61.599,46	13.504,05	93.298,60	26.370,05	4.307,406	41.466,05	7.895,083	.855,763	10.366,51	.993,052,0	.053,670
34	176.068,9	67.232,11	15.199,55	99.038,74	28.807,43	4.850,633	44.017,22	8.630,372	.964,030	11.004,30	1.085,954	.060,473
35	186.578,2	73.184,93	17.048,22	104.950,2	31.387,41	5.443,39	46.644,54	9.409,533	1.082,231	11.661,14	1.184,464	.067,903
36	197.392,1	79.464,88	19.058,73	111.033,1	34.113,57	6.088,54	49.348,02	10.233,78	1.210,954	12.337,01	1.288,743	.075,997
37	208.510,6	86.078,71	21.239,93	117.287,2	36.989,46	6.789,04	52.127,66	11.104,31	1.350,798	13.031,91	1.398,955	.084,792
38	219.933,8	93.032,95	23.600,87	123.712,8	40.018,57	7.547,88	54.983,44	12.022,32	1.502,380	13.745,86	1.515,260	.094,330
39	231.661,6	100.333,9	26.150,79	130.309,6	43.204,35	8.368,17	57.915,39	12.989,00	1.666,335	14.478,85	1.637,822	.104,650
40	243.693,9	107.987,7	28.899,09	137.077,8	46.550,16	9.253,05	60.923,48	14.005,54	1.843,311	15.230,87	1.766,800	.115,794
41	256.030,9	116.000,2	31.855,35	144.017,4	50.059,34	10.205,74	64.007,74	15.073,10	2.033,972	16.001,93	1.902,357	.127,804
42	268.672,6	124.377,0	35.029,38	151.128,3	53.735,15	11.229,57	67.168,14	16.192,85	2.239,001	16.792,04	2.044,653	.140,723
43	281.618,8	133.123,6	38.431,07	158.410,6	57.580,81	12.327,86	70.404,70	17.365,95	2.459,091	17.601,18	2.193,849	.154,598
44	294.869,7	142.245,1	42.070,53	165.864,2	61.599,46	13.504,05	73.717,42	18.593,55	2.694,954	18.429,35	2.350,105	.169,473
45	308.425,1	151.746,4	45.957,99	173.489,1	65.794,20	14.761,65	77.106,29	19.876,79	2.947,320	19.276,57	2.513,583	.185,395
46	322.285,2	161.632,3	50.103,86	181.285,4	70.168,05	16.104,20	80.571,31	21.216,80	3.216,926	20.142,83	2.684,440	.202,413
47	336.449,9	171.907,2	54.518,67	189.253,1	74.723,98	17.535,33	84.112,49	22.614,70	3.504,533	21.028,12	2.862,839	.220,575
48	350.919,3	182.575,3	59.213,10	197.392,1	79.464,88	19.058,73	87.729,82	24.071,62	3.810,913	21.932,45	3.048,936	.239,931
49	365.693,2	193.640,5	64.197,99	205.702,4	84.393,59	20.678,13	91.423,30	25.588,65	4.136,852	22.855,83	3.242,893	.260,532
50	380.771,8	205.106,4	69.484,27	214.184,1	89.512,88	22.397,36	95.192,94	27.166,89	4.483,153	23.798,24	3.444,867	.282,430
51	396.155,0	216.976,6	75.083,02	222.837,2	94.825,45	24.220,27	99.038,74	28.807,43	4.850,633	24.759,69	3.655,016	.305,679
52	411.842,8	229.254,1	81.005,40	231.661,6	100.333,9	26.150,79	102.960,7	30.511,35	5.240,13	25.740,17	3.873,499	.330,332
53	427.835,2	241.941,9	87.262,73	240.657,3	106.040,9	28.192,89	106.958,8	32.279,71	5.652,47	26.739,70	4.100,475	.356,445
54	444.132,2	255.042,5	93.866,39	249.824,4	111.948,7	30.350,62	111.033,1	34.113,57	6.088,54	27.758,26	4.336,099	.384,074
55	460.733,9	268.558,2	100.827,9	259.162,8	118.060,0	32.628,07	115.183,5	36.013,98	6.549,20	28.795,87	4.580,530	.413,276
56	477.640,1	282.491,2	108.158,8	268.672,6	124.377,0	35.029,38	119.410,0	37.981,97	7.035,34	29.852,51	4.833,924	.444,110
57	494.851,0	296.843,1	115.870,9	278.353,7	130.902,0	37.558,76	123.712,8	40.018,57	7.547,88	30.928,19	5.096,437	.476,635
58	512.366,5	311.615,6	123.975,8	288.206,2	137.637,1	40.220,45	128.091,6	42.124,81	8.087,71	32.022,91	5.368,226	.510,912
59	530.186,6	326.809,8	132.485,4	298.230,0	144.584,6	43.018,74	132.546,7	44.301,67	8.655,79	33.136,66	5.649,447	.547,002
60	548.311,4	342.426,6	141.411,7	308.425,1	151.746,4	45.957,99	137.077,8	46.550,16	9.253,05	34.269,46	5.940,255	.584,966
61	566.740,7	358.466,7	150.766,5	318.791,7	159.124,5	49.042,59	141.685,2	48.871,26	9.880,45	35.421,29	6.240,805	.624,869
62	585.474,7	374.930,5	160.561,9	329.329,5	166.720,9	52.276,98	146.368,7	51.265,93	10.538,95	36.592,17	6.551,251	.666,777
63	604.513,3	391.817,9	170.809,9	340.038,7	174.537,2	55.665,64	151.128,3	53.735,15	11.229,57	37.782,08	6.871,749	.710,752
64	623.856,5	409.128,8	181.522,7	350.919,3	182.575,3	59.213,10	155.964,1	56.279,86	11.953,27	38.991,03	7.202,451	.756,864
65	643.504,3	426.862,6	192.712,5	361.971,2	190.836,7	62.923,95	160.876,1	58.900,99	12.711,09	40.219,02	7.543,511	.805,178
66	663.456,7	445.018,5	204.391,3	373.194,4	199.323,1	66.802,78	165.864,2	61.599,46	13.504,05	41.466,05	7.895,083	.855,763
67	683.713,8	463.595,4	216.571,4	384.589,0	208.036,0	70.854,23	170.928,5	64.376,20	14.333,19	42.732,11	8.257,319	.908,691
68	704.275,5	482.591,8	229.264,9	396.155,0	216.976,6	75.083,02	176.068,9	67.232,11	15.199,55	44.017,22	8.630,372	.964,030
69	725.141,8	502.006,1	242.484,2	407.892,2	226.146,4	79.493,84	181.285,4	70.168,05	16.104,20	45.321,36	9.014,393	1.021,852
70	746.312,7	521.836,2	256.241,4	419.800,9	235.546,6	84.091,49	186.578,2	73.184,93	17.048,22	46.644,54	9.409,533	1.082,231
71	767.788,2	542.079,8	270.548,8	431.880,9	245.178,2	88.880,73	191.947,1	76.283,59	18.032,69	47.986,76	9.815,945	1.145,241
72	789.568,4	562.734,3	285.418,5	444.132,2	255.042,5	93.866,39	197.392,1	79.464,88	19.058,73	49.348,02	10.233,78	1.210,954
73	811.653,1	583.796,8	300.862,7	456.554,9	265.140,2	99.053,34	202.913,3	82.729,65	20.127,43	50.728,32	10.663,18	1.279,448
74	834.042,5	605.264,0	316.893,7	469.148,9	275.472,4	104.446,5	208.510,6	86.078,71	21.239,93	52.127,66	11.104,31	1.350,798
75	856.736,5	627.132,5	333.523,5	481.914,3	286.039,8	110.050,7	214.184,1	89.512,88	22.397,36	53.546,03	11.557,31	1.425,082
76	879.735,1	649.398,4	350.764,2	494.851,0	296.843,1	115.870,9	219.933,8	93.032,95	23.600,87	54.983,44	12.022,32	1.502,380
77	903.038,3	672.057,6	368.627,9	507.959,1	307.883,0	121.912,1	225.759,6	96.639,71	24.851,62	56.439,90	12.499,51	1.582,771
78	926.646,2	695.105,6	387.126,6	521.238,5	319.159,9	128.179,3	231.661,6	100.333,9	26.150,79	57.915,39	12.989,00	1.666,335
79	950.558,7	718.537,8	406.272,2	534.689,2	330.674,4	134.677,5	237.639,7	104.116,3	27.499,54	59.409,92	13.490,97	1.753,154
80	974.775,7	742.349,2	426.076,5	548.311,4	342.426,6	141.411,7	243.693,9	107.987,7	28.899,09	60.923,48	14.005,54	1.843,311
81	999.297,5	766.534,3	446.551,5	562.104,8	354.417,0	148.386,9	249.824,4	111.948,7	30.350,62	62.456,09	14.532,87	1.936,890
82	1,024.124	791.087,7	467.708,7	576.069,6	366.645,6	155.608,4	256.030,9	116.000,2	31.855,35	64.007,74	15.073,10	2.033,972
83	1,049.255	816.003,4	489.559,9	590.205,8	379.112,6	163.081,0	262.313,7	120.142,7	33.414,55	65.578,42	15.626,38	2.134,648
84	1,074.690	841.275,2	512.116,6	604.513,3	391.817,9	170.809,9	268.672,6	124.377,0	35.029,38	67.168,14	16.192,85	2.239,001
85	1,100.430	866.896,6	535.390,1	618.992,1	404.761,4	178.800,3	275.107,6	128.703,7	36.701,15	68.776,90	16.772,66	2.347,119
86	1,126.475	892.860,8	559.391,9	633.642,3	417.942,9	187.057,2	281.618,8	133.123,6	38.431,07	70.404,70	17.365,95	2.459,091
87	1,152.825	919.160,7	584.133,2	648.463,9	431.362,0	195.585,9	288.206,2	137.637,1	40.220,45	72.051,54	17.972,87	2.575,006
88	1,179.479	945.789,1	609.625,0	663.456,7	445.018,5	204.391,3	294.869,7	142.245,1	42.070,53	73.717,42	18.593,55	2.694,954
89	1,206.437	972.738,1	635.878,4	678.621,0	458.911,8	213.478,7	301.609,3	146.948,0	43.982,61	75.402,33	19.228,14	2.819,031
90	1,233.701	1,000.000	662.904,2	693.956,6	473.041,2	222.853,2	308.425,1	151.746,4	45.957,99	77.106,29	19.876,79	2.947,320

TABLE 21—Continued

UNIT TWIST LOAD NO. 2

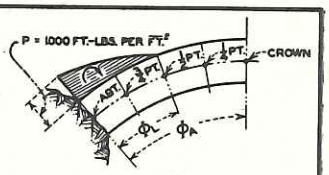
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



ϕ	Crown			¼ Point			½ Point			¾ Point		
°	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
10	.317,309,8	.051,686,51	.004,249,36	.317,309,8	.037,981,82	.002,292,79	.317,309,8	.024,204,82	$.0^3935,87$	.317,309,8	.010,381,74	$.0^3181,19$
11	.383,944,8	.068,729,68	.006,218,56	.383,944,8	.050,527,93	.003,356,04	.383,944,8	.032,209,80	.001,370,09	.383,944,8	.013,817,49	$.0^3265,31$
12	.456,926,1	.089,137,35	.008,802,75	.456,926,1	.065,562,22	.004,751,80	.456,926,1	.041,807,40	.001,940,23	.456,926,1	.017,937,98	$.0^3375,74$
13	.536,253,6	.113,202,6	.012,117,75	.536,253,6	.083,305,81	.006,542,96	.536,253,6	.053,141,05	.002,672,07	.536,253,6	.022,805,36	$.0^3517,53$
14	.621,927,2	.141,215,2	.016,289,03	.621,927,2	.103,978,4	.008,797,69	.621,927,2	.066,353,85	.003,593,57	.621,927,2	.028,481,72	$.0^3696,08$
15	.713,947,1	.173,461,3	.021,451,78	.713,947,1	.127,798,5	.011,589,56	.713,947,1	.081,588,50	.004,734,94	.713,947,1	.035,029,12	$.0^3917,27$
16	.812,313,1	.210,223,2	.027,750,67	.812,313,1	.154,982,8	.014,997,39	.812,313,1	.098,987,31	.006,128,56	.812,313,1	.042,509,59	.001,187,4
17	.917,025,4	.251,779,1	.035,339,88	.917,025,4	.185,746,4	.019,105,39	.917,025,4	.118,692,1	.007,809,11	.917,025,4	.050,985,11	.001,513,2
18	1.028,084	.298,402,9	.044,382,88	1.028,084	.220,302,8	.024,002,88	1.028,084	.140,844,4	.009,813,37	1.028,084	.060,517,64	.001,901,8
19	1.145,488	.350,363,9	.055,052,4	1.145,488	.258,863,5	.029,784,5	1.145,488	.165,585,0	.012,180,3	1.145,488	.071,169,06	.002,360,9
20	1.269,239	.407,926,2	.067,530,0	1.269,239	.301,638,0	.036,549,8	1.269,239	.193,054,3	.014,951,1	1.269,239	.083,001,24	.002,898,3
21	1.399,336	.471,349,0	.082,007,2	1.399,336	.348,834,0	.044,404,3	1.399,336	.223,392,1	.018,169,5	1.399,336	.096,075,98	.003,522,8
22	1.535,780	.540,886,3	.098,683,3	1.535,780	.400,656,6	.053,457,8	1.535,780	.256,737,8	.021,880,7	1.535,780	.110,455,0	.004,243,2
23	1.678,569	.616,786,0	.117,766,2	1.678,569	.457,308,8	.063,824,8	1.678,569	.293,229,8	.026,132,4	1.678,569	.126,200,1	.005,068,5
24	1.827,705	.699,290,5	.139,473,3	1.827,705	.518,991,5	.075,626,3	1.827,705	.333,006,4	.030,974,9	1.827,705	.143,372,8	.006,009,0
25	1.983,186	.788,635,8	.164,030,8	1.983,186	.585,902,6	.088,986,9	1.983,186	.376,204,7	.036,460,0	1.983,186	.162,034,6	.007,074,5
26	2.145,014	.885,051,9	.191,671,2	2.145,014	.658,237,9	.104,036,9	2.145,014	.422,961,5	.042,641,9	2.145,014	.182,247,3	.008,275,7
27	2.313,189	.988,761,8	.222,636,6	2.313,189	.736,190,1	.120,910,9	2.313,189	.473,412,5	.049,577,0	2.313,189	.204,072,1	.009,623,5
28	2.487,709	1.099,982	.257,176,9	2.487,709	.819,949,3	.139,749,0	2.487,709	.527,693,2	.057,323,6	2.487,709	.227,570,4	.011,129,7
29	2.668,576	1.218,922	.295,549,8	2.668,576	.909,702,8	.160,695,6	2.668,576	.585,937,8	.065,942,7	2.668,576	.252,803,6	.012,806,1
30	2.855,788	1.345,783	.338,020,1	2.855,788	1.005,635	.183,900,0	2.855,788	.648,279,9	.075,496,7	2.855,788	.279,832,9	.014,664,9
31	3.049,347	1.480,760	.384,860,3	3.049,347	1.107,926	.209,516,1	3.049,347	.714,852,4	.086,050,5	3.049,347	.308,719,5	.016,719,1
32	3.249,252	1.624,039	.436,350,2	3.249,252	1.216,755	.237,702,5	3.249,252	.785,787,0	.097,671,1	3.249,252	.339,524,4	.018,981,7
33	3.455,504	1.775,803	.492,776,8	3.455,504	1.332,296	.268,622,3	3.455,504	.861,214,8	.110,427,4	3.455,504	.372,308,6	.021,466,4
34	3.668,101	1.936,218	.554,433,4	3.668,101	1.454,721	.302,443,1	3.668,101	.941,265,9	.124,390,5	3.668,101	.407,133,0	.024,187,1
35	3.887,045	2.105,448	.621,620,3	3.887,045	1.584,197	.339,337,0	3.887,045	1.026,070	.139,633,6	3.887,045	.444,058,7	.027,158,6
36	4.112,335	2.283,648	.694,644,0	4.112,335	1.720,888	.379,480,1	4.112,335	1.115,754	.156,231,7	4.112,335	.483,146,1	.030,395,5
37	4.343,971	2.470,963	.773,817,7	4.343,971	1.864,955	.423,053,8	4.343,971	1.210,446	.174,262,3	4.343,971	.524,455,9	.033,913,5
38	4.581,954	2.667,528	.859,459,8	4.581,954	2.016,556	.470,242,4	4.581,954	1.310,272	.193,804,3	4.581,954	.568,048,8	.037,727,9
39	4.826,282	2.873,472	.951,894,9	4.826,282	2.175,842	.521,235,3	4.826,282	1.415,357	.214,939,3	4.826,282	.613,985,2	.041,855,2
40	5.076,958	3.088,913	1.051,453	5.076,958	2.342,964	.576,225,5	5.076,958	1.525,826	.237,750,2	5.076,958	.662,325,4	.046,311,8
41	5.333,978	3.313,960	1.158,470	5.333,978	2.518,066	.635,410,2	5.333,978	1.641,800	.262,322,5	5.333,978	.713,129,6	.051,114,8
42	5.597,345	3.548,711	1.273,286	5.597,345	2.701,290	.698,991,5	5.597,345	1.763,402	.288,743,5	5.597,345	.766,458,1	.056,281,9
43	5.867,059	3.793,256	1.396,245	5.867,059	2.892,772	.767,172,3	5.867,059	1.890,753	.317,101,8	5.867,059	.822,370,9	.061,830,4
44	6.143,118	4.047,675	1.527,698	6.143,118	3.092,644	.840,163,5	6.143,118	2.023,972	.347,489,4	6.143,118	.880,927,9	.067,779,3
45	6.425,524	4.312,037	1.667,999	6.425,524	3.301,036	.918,176,8	6.425,524	2.163,177	.379,998,7	6.425,524	.942,189,0	.074,146,9
46	6.714,276	4.586,403	1.817,506	6.714,276	3.518,071	1.001,429	6.714,276	2.308,486	.414,725,2	6.714,276	1.006,214	.080,952,5
47	7.009,374	4.870,820	1.976,580	7.009,374	3.743,868	1.090,138	7.009,374	2.460,013	.451,764,8	7.009,374	1.073,062	.088,215,0
48	7.310,818	5.165,329	2.145,589	7.310,818	3.978,542	1.184,529	7.310,818	2.617,874	.491,217,7	7.310,818	1.142,793	.095,955,6
49	7.618,609	5.469,956	2.324,898	7.618,609	4.222,204	1.284,827	7.618,609	2.782,182	.533,183,2	7.618,609	1.215,466	.104,193,7
50	7.932,746	5.784,720	2.514,881	7.932,746	4.474,958	1.391,263	7.932,746	2.953,048	.577,765,2	7.932,746	1.291,140	.112,951,1
51	8.253,228	6.109,627	2.715,913	8.253,228	4.736,907	1.504,071	8.253,228	3.130,584	.625,066,7	8.253,228	1.369,875	.122,247,6
52	8.580,058	6.444,674	2.928,369	8.580,058	5.008,144	1.623,485	8.580,058	3.314,897	.675,194,6	8.580,058	1.451,729	.132,106,0
53	8.913,232	6.789,845	3.152,629	8.913,232	5.288,762	1.749,745	8.913,232	3.506,097	.728,256,2	8.913,232	1.536,762	.142,547,6
54	9.252,754	7.145,114	3.389,073	9.252,754	5.578,845	1.883,093	9.252,754	3.704,288	.784,362,1	9.252,754	1.625,032	.153,595,6
55	9.598,622	7.510,445	3.638,085	9.598,622	5.878,476	2.023,775	9.598,622	3.909,576	.843,623,7	9.598,622	1.716,597	.165,272,7
56	9.950,836	7.885,788	3.900,048	9.950,836	6.187,729	2.172,037	9.950,836	4.122,065	.906,154,2	9.950,836	1.811,515	.177,602,2
57	10.309,40	8.271,085	4.175,347	10.309,40	6.506,675	2.328,131	10.309,40	4.341,855	.972,068,2	10.309,40	1.909,847	.190,607,2
58	10.674,30	8.666,263	4.464,367	10.674,30	6.835,379	2.492,309	10.674,30	4.569,048	1.041,483	10.674,30	2.011,648	.204,312
59	11.045,55	9.071,241	4.767,496	11.045,55	7.173,901	2.664,827	11.045,55	4.803,742	1.114,517	11.045,55	2.116,978	.218,743
60	11.423,15	9.485,926	5.085,117	11.423,15	7.522,296	2.845,942	11.423,15	5.046,034	1.191,290	11.423,15	2.225,894	.233,922
61	11.807,10	9.910,210	5.417,619	11.807,10	7.880,613	3.035,915	11.807,10	5.296,020	1.271,924	11.807,10	2.338,454	.249,877
62	12.197,39	10.343,98	5.765,385	12.197,39	8.248,895	3.235,006	12.197,39	5.553,794	1.356,543	12.197,39	2.454,715	.266,633
63	12.594,03	10.787,10	6.128,803	12.594,03	8.627,181	3.443,482	12.594,03	5.819,447	1.445,271	12.594,03	2.574,735	.284,215
64	12.997,01	11.239,44	6.508,255	12.997,01	9.015,502	3.661,608	12.997,01	6.093,072	1.538,236	12.997,01	2.698,570	.302,652
65	13.406,34	11.700,85	6.904,122	13.406,34	9.413,887	3.889,651	13.406,34	6.374,755	1.635,565	13.406,34	2.826,279	.321,968
66	13.822,02	12.171,15	7.316,789	13.822,02	9.822,356	4.127,882	13.822,02	6.664,586	1.737,389	13.822,02	2.957,918	.342,192
67	14.244,04	12.650,18	7.746,632	14.244,04	10.240,93	4.376,572	14.244,04	6.962,649	1.843,839	14.244,04	3.093,543	.363,353
68	14.672,41	13.137,75	8.194,027	14.672,41	10.669,60	4.635,994	14.672,41	7.269,028	1.955,048	14.672,41	3.233,211	.385,478
69	15.107,12	13.633,67	8.659,350	15.107,12	11.108,39	4.906,421	15.107,12	7.583,806	2.071,149	15.107,12	3.376,978	.408,594
70	15.548,18	14.137,72	9.142,971	15.548,18	11.557,30	5.188,131	15.548,18	7.907,061	2.192,280	15.548,18	3.524,902	.432,732
71	15.995,59	14.649,68	9.645,261	15.995,59	12.016,30	5.481,400	15.995,59	8.238,873	2.318,578	15.995,59	3.677,037	.457,923
72	16.449,34	15.169,32	10.166,58	16.449,34	12.485,40	5.786,503	16.449,34	8.579,319	2.450,180	16.449,34	3.833,439	.484,192
73	16.909,44	15.696,39	10.707,29	16.909,44	12.964,56	6.103,722	16.909,44	8.928,473	2.587,227	16.909,44	3.994,165	.511,572
74	17.375,89	16.230,65	11.267,75	17.375,89	13.453,77	6.433,336	17.375,89	9.286,407	2.729,862	17.375,89	4.159,269	.540,095
75	17.848,68	16.771,83	11.848,31	17.848,68	13.952,99	6.775,625	17.848,68	9.653,193	2.878,228	17.848,68	4.328,806	.569,790
76	18.327,82	17.319,64	12.449,32	18.327,82	14.462,19	7.130,870	18.327,82	10.028,90	3.032,468	18.327,82	4.502,834	.600,690
77	18.813,30	17.873,80	13.071,13	18.813,30	14.981,32	7.499,354	18.813,30	10.413,60	3.192,728	18.813,30	4.681,404	.632,825
78	19.305,13	18.434,01	13.714,06	19.305,13	15.510,33	7.881,357	19.305,13	10.807,34	3.359,155	19.305,13	4.864,574	.666,227
79	19.803,31	18.999,96	14.378,45	19.803,31	16.049,16	8.277,162	19.803,31	11.210,21	3.531,897	19.803,31	5.052,396	.700,930
80	20.307,83	19.571,33	15.064,64	20.307,83	16.597,75	8.687,052	20.307,83	11.622,25	3.711,105	20.307,83	5.244,926	.736,967
81	20.818,70	20.147,78	15.772,93	20.818,70	17.156,04	9.111,309	20.818,70	12.043,52	3.896,928	20.818,70	5.442,218	.774,371
82	21.335,91	20.728,97	16.503,64	21.335,91	17.723,95	9.550,216	21.335,91	12.474,10	4.089,519	21.335,91	5.644,325	.813,175
83	21.859,47	21.314,55	17.257,08	21.859,47	18.301,40	10.004,05	21.859,47	12.914,02	4.289,031	21.859,47	5.851,300	.853,414
84	22.389,38	21.904,16	18.033,54	22.389,38	18.888,30	10.473,11	22.389,38	13.363,34	4.495,617	22.389,38	6.063,199	.895,122
85	22.925,63	22.497,42	18.833,33	22.925,63	19.484,56	10.957,65	22.925,63	13.822,11	4.709,435	22.925,63	6.280,073	.938,334
86	23.468,23	23.093,94	19.656,72	23.468,23	20.090,08	11.457,97	23.468,23	14.290,39	4.930,639	23.468,23	6.501,976	.983,086
87	24.017,18	23.693,33	20.503,99	24.017,18	20.704,75	11.974,35	24.017,18	14.768,21	5.159,389	24.017,18	6.728,961	1.029,412
88	24.572,47	24.295,19	21.375,41	24.572,47	21.328,47	12.507,07	24.572,47	15.255,63	5.395,841	24.572,47	6.961,080	1.077,350
89	25.134,11	24.899,10	22.271,24	25.134,11	21.961,10	13.056,40	25.134,11	15.752,68	5.640,156	25.134,11	7.198,387	1.126,935
90	25.702,10	25.504,64	23.191,73	25.702,10	22.602,54	13.622,62	25.702,10	16.259,41	5.892,496	25.702,10	7.440,932	1.178,205

TABLE 21—Continued

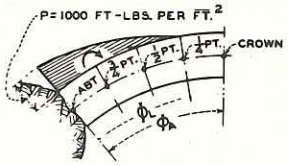
UNIT TWIST LOAD NO. 3
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



φ	Crown			¼ Point			½ Point			¾ Point		
°	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
10	1.269,239	.193,054,3	.014,951,1	1.269,239	.138,159,3	.007,724,06	1.269,239	.083,001,24	.002,898,3	.634,619,6	.019,033,49	$.0^3$317,11
11	1.535,780	.256,737,8	.021,880,7	1.535,780	.183,808,1	.011,306,3	1.535,780	.110,455,0	.004,243,2	.767,889,7	.025,332,53	$.0^3$464,31
12	1.827,705	.333,006,4	.030,974,9	1.827,705	.238,516,4	.016,009,1	1.827,705	.143,372,8	.006,009,0	.913,852,3	.032,887,03	$.0^3$657,57
13	2.145,014	.422,961,5	.042,641,9	2.145,014	.303,091,9	.022,044,4	2.145,014	.182,247,3	.008,275,7	1.072,507	.041,810,92	$.0^3$905,66
14	2.487,709	.527,693,2	.057,323,6	2.487,709	.378,337,5	.029,641,9	2.487,709	.227,570,4	.011,129,7	1.243,855	.052,218,05	.001,218,2
15	2.855,788	.648,279,9	.075,496,7	2.855,788	.465,052,1	.039,050,0	2.855,788	.279,832,9	.014,664,9	1.427,894	.064,222,27	.001,605,2
16	3.249,252	.785,787,0	.097,671,1	3.249,252	.564,029,6	.050,534,4	3.249,252	.339,524,4	.018,981,7	1.624,626	.077,937,31	.002,077,9
17	3.668,101	.941,265,9	.124,390,5	3.668,101	.676,058,6	.064,379,3	3.668,101	.407,133,0	.024,187,1	1.834,051	.093,476,86	.002,648,1
18	4.112,335	1.115,754	.156,231,7	4.112,335	.801,922,0	.080,886,0	4.112,335	.483,146,1	.030,395,5	2.056,168	.110,954,5	.003,328,3
19	4.581,954	1.310,272	.193,804,3	4.581,954	.942,397,1	.100,374,1	4.581,954	.568,048,8	.037,727,9	2.290,977	.130,483,9	.004,131,7
20	5.076,958	1.525,826	.237,750,2	5.076,958	1.098,255	.123.1804	5.076,958	.662,325,4	.046,311,8	2.538,478	.152,178,3	.005,072,3
21	5.597,345	1.763,402	.288,743,5	5.597,345	1.270,259	.149,659,3	5.597,345	.766,458,1	.056,281,9	2.798,673	.176,151,3	.006,165,2
22	6.143,118	2.023,972	.347,489,4	6.143,118	1.459,168	.180,182,2	6.143,118	.880,927,9	.067,779,3	3.071,559	.202,516,0	.007,425,8
23	6.714,276	2.308,486	.414,725,2	6.714,276	1.665,731	.215,138,5	6.714,276	1.006,214	.080,952,5	3.357,138	.231,385,6	.008,807,3
24	7.310,818	2.617,874	.491,217,7	7.310,818	1.890,691	.254,933,8	7.310,818	1.142,793	.095,955,6	3.655,409	.262,873,3	.010,516,0
25	7.932,746	2.953,048	.577,765,2	7.932,746	2.134,783	.299,991,6	7.932,746	1.291,140	.112,951	3.966,373	.297,092,1	.012,380,8
26	8.580,058	3.314,897	.675,194,6	8.580,058	2.398,733	.350,751,0	8.580,058	1.451,729	.132,106	4.290,029	.334,154,7	.014,482,9
27	9.252,754	3.704,288	.784,362,1	9.252,754	2.683,259	.407,668,7	9.252,754	1.625,032	.153,596	4.626,377	.374,174,1	.016,841,9
28	9.950,836	4.122,065	.906,154,2	9.950,836	2.989,070	.471,218,6	9.950,836	1.811,515	.177,602	4.975,418	.417,262,8	.019,477,9
29	10.674,30	4.569,048	1.041,483	10.674,30	3.316,867	.541,889	10.674,30	2.011,648	.204,312	5.337,151	.463,533,3	.022,411,8
30	11.423,15	5.046,034	1.191,290	11.423,15	3.667,339	.620,185	11.423,15	2.225,894	.233,922	5.711,577	.513,098,1	.025,664,9
31	12.197,39	5.553,794	1.356,543	12.197,39	4.041,167	.706,630	12.197,39	2454,715	.266,633	6.098,695	.566,069,5	.029,259,9
32	12.997,01	6.093,072	1.538,236	12.997,01	4.439,021	.801,761	12.997,01	2.698,570	.302,652	6.498,505	.622,559,6	.033,219,7
33	13.822,02	6.664,586	1.737,389	13.822,02	4.861,562	.906,129	13.822,02	2.957,918	.342,192	6.911,007	.682,680,3	.037,568,2
34	14.672,41	7.269,028	1.955,048	14.672,41	5.309,439	1.020,306	14.672,41	3.233,211	.385,478	7.336,203	.746,543,7	.042,330,6
35	15.548,18	7.907,061	2.192,280	15.548,18	5.783,290	1.144,873	15.548,18	3.524,902	.432,732	7.774,091	.814,261,3	.047,530,8
36	16.449,34	8.579,319	2.450,180	16.449,34	6.283,742	1.280,431	16.449,34	3.833,439	.484,192	8.224,671	.885,944,7	.053,196,2
37	17.375,89	9.286,407	2.729,862	17.375,89	6.811,411	1.427,593	17.375,89	4.159,269	.540,095	8.687,943	.961,705,4	.059,353,0
38	18.327,82	10.028,90	3.032,468	18.327,82	7.366,900	1.586,988	18.327,82	4.502,834	.600,690	9.163,907	1.041,655	.066,028,9
39	19.305,13	10.807,34	3.359,155	19.305,13	7.950,799	1.759,260	19.305,13	4.864,574	.666,227	9.652,565	1.125,903	.073,2525
40	20.307,83	11.622,25	3.711,105	20.307,83	8.563,689	1.945,064	20.307,83	5.244,926	.736,967	10.153,91	1.214,563	.081,052
41	21.335,91	12.474,10	4.089,519	21.335,91	9.206,134	2.145,075	21.335,91	5.644,325	.813,175	10.667,96	1.307,743	.089,459
42	22.389,38	13.363,34	4.495,617	22.389,38	9.878,688	2.359,975	22.389,38	6.063,199	.895,122	11.194,69	1.405,555	.098,502
43	23.468,23	14.290,39	4.930,639	23.468,23	10.581,89	2.590,466	23.468,23	6.501,976	.983,086	11.734,12	1.508,110	.108,214
44	24.572,47	15.255,63	5.395,841	24.572,47	11.316,27	2.837,259	24.572,47	6.961,080	1.077,350	12.286,24	1.615,517	.118,626
45	25.702,10	16.259,41	5.892,496	25.702,10	12.082,33	3.101,078	25.702,10	7.440,932	1.178,205	12.851,05	1.727,886	.129,771
46	26.857,10	17.302,05	6.421,893	26.857,10	12.880,58	3.382,662	26.857,10	7.941,946	1.285,946	13.428,55	1.845,327	.141,682
47	28.037,50	18.383,82	6.985,340	28.037,50	13.711,50	3.682,766	28.037,50	8.464,537	1.400,875	14.018,75	1.967,950	.154,395
48	29.243,27	19.504,96	7.584,153	29.243,27	14.575,55	4.002,147	29.243,27	9.009,113	1.523,299	14.621,64	2.095,865	.167,943
49	30.474,43	20.665,69	8.219,670	30.474,43	15.473,19	4.341,588	30.474,43	9.576,080	1.653,537	15.237,22	2.229,179	.182,363
50	31.730,98	21.866,18	8.893,228	31.730,98	16.404,87	4.701,866	31.730,98	10.165,84	1.791,900	15.865,49	2.368,003	.197,691
51	33.012,91	23.106,56	9.606,198	33.012,91	17.371,00	5.083,794	33.012,91	10.778,79	1.938,725	16.506,46	2.512,446	.213,964
52	34.320,23	24.386,93	10.359,93	34.320,23	18.372,00	5.488,167	34.320,23	11.415,32	2.094,331	17.160,12	2.662,614	.231,219
53	35.652,94	25.707,35	11.155,83	35.652,94	19.408,25	5.915,825	35.652,94	12.075,83	2.259,070	17.826,47	2.818,618	.249,496
54	37.011,02	27.067,83	11.995,25	37.011,02	20.480,13	6.367,573	37.011,02	12.760,69	2.433,262	18.505,51	2.980,565	.268,835
55	38.394,49	28.468,38	12.879,61	38.394,49	21.588,01	6.844,278	38.394,49	13.470,30	2.617,277	19.197,24	3.148,563	.289,274
56	39.803,34	29.908,92	13.810,30	39.803,34	22.732,21	7.346,785	39.803,34	14.205,02	2.811,463	19.901,67	3.322,720	.310,856
57	41.237,58	31.389,38	14.788,75	41.237,58	23.913,09	7.875,955	41.237,58	14.965,23	3.016,177	20.618,79	3.503,142	.333,621
58	42.697,21	32.909,61	15.816,35	42.697,21	25.130,93	8.432,660	42.697,21	15.751,30	3.231,784	21.348,60	3.689,937	.357,612
59	44.182,22	34.469,45	16.894,54	44.182,22	26.386,04	9.017,786	44.182,22	16.563,58	3.458,659	22.091,11	3.883,213	.382,872
60	45.692,61	36.068,68	18.024,72	45.692,61	27.678,69	9.632,217	45.692,61	17.402,45	3.697,172	22.846,31	4.083,075	.409,445
61	47.228,39	37.707,06	19.208,33	47.228,39	29.009,14	10.276,86	47.228,39	18.268,25	3.947,713	23.614,20	4.289,629	.437,373
62	48.789,56	39.384,29	20.446,80	48.789,56	30.377,63	10.952,62	48.789,56	19.161,33	4.210,661	24.394,78	4.502,983	.466,704
63	50.376,11	41.100,06	21.741,55	50.376,11	31.784,38	11.660,41	50.376,11	20.082,04	4.486,410	25.188,05	4.723,242	.497,484
64	51.988,04	42.853,97	23.094,00	51.988,04	33.229,61	12.401,15	51.988,04	21.030,72	4.775,360	25.994,02	4.950,511	.529,758
65	53.625,36	44.645,64	24.505,59	53.625,36	34.713,48	13.175,78	53.625,36	22.007,71	5.077,910	26.812,69	5.184,895	.563,582
66	55.288,06	46.474,60	25.977,73	55.288,06	36.236,17	13.985,24	55.288,06	23.013,32	5.394,469	27.644,03	5.426,500	.598,978
67	56.976,15	48.340,36	27.511,85	56.976,15	37.797,84	14.830,47	56.976,15	24.047,90	5.725,452	28.488,08	5.675,431	.636,022
68	58.689,62	50.242,39	29.109,35	58.689,62	39.398,60	15.712,41	58.689,62	25.111,75	6.071,275	29.344,81	5.931,791	.674,753
69	60.428,48	52.180,13	30.771,65	60.428,48	41.038,58	16.632,03	60.428,48	26.205,20	6.432,354	30.214,24	6.195,686	.715,223
70	62.192,72	54.152,96	32.500,15	62.192,72	42.717,87	17.590,30	62.192,72	27.328,55	6.809,127	31.096,36	6.467,217	.757,481
71	63.982,35	56.160,23	34.296,25	63.982,35	44.436,54	18.588,18	63.982,35	28.482,11	7.202,019	31.991,18	6.746,491	.801,582
72	65.797,36	58.201,25	36.161,33	65.797,36	46.194,64	19.626,64	65.797,36	29.666,17	7.611,469	32.898,68	7.033,608	.847,572
73	67.637,76	60.275,28	38.096,77	67.637,76	47.992,21	20.706,67	67.637,76	30.881,04	8.037,920	33.818,88	7.328,674	.895,510
74	69.503,54	62.381,55	40.103,93	69.503,54	49.829,27	21.829,24	69.503,54	32.126,99	8.481,813	34.751,77	7.631,788	.945,447
75	71.394,71	64.519,25	42.184,18	71.394,71	51.705,80	22.995,34	71.394,71	33.404,32	8.943,603	35.697,35	7.943,056	.997,435
76	73.311,26	66.687,52	44.338,85	73.311,26	53.621,80	24.205,97	73.311,26	34.713,29	9.423,749	36.655,63	8.262,577	1.051,534
77	75.253,19	68.885,49	46.569,27	75.253,19	55.577,21	25.462,12	75.253,19	36.054,18	9.922,701	37.626,60	8.590,455	1.107,797
78	77.220,52	71.112,20	48.876,77	77.220,52	57.571,97	26.764,79	77.220,52	37.427,26	10.440,93	38.610,26	8.926,789	1.166,281
79	79.213,22	73.366,69	51.262,63	79.213,22	59.606,00	28.114,96	79.213,22	38.832,78	10.978,90	39.606,61	9.271,682	1.227,043
80	81.231,31	75.647,96	53.728,15	81.231,31	61.679,19	29.513,67	81.231,31	40.271,01	11.537,10	40.615,66	9.625,233	1.290,139
81	83.274,79	77.954,95	56.274,58	83.274,79	63.791,43	30.961,88	83.274,79	41.742,18	12.115,98	41.637,39	9.987,543	1.355,630
82	85.343,65	80.286,58	58.903,17	85.343,65	65.942,56	32.460,64	85.343,65	43.246,54	12.716,03	42.671,82	10.358,71	1.423,573
83	87.437,89	82.641,72	61.615,15	87.437,89	68.132,42	34.010,92	87.437,89	44.784,32	13.337,75	43.718,95	10.738,84	1.494,031
84	89.557,52	85.019,21	64.411,73	89.557,52	70.360,82	35.613,75	89.557,52	46.355,77	13.981,61	44.778,76	11.128,03	1.567,062
85	91.702,54	87.417,85	67.294,10	91.702,54	72.627,57	37.270,13	91.702,54	47.961,09	14.648,11	45.851,27	11.526,37	1.642,726
86	93.872,94	89.836,39	70.263,41	93.872,94	74.932,44	38.981,07	93.872,94	49.600,52	15.337,75	46.936,47	11.933,96	1.721,088
87	96.068,72	92.273,58	73.320,80	96.068,72	77.275,17	40.747,58	96.068,72	51.274,26	16.051,02	48.034,36	12.350,92	1.802,209
88	98.289,89	94.728,09	76.467,40	98.289,89	79.655,51	42.570,66	98.289,89	52.982,51	16.788,44	49.144,94	12.777,32	1.886,152
89	100.536,4	97.198,59	79.704,28	100.536,4	82.073,16	44.451,33	100.536,4	54.725,49	17.550,50	50.268,22	13.213,26	1.972,982
90	102.808,4	99.683,68	83.032,53	102.808,4	84.527,82	46.390,58	102.808,4	56.503,37	18.337,73	51.404,19	13.658,86	2.062,762

TABLE 21—Continued

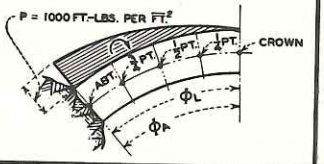
UNIT TWIST LOAD NO. 4
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



φ	Crown			¼ Point			½ Point			¾ Point		
°	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
10	2.855,788	.403,497,7	.029,575,3	2.855,788	.279,832,9	.014,664,9	2.115,399	.129,119,3	.004,347,6	.740,389,6	.021,917,40	$.0^3362,42$
11	3.455,504	.536,638,6	.043,283,7	3.455,504	.372,308,6	.021,466,4	2.559,633	.171,829,0	.006,364,9	.895,871,3	.029,170,88	$.0^3530,64$
12	4.112,335	.696,116,2	.061,275,6	4.112,335	.483,146,1	.030,395,5	3.046,174	.223,039,8	.009,013,7	1.066,161	.037,870,04	$.0^3751,51$
13	4.826,282	.884,240,2	.084,358,6	4.826,282	.613,985,2	.041,855,2	3.575,024	.283,518,9	.012,413,8	1.251,258	.048,146,10	.001,035,0
14	5.597,345	1.103,302	.113,408,3	5.597,345	.766,458,1	.056,281,9	4.146,182	.354,032,0	.016,695,1	1.451,164	.060,130,17	.001,392,2
15	6.425,524	1.355,572	.149,367,8	6.425,524	.942,189,0	.074,146,9	4.759,647	.435,343,1	.021,998,3	1.665,876	.073,953,32	.001,834,6
16	7.310,818	1.643,292	.193,247,9	7.310,818	1.142,793	.095,955,6	5.415,421	.528,214,5	.028,473,8	1.895,397	.089,746,56	.002,374,8
17	8.253,228	1.968,683	.246,125,7	8.253,228	1.369,875	.122,247,6	6.113,502	.633,406,8	.036,282,8	2.139,726	.107,640,8	.003,026,5
18	9.252,754	2.333,934	.309,143,8	9.252,754	1.625,032	.153,595,6	6.853,892	.751,678,5	.045,596,0	2.398,862	.127,766,8	.003,803,7
19	10.309,40	2.741,206	.383,511,5	10.309,40	1.909,847	.190,607,2	7.636,589	.883,786,0	.056,595,3	2.672,806	.150,255,5	.004,721,9
20	11.423,15	3.192,629	.470,500	11.423,15	2.225,894	.233,922	8.461,595	1.030,483	.069,472,5	2.961,558	.175,237,3	.005,796,8
21	12.594,03	3.690,301	.571,448	12.594,03	2.574,735	.284,215	9.328,908	1.192,523	.084,428,9	3.265,118	.202,843,0	.007,045,9
22	13.822,02	4.236,284	.687,755	13.822,02	2.957,918	.342,192	10.238,53	1.370,654	.101,677,2	3.583,485	.233,202,9	.008,486,7
23	15.107,12	4.832,604	.820,882	15.107,12	3.376,978	.408,594	11.190,46	1.565,623	.121,439,3	3.916,661	.266,447,5	.010,137,5
24	16.449,34	5.481,251	.972,354	16.449,34	3.833,439	.484,192	12.184,70	1.778,174	.143,947,4	4.264,644	.302,706,9	.012,018,4
25	17.848,68	6.184,176	1.143,754	17.848,68	4.328,806	.569,790	13.221,24	2.009,049	.169,444,0	4.627,435	.342,111,2	.014,149,5
26	19.305,13	6.943,287	1.336,727	19.305,13	4.864,574	.666,227	14.300,10	2.258,986	.198,182	5.005,033	.384,790,6	.016,551,9
27	20.818,70	7.760,453	1.552,973	20.818,70	5.442,218	.774,371	15.421,26	2.528,721	.230,422	5.397,440	.430,874,8	.019,248,0
28	22.389,38	8.637,496	1.794,255	22.389,38	6.063,199	.895,122	16.584,73	2.818,985	.266,438	5.804,654	.480,493,6	.022,260,6
29	24.017,18	9.576,196	2.062,389	24.017,18	6.728,961	1.029,412	17.790,50	3.130,508	.306,512	6.226,676	.533,776,6	.025,613,5
30	25.702,10	10.578,28	2.359,246	25.702,10	7.440,932	1.178,205	19.038,59	3.464,014	.350,937	6.663,506	.590,853,2	.029,331,6
31	27.444,13	11.645,44	2.686,755	27.444,13	8.200,519	1.342,493	20.328,98	3.820,227	.400,014	7.115,144	.651,852,8	.033,440,2
32	29.243,27	12.779,31	3.046,894	29.243,27	9.009,113	1.523,299	21.661,68	4.199,864	.454,055	7.581,589	.716,904,6	.037,965,7
33	31.099,54	13.981,47	3.441,702	31.099,54	9.868,086	1.721,684	23.036,69	4.603,639	.513,382	8.062,842	.786,137,0	.042,935,5
34	33.012,91	15.253,45	3.873,255	33.012,91	10.778,79	1.938,725	24.454,01	5.032,264	.578,329	8.558,903	.859,678,5	.048,378,4
35	34.983,41	16.596,73	4.343,686	34.983,41	11.742,56	2.175,535	25.913,64	5.486,445	.649,232	9.069,772	.937,662,0	.054,321,4
36	37.011,02	18.012,74	4.855,184	37.011,02	12.760,69	2.433,262	27.415,57	5.966,885	.726,446	9.595,449	1.020,211	.060,795,9
37	39.095,74	19.502,84	5.409,975	39.095,74	13.834,50	2.713,076	28.959,81	6.474,282	.810,329	10.135,93	1.107,455	.067,832
38	41.237,58	21.068,34	6.010,336	41.237,58	14.965,23	3.016,177	30.546,36	7.009,329	.901,253	10.691,23	1.199,523	.075,462
39	43.436,54	22.710,50	6.658,582	43.436,54	16.154,14	3.343,790	32.175,22	7.572,717	.999,596	11.261,33	1.296,543	.083,718
40	45.692,61	24.430,50	7.357,083	45.692,61	17.402,45	3.697,172	33.846,38	8.165,131	1.105,749	11.846,23	1.398,642	.092,634
41	48.005,80	26.229,48	8.108,236	48.005,80	18.711,36	4.077,607	35.559,85	8.787,250	1.220,107	12.445,95	1.505,948	.102,241
42	50.376,11	28.108,50	8.914,502	50.376,11	20.082,04	4.486,410	37.315,63	9.439,750	1.343,080	13.060,47	1.618,588	.112,576
43	52.803,53	30.068,58	9.778,347	52.803,53	21.515,66	4.924,909	39.113,72	10.123,30	1.475,087	13.689,80	1.736,689	.123,675
44	55.288,06	32.110,65	10.702,30	55.288,06	23.013,32	5.394,469	40.954,12	10.838,57	1.616,551	14.333,94	1.860,379	.135,575
45	57.829,71	34.235,59	11.688,93	57.829,71	24.576,15	5.896,481	42.836,83	11.586,22	1.767,910	14.992,89	1.989,785	.148,313
46	60.428,48	36.444,22	12.740,82	60.428,48	26.205,20	6.432,354	44.761,84	12.366,90	1.929,606	15.666,64	2.125,032	.161,926
47	63.084,36	38.737,27	13.860,60	63.084,36	27.901,53	7.003,530	46.729,16	13.181,26	2.102,097	16.355,21	2.266,246	.176,455
48	65.797,36	41.115,42	15.050,91	65.797,36	29.666,17	7.611,469	48.738,79	14.029,95	2.285,837	17.058,58	2.413,555	.191,939
49	68.567,48	43.579,29	16.314,46	68.567,48	31.500,11	8.257,656	50.790,72	14.913,60	2.481,309	17.776,75	2.567,084	.208,419
50	71.394,71	46.129,39	17.653,96	71.394,71	33.404,32	8.943,603	52.884,97	15.832,86	2.688,984	18.509,74	2.726,958	.225,938
51	74.279,05	48.766,21	19.072,15	74.279,05	35.379,73	9.670,845	55.021,52	16.788,34	2.909,361	19.257,53	2.893,302	.244,535
52	77.220,52	51.490,14	20.571,79	77.220,52	37.427,26	10.440,93	57.200,38	17.780,66	3.142,929	20.020,13	3.066,243	.264,257
53	80.219,09	54.301,49	22.155,68	80.219,09	39.547,79	11.255,44	59.421,55	18.810,45	3.390,201	20.797,54	3.245,904	.285,145
54	83.274,79	57.200,53	23.826,62	83.274,79	41.742,18	12.115,98	61.685,03	19.878,32	3.651,689	21.589,76	3.432,410	.307,247
55	86.387,60	60.187,42	25.587,45	86.387,60	44.011,24	13.024,15	63.990,81	20.984,86	3.927,919	22.396,78	3.625,885	.330,608
56	89.557,52	63.262,27	27.441,01	89.557,52	46.355,77	13.981,61	66.338,91	22.130,67	4.219,424	23.218,62	3.826,454	.355,273
57	92.784,56	66.425,11	29.390,18	92.784,56	48.776,53	14.990,01	68.729,31	23.316,34	4.526,745	24.055,26	4.034,241	.381,292
58	96.068,72	69.675,88	31.437,82	96.068,72	51.274,26	16.051,02	71.162,02	24.542,47	4.850,432	24.906,71	4.249,367	.408,713
59	99.409,99	73.014,47	33.586,84	99.409,99	53.849,65	17.166,36	73.637,03	25.809,61	5.191,036	25.772,96	4.471,957	.437,582
60	102.808,4	76.440,68	35.840,15	102.808,4	56.503,37	18.337,73	76.154,36	27.118,35	5.549,132	26.654,02	4.702,135	.467,951
61	106.263,9	79.954,22	38.200,63	106.263,9	59.236,06	19.566,87	78.713,99	28.469,25	5.925,292	27.549,89	4.940,021	.499,871
62	109.776,5	83.554,75	40.671,24	109.776,5	62.048,32	20.855,53	81.315,93	29.862,87	6.320,090	28.460,57	5.185,739	.533,396
63	113.346,2	87.241,83	43.254,88	113.346,2	64.940,72	22.205,48	83.960,18	31.299,75	6.734,131	29.386,06	5.439,411	.568,573
64	116.973,1	91.014,95	45.954,48	116.973,1	67.913,81	23.618,50	86.646,73	32.780,43	7.167,996	30.326,36	5.701,157	.605,460
65	120.657,1	94.873,53	48.772,98	120.657,1	70.968,07	25.096,40	89.375,59	34.305,47	7.622,303	31.281,46	5.971,100	.644,108
66	124.398,1	98.816,91	51.713,31	124.398,1	74.104,00	26.640,97	92.146,77	35.875,37	8.097,663	32.251,37	6.249,361	.684,573
67	128.196,3	102.844,3	54.778,39	128.196,3	77.322,01	28.254,07	94.960,25	37.490,67	8.594,698	33.236,09	6.536,060	.726,912
68	132.051,7	106.955,0	57.971,16	132.051,7	80.622,52	29.937,52	97.816,04	39.151,87	9.114,033	34.235,61	6.831,318	.771,179
69	135.964,1	111.148,0	61.294,53	135.964,1	84.005,88	31.693,19	100.714,1	40.859,48	9.656,303	35.249,95	7.135,255	.817,433
70	139.933,6	115.422,3	64.751,42	139.933,6	87.472,44	33.522,94	103.654,5	42.614,01	10.222,16	36.279,09	7.447,989	.865,731
71	143.960,3	119.776,9	68.344,72	143.960,3	91.022,49	35.428,66	106.637,2	44.415,94	10.812,24	37.323,04	7.769,642	.916,135
72	148.044,1	124.210,6	72.077,35	148.044,1	94.656,29	37.412,23	109.662,3	46.265,75	11.427,22	38.381,80	8.100,332	.968,700
73	152.185,0	128.722,3	75.952,14	152.185,0	98.374,06	39.475,56	112.729,6	48.163,91	12.067,76	39.455,36	8.440,177	1.023,489
74	156.383,0	133.310,6	79.971,99	156.383,0	102.176,0	41.620,56	115.839,2	50.110,90	12.734,51	40.543,73	8.789,295	1.080,564
75	160.638,1	137.974,1	84.139,72	160.638,1	106.062,3	43.849,14	118.991,2	52.107,17	13.428,19	41.646,91	9.147,805	1.139,985
76	164.950,3	142.711,5	88.458,21	164.950,3	110.033,0	46.163,29	122.185,4	54.153,18	14.149,45	42.764,90	9.515,825	1.201,817
77	169.319,7	147.521,1	92.930,20	169.319,7	114.088,2	48.564,87	125.422,0	56.249,36	14.899,01	43.897,70	9.893,471	1.266,122
78	173.746,2	152.401,4	97.558,51	173.746,2	118.228,0	51.055,88	128.700,9	58.396,15	15.677,55	45.045,30	10.280,86	1.332,966
79	178.229,7	157.350,7	102.345,9	178.229,7	122.452,3	53.638,24	132.022,0	60.593,97	16.485,78	46.207,71	10.678,11	1.402,412
80	182.770,5	162.367,2	107.295,1	182.770,5	126.761,2	56.313,96	135.385,5	62.843,24	17.324,42	47.384,93	11.085,33	1.474,530
81	187.368,3	167.449,1	112.408,8	187.368,3	131.154,5	59.084,95	138.791,3	65.144,37	18.194,19	48.576,96	11.502,65	1.549,384
82	192.023,2	172.594,4	117.689,7	192.023,2	135.632,2	61.953,22	142.239,4	67.497,75	19.095,80	49.783,79	11.930,17	1.627,040
83	196.735,3	177.801,2	123.140,4	196.735,3	140.194,2	64.920,72	145.729,8	69.903,78	20.030,01	51.005,44	12.368,02	1.707,570
84	201.504,4	183.067,4	128.763,7	201.504,4	144.840,1	67.989,47	149.262,5	72.362,84	20.997,54	52.241,89	12.816,30	1.791,041
85	206.330,7	188.390,9	134.561,7	206.330,7	149.569,9	71.161,39	152.837,6	74.875,31	21.999,12	53.493,15	13.275,13	1.877,524
86	211.214,1	193.769,3	140.537,4	211.214,1	154.383,3	74.438,53	156.454,9	77.441,54	23.035,55	54.759,21	13.744,63	1.967,089
87	216.154,6	199.200,5	146.693,1	216.154,6	159.279,9	77.822,85	160.114,5	80.061,88	24.107,50	56.040,09	14.224,90	2.059,808
88	221.152,3	204.682,2	153.031,3	221.152,3	164.259,5	81.316,34	163.816,5	82.736,69	25.215,80	57.335,77	14.716,06	2.155,752
89	226.207,0	210.211,7	159.554,4	226.207,0	169.321,5	84.920,97	167.560,7	85.466,31	26.361,20	58.646,26	15.218,22	2.254,997
90	231.318,9	215.786,7	166.264,8	231.318,9	174.465,8	88.638,76	171.347,3	88.251,05	27.544,48	59.971,56	15.731,50	2.357,615

TABLE 21—Continued

UNIT TWIST LOAD NO. 5
TWIST, RADIAL AND TANGENTIAL DEFLECTIONS



φ	Crown			¼ Point			½ Point			¾ Point		
°	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
10	5.076,958	.662.325,4	.046,311,8	4.283,682	.396,465,5	.020,164,9	2.538,478	.152,178,3	.005,072,3	.793,274,5	.023,359,36	$.0^3$385,1
11	6.143,118	.880,927,9	.067,779,3	5.183,256	.527,494,5	.029,517,3	3.071,559	.202,516,0	.007,425,8	.959,862,2	.031,090,05	$.0^3$563,8
12	7.310,818	1.142,793	.095,955,6	6.168,503	.684,545,4	.041,795,8	3.655,409	.262,873,3	.010,516,0	1.142,315	.040,361,55	$.0^3$798,4
13	8.580,058	1.451,729	.132,106,0	7.239,423	.869,944,2	.057,553,9	4.290,029	.334,154,7	.014,482,9	1.340,634	.051,313,69	.001,099,7
14	9.950,836	1.811,515	.177,602,2	8.396,018	1.086,007	.077,392,2	4.975,418	.417,262,8	.019,477,9	1.554,818	.064,086,22	.001,479,2
15	11.423,15	2.225,894	.233,922	9.638,285	1.335,037	.101,958,9	5.711,577	.513,098,1	.025,664,9	1.784,868	.078,818,85	.001,949,2
16	12.997,01	2.698,570	.302,652	10.966,23	1.619,329	.131,949	6.498,505	.622,559,6	.033,219,7	2.030,783	.095,651,18	.002,523,3
17	14.672,41	3.233,211	.385,478	12.379,84	1.941,160	.168,105	7.336,203	.746,543,7	.042,330,6	2.292,563	.114,722,7	.003,215,6
18	16.449,34	3.833,439	.484,192	13.879,13	2.302,798	.211,215	8.224,671	.885,944,7	.053,196,2	2.570,210	.136,173,0	.004,041,5
19	18.327,82	4.502,834	.600,690	15.464,09	2.706,494	.262,114	9.163,907	1.041,655	.066,028,9	2.863,721	.160,141,3	.005,017,0
20	20.307,83	5.244,926	.736,967	17.134,73	3.154,484	.321,686	10.153,91	1.214,563	.081,052	3.173,098	.186,766,8	.006,159,2
21	22.389,38	6.063,199	.895,122	18.891,04	3.648,988	.390,850	11.194,69	1.405,555	.098,502	3.498,341	.216,188,9	.007,486,4
22	24.572,47	6.961,080	1.077,350	20.733,02	4.192,209	.470,585	12.286,24	1.615,517	.118,626	3.839,449	.248,546,4	.009,017,2
23	26.857,10	7.941,946	1.285,946	22.660,68	4.786,332	.561,908	13.428,55	1.845,327	.141,682	4.196,422	.283,978,4	.010,771,2
24	29.243,27	9.009,113	1.523,299	24.674,01	5.433,524	.665,882	14.621,64	2.095,865	.167,943	4.569,261	.322,623,6	.012,769,6
25	31.730,98	10.165,84	1.791,900	26.773,02	6.135,931	.783,613	15.865,49	2.368,003	.197,691	4.957,966	.364,620,8	.015,033,8
26	34.320,23	11.415,32	2.094,331	28.957,69	6.895,681	.916,254	17.160,12	2.662,614	.231,219	5.362,536	.410,108,5	.017,586,4
27	37.011,02	12.760,69	2.433,262	31.228,05	7.714,881	1.065,000	18.505,51	2.980,565	.268,835	5.782,971	.459,225,1	.020,451,0
28	39.803,34	14.205,02	2.811,463	33.584,07	8.595,612	1.231,091	19.901,67	3.322,720	.310,856	6.219,272	.512,109,0	.023,652,0
29	42.697,21	15.751,30	3.231,784	36.025,77	9.539,938	1.415,810	21.348,60	3.689,937	.357,612	6.671,439	.568,898,2	.027,214,6
30	45.692,61	17.402,45	3.697,172	38.553,14	10.549,90	1.620,483	22.846,31	4.083,075	.409,445	7.139,471	.629,730,7	.031,165,0
31	48.789,56	19.161,33	4.210,661	41.166,19	11.627,50	1.846,478	24.394,78	4.502,983	.466,704	7.623,368	.694,744,5	.035,530,4
32	51.988,04	21.030,72	4.775,360	43.864,91	12.774,74	2.095,202	25.994,02	4.950,511	.529,758	8.123,131	.764,077,2	.040,338,9
33	55.288,06	23.013,32	5.394,469	46.649,30	13.993,58	2.368,115	27.644,03	5.426,500	.598,978	8.638,759	.837,866,2	.045,619,1
34	58.689,62	25.111,75	6.071,275	49.519,37	15.285,95	2.666,700	29.344,81	5.931,791	.674,753	9.170,253	.916,247,5	.051,402,3
35	62.192,72	27.328,55	6.809,127	52.475,11	16.653,77	2.992,499	31.096,36	6.467,217	.757,481	9.717,613	.999,362,5	.057,716,9
36	65.797,36	29.666,17	7.611,469	55.516,53	18.098,97	3.347,081	32.898,68	7.033,608	.847,572	10.280,84	1.087,344	.064,597
37	69.503,54	32.126,99	8.481,813	58.643,61	19.623,24	3.732,067	34.751,77	7.631,788	.945,447	10.859,93	1.180,330	.072,073
38	73.311,26	34.713,29	9.423,749	61.856,37	21.228,57	4.149,102	36.655,63	8.262,577	1.051,534	11.454,88	1.278,457	.080,179
39	77.220,52	37.427,26	10.440,93	65.154,81	22.916,69	4.599,885	38.610,26	8.926,789	1.166,281	12.065,71	1.381,862	.088,951
40	81.231,31	40.271,01	11.537,10	68.538,92	24.689,38	5.086,142	40.615,66	9.625,233	1.290,139	12.692,39	1.490,681	.098,423
41	85.343,65	43.246,54	12.716,03	72.008,70	26.548,35	5.609,640	42.671,82	10.358,71	1.423,573	13.334,95	1.605,050	.108,632
42	89.557,52	46.355,77	13.981,61	75.564,16	28.495,32	6.172,201	44.778,76	11.128,03	1.567,062	13.993,36	1.725,104	.119,613
43	93.872,94	49.600,52	15.337,75	79.205,29	30.531,95	6.775,646	46.936,47	11.933,96	1.721,088	14.667,65	1.850,979	.131,406
44	98.289,89	52.982,51	16.788,44	82.932,09	32.659,86	7.421,864	49.144,94	12.777,32	1.886,152	15.357,80	1.982,811	.144,050
45	102.808,4	56.503,37	18.337,73	86.744,57	34.880,66	8.112,773	51.404,19	13.658,86	2.062,762	16.063,81	2.120,734	.157,583
46	107.428,4	60.164,61	19.989,72	90.642,72	37.195,91	8.850,317	53.714,20	14.579,37	2.251,435	16.785,69	2.264,884	.172,048
47	112.150,0	63.967,65	21.748,58	94.626,54	39.607,15	9.636,480	56.074,99	15.539,62	2.452,703	17.523,43	2.415,395	.187,485
48	116.973,1	67.913,81	23.618,50	98.696,04	42.115,85	10.473,28	58.486,54	16.540,36	2.667,106	18.277,05	2.572,400	.203,937
49	121.897,7	72.004,29	25.603,76	102.851,2	44.723,48	11.362,77	60.948,87	17.582,36	2.895,195	19.046,52	2.736,036	.221,448
50	126.923,9	76.240,19	27.708,64	107.092,1	47.431,46	12.307,04	63.461,96	18.666,36	3.137,527	19.831,86	2.906,435	.240,060
51	132.051,7	80.622,52	29.937,52	111.418,6	50.241,16	13.308,20	66.025,83	19.793,11	3.394,679	20.633,07	3.083,731	.259,821
52	137.280,9	85.152,14	32.294,79	115.830,8	53.153,92	14.368,40	68.640,46	20.963,34	3.667,229	21.450,14	3.268,057	.280,775
53	142.611,7	89.829,85	34.784,86	120.328,6	56.171,06	15.489,80	71.305,86	22.177,77	3.955,766	22.283,08	3.459,546	.302,971
54	148.044,1	94.656,29	37.412,23	124.912,2	59.293,82	16.674,64	74.022,03	23.437,13	4.260,902	23.131,88	3.658,332	.326,454
55	153.578,0	99.632,02	40.181,39	129.581,4	62.523,43	17.925,13	76.788,97	24.742,14	4.583,239	23.996,55	3.864,546	.351,275
56	159.213,4	104.757,5	43.096,88	134.336,3	65.861,08	19.243,55	79.606,69	26.093,49	4.923,407	24.877,09	4.078,322	.377,483
57	164.950,3	110.033,0	46.163,29	139.176,8	69.307,89	20.632,19	82.475,17	27.491,90	5.282,028	25.773,49	4.299,790	.405,128
58	170.788,9	115.458,7	49.385,21	144.103,1	72.864,98	22.093,38	85.394,41	28.938,05	5.659,747	26.685,76	4.529,082	.434,262
59	176.728,9	121.034,8	52.767,19	149.115,0	76.533,39	23.629,46	88.364,43	30.432,63	6.057,224	27.613,89	4.766,330	.464,938
60	182.770,5	126.761,2	56.313,96	154.212,6	80.314,13	25.242,80	91.385,23	31.976,30	6.475,111	28.557,88	5.011,665	.497,205
61	188.913,6	132.637,7	60.030,13	159.395,8	84.208,18	26.935,81	94.456,79	33.569,75	6.914,081	29.517,75	5.265,217	.531,121
62	195.158,2	138.664,2	63.920,41	164.664,8	88.216,46	28.710,90	97.579,11	35.213,63	7.374,806	30.493,47	5.527,117	.566,741
63	201.504,4	144.840,1	67.989,47	170.019,4	92.339,84	30.570,52	100.752,21	36.908,60	7.857,990	31.485,07	5.797,495	.604,118
64	207.952,2	151.165,1	72.241,99	175.459,6	96.579,18	32.517,16	103.976,1	38.655,29	8.364,315	32.492,53	6.076,481	.643,311
65	214.501,4	157.638,5	76.682,71	180.985,6	100.935,2	34.553,29	107.250,7	40.454,35	8.894,500	33.515,85	6.364,203	.684,375
66	221.152,3	164.259,5	81.316,34	186.597,2	105.408,8	36.681,42	110.576,1	42.306,39	9.449,26	34.555,04	6.660,792	.727,371
67	227.904,6	171.027,2	86.147,56	192.294,5	110.000,5	38.904,10	113.952,3	44.212,05	10.029,32	35.610,09	6.966,375	.772,356
68	234.758,5	177.940,7	91.181,12	198.077,5	114.711,0	41.223,90	117.379,3	46.171,93	10.635,42	36.681,01	7.281,082	.819,392
69	241.713,9	184.998,9	96.421,72	203.946,1	119.541,0	43.643,35	120.857,0	48.186,63	11.268,28	37.767,80	7.605,039	.868,537
70	248.770,9	192.200,6	101.874,1	209.900,4	124.491,0	46.165,08	124.385,5	50.256,74	11.928,68	38.870,45	7.938,375	.919,857
71	255.929,4	199.544,3	107.542,9	215.940,4	129.561,4	48.791,67	127.964,7	52.382,85	12.617,35	39.988,97	8.281,218	.973,411
72	263.189,5	207.028,6	113.432,8	222.066,1	134.752,8	51.525,77	131.594,7	54.565,53	13.335,10	41.123,35	8.633,693	1.029,264
73	270.551,0	214.651,9	119.548,4	228.277,4	140.065,6	54.370,01	135.275,5	56.805,34	14.082,67	42.273,60	8.995,928	1.087,479
74	278.014,2	222.412,6	125.894,6	234.574,5	145.500,1	57.327,04	139.007,1	59.102,85	14.860,87	43.439,71	9.368,048	1.148,122
75	285.578,8	230.308,6	132.475,8	240.957,1	151.056,7	60.399,52	142.789,4	61.458,60	15.670,48	44.621,69	9.750,180	1.211,260
76	293.245,0	238.338,2	139.296,7	247.425,5	156.735,5	63.590,19	146.622,5	63.873,12	16.512,31	45.819,54	10.142,45	1.276,958
77	301.012,8	246.499,1	146.361,8	253.979,5	162.536,9	66.901,68	150.506,4	66.346,94	17.387,16	47.033,25	10.544,98	1.345,285
78	308.882,1	254.789,2	153.675,8	260.619,2	168.461,0	70.336,73	154.441,0	68.880,59	18.295,86	48.262,82	10.957,90	1.416,308
79	316.852,9	263.206,2	161.243,1	267.344,6	174.507,8	73.898,06	158.426,4	71.474,56	19.239,22	49.508,26	11.381,32	1.490,098
80	324.925,3	271.747,6	169.068,2	274.155,7	180.677,5	77.588,39	162.462,6	74.129,35	20.218,10	50.769,57	11.815,39	1.566,725
81	333.099,2	280.410,8	177.155,5	281.052,4	186.970,2	81.410,46	166.549,6	76.845,46	21.233,29	52.046,74	12.260,20	1.646,260
82	341.374,6	289.193,2	185.509,6	288.034,8	193.385,6	85.367,00	170.687,3	79.623,36	22.285,69	53.339,78	12.715,91	1.728,772
83	349.751,6	298.091,9	194.134,6	295.102,9	199.923,8	89.460,78	174.875,8	82.463,51	23.376,15	54.648,68	13.182,61	1.814,340
84	358.230,1	307.104,0	203.035,0	302.256,6	206.584,6	93.694,58	179.115,0	85.366,38	24.505,49	55.973,45	13.660,44	1.903,031
85	366.810,2	316.226,5	212.215,1	309.496,1	213.367,8	98.071,12	183.405,1	88.332,41	25.674,63	57.314,09	14.149,51	1.994,923
86	375.491,8	325.456,3	221.679,0	316.821,2	220.273,2	102.593,2	187.745,9	91.362,04	26.884,41	58.670,58	14.649,96	2.090,089
87	384.274,9	334.790,0	231.430,8	324.231,9	227.300,4	107.263,6	192.137,4	94.455,69	28.135,74	60.042,95	15.161,89	2.188,607
88	393.159,6	344.224,2	241.474,8	331.728,4	234.449,2	112.085,1	196.579,8	97.613,79	29.429,48	61.431,18	15.685,43	2.290,553
89	402.145,8	353.755,5	251.814,9	339.310,5	241.719,1	117.060,4	201.072,9	100.836,7	30.766,56	62.835,28	16.220,71	2.396,006
90	411.233,5	363.380,2	262.455,1	346.978,3	249.109,7	122.192,4	205.616,8	104.124,9	32.147,85	64.255,24	16.767,82	2.505,041

TRIGONOMETRIC TABLE FOR ARCH ANALYSIS

206. General Statement.—A convenient table of trigonometric functions is given in table 22. These are used in calculating arch data as described in chapter VI. The trigonometric quantities in the table occur at various places in arch computation forms. Some are necessary in evaluating moments, thrusts, and shears due to applied loads, using formulas for M_L, H_L, and V_L as given in chapter V.

Although all quantities tabulated, except $sin^2\ \phi$ and $cos^2\ \phi$, are given in preceding tables, the values are repeated in table 22 for convenience in making arch computations. After arch and load constants are copied from tables 10 and 17 to 21, inclusive, all remaining trigonometric functions needed may be taken from table 22.

TABLE 22—TRIGONOMETRIC TABLE FOR ARCH ANALYSIS

ϕ Degrees	ϕ Radians	SIN ϕ	COS ϕ	VERS ϕ	ϕ-SIN ϕ	$\frac{\phi^2}{2}$-VERS ϕ	$\frac{\phi^2}{2}$	SIN^2 ϕ	COS^2 ϕ
0 15	.004,363,323	.004,363,309	.999,990,5	.^{5}9,519,279	.^{7}13,845,24	.^{10}15,102,82	.^{5}9,519,294	.^{4}19,038,47	.999,981,0
0 30	.008,726,646	.008,726,535	.999,961,9	.^{4}38,076,94	.^{6}110,761,6	.^{9}241,644,6	.^{4}38,077,18	.^{4}76,152,42	.999,923,8
0 45	.013,089,97	.013,089,60	.999,914,3	.^{4}85,672,43	.^{6}373,818,6	.^{8}1,223,322	.^{4}85,673,65	.^{3}171,337,5	.999,828,7
1 00	.017,453,29	.017,452,41	.999,847,7	.^{3}152,304,8	.^{6}886,082,6	.^{8}3,866,284	.^{3}152,308,7	.^{3}304,586,5	.999,695,4
1 15	.021,816,62	.021,814,89	.999,762,0	.^{3}237,972,9	.^{5}1,730,615	.^{8}9,439,117	.^{3}237,982,4	.^{3}475,889,2	.999,524,1
1 30	.026,179,94	.026,176,95	.999,657,3	.^{3}342,675,0	.^{5}2,990,472	.^{7}19,572,82	.^{3}342,694,6	.^{3}685,232,6	.999,314,8
1 45	.030,543,26	.030,538,51	.999,533,6	.^{3}466,409,2	.^{5}4,748,700	.^{7}36,260,76	.^{3}466,445,4	.^{3}932,600,8	.999,067,4
2 00	.034,906,59	.034,899,50	.999,390,8	.^{3}609,173,0	.^{5}7,088,337	.^{7}61,858,67	.^{3}609,234,8	.001,217,975	.998,782,0
2 15	.039,269,91	.039,259,82	.999,229,0	.^{3}770,963,8	.^{4}10,092,41	.^{7}99,084,56	.^{3}771,062,8	.001,541,333	.998,458,7
2 30	.043,633,23	.043,619,39	.999,048,2	.^{3}951,778,4	.^{4}13,843,93	.^{6}151,018,7	.^{3}951,929,4	.001,902,651	.998,097,3
2 45	.047,996,55	.047,978,13	.998,848,4	.001,151,614	.^{4}18,425,91	.^{6}221,103,5	.001,151,835	.002,301,901	.997,698,1
3 00	.052,359,88	.052,335,96	.998,629,5	.001,370,465	.^{4}23,921,32	.^{6}313,143,6	.001,370,778	.002,739,052	.997,260,9
3 15	.056,723,20	.056,692,79	.998,391,7	.001,608,329	.^{4}30,413,13	.^{6}431,305,6	.001,608,761	.003,214,072	.996,785,9
3 30	.061,086,52	.061,048,54	.998,134,8	.001,865,202	.^{4}37,984,28	.^{6}580,118,1	.001,865,782	.003,726,924	.996,273,1
3 45	.065,449,85	.065,403,13	.997,858,9	.002,141,077	.^{4}46,717,72	.^{6}764,471,5	.002,141,841	.004,277,569	.995,722,4
4 00	.069,813,17	.069,756,47	.997,564,1	.002,435,950	.^{4}56,696,34	.^{6}989,618,1	.002,436,939	.004,865,966	.995,134,0
4 15	.074,176,49	.074,108,49	.997,250,2	.002,749,815	.^{4}68,003,01	.^{5}1,261,172	.002,751,076	.005,492,068	.994,507,9
4 30	.078,539,82	.078,459,10	.996,917,3	.003,082,666	.^{4}80,720,61	.^{5}1,585,108	.003,084,251	.006,155,830	.993,844,2
4 45	.082,903,14	.082,808,21	.996,565,5	.003,434,498	.^{4}94,931,96	.^{5}1,967,765	.003,436,465	.006,857,199	.993,142,8
5 00	.087,266,46	.087,155,74	.996,194,7	.003,805,302	.^{3}110,719,9	.^{5}2,415,839	.003,807,718	.007,596,124	.992,403,9
5 15	.091,629,79	.091,501,62	.995,804,9	.004,195,072	.^{3}128,167,1	.^{5}2,936,391	.004,198,009	.008,372,546	.991,627,5
5 30	.095,993,11	.095,845,75	.995,396,2	.004,603,802	.^{3}147,356,3	.^{5}3,536,841	.004,607,338	.009,186,408	.990,813,6
5 45	.100,356,4	.100,188,1	.994,968,5	.005,031,482	.^{3}168,370,4	.^{5}4,224,972	005.035,707	.010,037,65	.989,962,4
6 00	.104,719,8	.104,528,5	.994,521,9	.005,478,105	.^{3}191,291,9	.^{5}5,008,924	.005,483,114	.010,926,20	.989,073,8
6 15	.109,083,1	.108,866,9	.994,056,3	.005,943,662	.^{3}216,203,4	.^{5}5,897,203	.005,949,559	.011,852,00	.988,148,0
6 30	.113,446,4	.113,203,2	.993,571,9	.006,428,144	.^{3}243,187,6	.^{5}6,898,670	.006,435,043	.012,814,97	.987,185,0
6 45	.117,809,7	.117,537,4	.993,068,5	.006,931,543	.^{3}272,327,1	.^{5}8,022,549	.006,939,566	.013,815,04	.986,185,0
7 00	.122,173,0	.121,869,3	.992,546,2	.007,453,848	.^{3}303,704,2	.^{5}9,278,426	.007,463,127	.014,852,14	.985,147,9
7 15	.126,536,4	.126,199,0	.992,004,9	.007,995,050	.^{3}337,401,6	.^{4}10,676,24	.008,005,727	.015,926,18	.984,073,8
7 30	.130,899,7	.130,526,2	.991,444,9	.008,555,139	.^{3}373,501,7	.^{4}12,226,31	.008,567,365	.017,037,09	.982,962,9
7 45	.135,263,0	.134,850,9	.990,865,9	.009,134,103	.^{3}412,086,8	.^{4}13,939,27	.009,148,042	.018,184,77	.981,815,2
8 00	.139,626,3	.139,173,1	.990,268,1	.009,731,931	.^{3}453,239,2	.^{4}15,826,17	.009,747,757	.019,369,15	.980,630,9
8 15	.143,989,7	.143,492,6	.989,651,4	.010,348,61	.^{3}497,041,3	.^{4}17,898,39	.010,366,51	.020,590,13	.979,409,9
8 30	.148,353,0	.147,809,4	.989,015,9	.010,984,14	.^{3}543,575,3	.^{4}20,167,65	.011,004,30	.021,847,62	.978,152,4
8 45	.152,716,3	.152,123,4	.988,361,5	.011,638,49	.^{3}592,923,4	.^{4}22,646,07	.011,661,14	.023,141,52	.976,858,5
9 00	.157,079,6	.156,434,5	.987,688,3	.012,311,66	.^{3}645,167,6	.^{4}25,346,10	.012,337,01	.024,471,74	.975,528,3
9 15	.161,443,0	.160,742,6	.986,996,4	.013,003,63	.^{3}700,390,2	.^{4}28,280,55	.013,031,91	.025,838,17	.974,161,8
9 30	.165,806,3	.165,047,6	.986,285,6	.013,714,40	.^{3}758,673,1	.^{4}31,462,61	.013,745,86	.027,240,71	.972,759,3
9 45	.170,169,6	.169,349,5	.985,556,1	.014,443,94	.^{3}820,098,2	.^{4}34,905,79	.014,478,85	.028,679,25	.971,320,7
10 00	.174,532,9	.173,648,2	.984,807,8	.015,192,25	.^{3}884,747,5	.^{4}38,624,00	.015,230,87	.030,153,69	.969,846,3
10 15	.178,896,2	.177,943,5	.984,040,7	.015,959,30	.^{3}952,702,9	.^{4}42,631,48	.016,001,93	.031,663,91	.968,336,1
10 30	.183,259,6	.182,235,5	.983,254,9	.016,745,09	.001,024,046	.^{4}46,942,83	.016,792,04	.033,209,79	.966,790,2
10 45	.187,622,9	.186,524,0	.982,450,4	.017,549,60	.001,098,859	.^{4}51,573,02	.017,601,18	.034,791,22	.965,208,8
11 00	.191,986,2	.190,809,0	.981,627,2	.018,372,82	.001,177,222	.^{4}56,537,34	.018,429,35	.036,408,07	.963,591,9
11 15	.196,349,5	.195,090,3	.980,785,3	.019,214,72	.001,259,219	.^{4}61,851,50	.019,276,57	.038,060,23	.961,939,8
11 30	.200,712,9	.199,367,9	.979,924,7	.020,075,30	.001,344,930	.^{4}67,531,50	.020,142,83	.039,747,57	.960,252,4
11 45	.205,076,2	.203,641,8	.979,045,5	.020,954,53	.001,434,436	.^{4}73,593,74	.021,028,12	.041,469,96	.958,530,0
12 00	.209,439,5	.207,911,7	.978,147,6	.021,852,40	.001,527,819	.^{4}80,054,96	.021,932,45	.043,227,27	.956,772,7
12 15	.213,802,8	.212,177,7	.977,231,1	.022,768,89	.001,625,161	.^{4}86,932,24	.022,855,83	.045,019,36	.954,980,6
12 30	.218,166,2	.216,439,6	.976,296,0	.023,703,99	.001,726,543	.^{4}94,243,04	.023,798,24	.046,846,11	.953,153,9
12 45	.222,529,5	.220,697,4	.975,342,3	.024,657,68	.001,832,045	.^{3}102,005,2	.024,759,68	.048,707,36	.951,292,6
13 00	.226,892,8	.224,951,1	.974,370,1	.025,629,94	.001,941,748	.^{3}110,236,8	.025,740,17	.050,602,98	.949,397,0
13 15	.231,256,1	.229,200,4	.973,379,3	.026,620,74	.002,055,735	.^{3}118,956,3	.026,739,70	.052,532,82	.947,467,2
13 30	.235,619,4	.233,445,4	.972,369,9	.027,630,08	.002,174,085	.^{3}128,182,8	.027,758,26	.054,496,74	.945,503,3
13 45	.239,982,8	.237,685,9	.971,342,1	.028,657,93	.002,296,880	.^{3}137,935,3	.028,795,87	.056,494,58	.943,505,4
14 00	.244,346,1	.241,921,9	.970,295,7	.029,704,27	.002,424,200	.^{3}148,233,4	.029,852,51	.058,526,20	.941,473,8
14 15	.248,709,4	.246,153,3	.969,230,9	.030,769,09	.002,556,125	.^{3}159,097,1	.030,928,19	.060,591,44	.939,408,6
14 30	.253,072,7	.250,380,0	.968,147,6	.031,852,36	.002,692,737	.^{3}170,546,6	.032,022,91	.062,690,15	.937,309,9
14 45	.257,436,1	.254,601,9	.967,045,9	.032,954,06	.002,834,117	.^{3}182,602,6	.033,136,66	.064,822,15	.935,177,8
15 00	.261,799,4	.258,819,0	.965,925,8	.034,074,17	.002,980,343	.^{3}195,286,0	.034,269,46	.066,987,30	.933,012,7
15 15	.266,162,7	.263,031,2	.964,787,3	.035,212,68	.003,131,496	.^{3}208,618,2	.035,421,29	.069,185,42	.930,814,6
15 30	.270,526,0	.267,238,4	.963,630,5	.036,369,55	.003,287,658	.^{3}222,620,8	.036,592,17	.071,416,35	.928,583,7
15 45	.274,889,4	.271,440,4	.962,455,2	.037,544,76	.003,448,907	.^{3}237,315,8	.037,782,08	.073,679,92	.926,320,1
16 00	.279,252,7	.275,637,4	.961,261,7	.038,738,30	.003,615,325	.^{3}252,725,7	.038,991,03	.075,975,95	.924,024,1
16 15	.283,616,0	.279,829,0	.960,049,9	.039,950,15	.003,786,989	.^{3}268,873,1	.040,219,02	.078,304,28	.921,695,7
16 30	.287,979,3	.284,015,3	.958,819,7	.041,180,27	.003,963,982	.^{3}285,781,1	.041,466,05	.080,664,72	.919,335,3
16 45	.292,342,6	.288,196,3	.957,571,4	.042,428,64	.004,146,382	.^{3}303,473,2	.042,732,11	.083,057,09	.916,942,9
17 00	.296,706,0	.292,371,7	.956,304,8	.043,695,24	.004,334,266	.^{3}321,973,1	.044,017,22	.085,481,21	.914,518,8
17 15	.301,069,3	.296,541,6	.955,019,9	.044,980,06	.004,527,721	.^{3}341,304,9	.045,321,36	.087,936,91	.912,063,1
17 30	.305,432,6	.300,705,8	.953,717,0	.046,283,05	.004,726,820	.^{3}361,493,2	.046,644,54	.090,423,98	.909,576,0
17 45	.309,795,9	.304,864,3	.952,395,8	.047,604,20	.004,931,643	.^{3}382,562,6	.047,986,76	.092,942,24	.907,057,8
18 00	.314,159,3	.309,017,0	.951,056,5	.048,943,48	.005,142,271	.^{3}404,538,3	.049,348,02	.095,491,50	.904,508,5
18 15	.318,522,6	.313,163,8	.949,699,1	.050,300,87	.005,358,782	.^{3}427,445,9	.050,728,32	.098,071,57	.901,928,4
18 30	.322,885,9	.317,304,7	.948,323,7	.051,676,34	.005,581,255	.^{3}451,311,2	.052,127,66	.100,682,2	.899,317,8
18 45	.327,249,2	.321,439,5	.946,930,1	.053,069,87	.005,809,769	.^{3}476,160,3	.053,546,03	.103,323,3	.896,676,7
19 00	.331,612,6	.325,568,2	.945,518,6	.054,481,42	.006,044,403	.^{3}502,019,9	.054,983,44	.105,994,6	.894,005,4
19 15	.335,975,9	.329,690,6	.944,089,0	.055,910,98	.006,285,236	.^{3}528,916,7	.056,439,90	.108,695,9	.891,304,1
19 30	.340,339,2	.333,806,9	.942,641,5	.057,358,51	.006,532,345	.^{3}556,878,0	.057,915,39	.111,427,0	.888,573,0
19 45	.344,702,5	.337,916,7	.941,176,0	.058,823,98	.006,785,809	.^{3}585,931,4	.059,409,92	.114,187,7	.885,812,3
20 00	.349,065,9	.342,020,1	.939,692,6	.060,307,38	.007,045,707	.^{3}616,104,7	.060,923,48	.116,977,8	.883,022,2
20 15	.353,429,2	.346,117,1	.938,191,3	.061,808,66	.007,312,116	.^{3}647,426,3	.062,456,09	.119,797,0	.880,203,0
20 30	.357,792,5	.350,207,4	.936,672,2	.063,327,81	.007,585,115	.^{3}679,924,6	.064,007,74	.122,645,2	.877,354,8
20 45	.362,155,8	.354,291,0	.935,135,2	.064,864,79	.007,864,782	.^{3}713,628,6	.065,578,42	.125,522,1	.874,477,9
21 00	.366,519,1	.358,367,9	.933,580,4	.066,419,57	.008,151,193	.^{3}748,567,6	.067,168,14	.128,427,6	.871,572,4
21 15	.370,882,5	.362,438,0	.932,007,9	.067,992,13	.008,444,428	.^{3}784,771,1	.068,776,90	.131,361,3	.868,638,7
21 30	.375,245,8	.366,501,2	.930,417,6	.069,582,43	.008,744,562	.^{3}822,269,1	.070,404,70	.134,323,1	.865,676,9
21 45	.379,609,1	.370,557,4	.928,809,6	.071,190,45	.009,051,675	.^{3}861,091,9	.072,051,54	.137,312,8	.862,687,2
22 00	.383,972,4	.374,606,6	.927,183,9	.072,816,15	.009,365,842	.^{3}901,270,2	.073,717,42	.140,330,1	.859,669,9
22 15	.388,335,8	.378,648,6	.925,540,5	.074,459,50	.009,687,141	.^{3}942,834,7	.075,402,33	.143,374,8	.856,625,2
22 30	.392,699,1	.382,683,4	.923,879,5	.076,120,47	.010,015,65	.^{3}985,816,9	.077,106,28	.146,446,6	.853,553,4

.^{4}10,092,41 = .000,010,092,41

TABLE 22—Continued

ϕ Degrees	ϕ Radians	SIN ϕ	COS ϕ	VERS ϕ	ϕ-SIN ϕ	$\frac{\phi^2}{2}$-VERSϕ	$\frac{\phi^2}{2}$	SIN^2 ϕ	COS^2 ϕ
22 45	.397,062,4	.386,711,0	.922,201,0	.077,799,03	.010,351,44	.001,030,248	.078,829,28	.149,545,4	.850,454,6
23 00	.401,425,7	.390,731,1	.920,504,9	.079,495,15	.010,694,60	.001,076,161	.080,571,31	.152,670,8	.847,329,2
23 15	.405,789,1	.394,743,9	.918,791,2	.081,208,79	.011,045,19	.001,123,587	.082,332,38	.155,822,7	.844,177,3
23 30	.410,152,4	.398,749,1	.917,060,1	.082,939,93	.011,403,31	.001,172,559	.084,112,49	.159,000,8	.840,999,2
23 45	.414,515,7	.402,746,7	.915,311,5	.084,688,52	.011,769,01	.001,223,111	.085,911,63	.162,204,9	.837,795,1
24 00	.418,879,0	.406,736,6	.913,545,5	.086,454,54	.012,142,38	.001,275,275	.087,729,82	.165,434,7	.834,565,3
24 15	.423,242,3	.410,718,9	.911,762,0	.088,237,96	.012,523,49	.001,329,084	.089,567,04	.168,690,0	.831,310,0
24 30	.427,605,7	.414,693,2	.909,961,3	.090,038,73	.012,912,42	.001,384,574	.091,423,30	.171,970,5	.828,029,5
24 45	.431,969,0	.418,659,7	.908,143,2	.091,856,83	.013,309,25	.001,441,778	.093,298,60	.175,276,0	.824,724,0
25 00	.436,332,3	.422,618,3	.906,307,8	.093,692,21	.013,714,05	.001,500,731	.095,192,94	.178,606,2	.821,393,8
25 15	.440,695,6	.426,568,7	.904,455,1	.095,544,85	.014,126,90	.001,561,467	.097,106,32	.181,960,9	.818,039,1
25 30	.445,059,0	.430,511,1	.902,585,3	.097,414,72	.014,547,86	.001,624,023	.099,038,74	.185,339,8	.814,660,2
25 45	.449,422,3	.434,445,3	.900,698,2	.099,301,76	.014,977,02	.001,688,433	.100,990,2	.188,742,7	.811,257,3
26 00	.453,785,6	.438,371,1	.898,794,0	.101,206,0	.015,414,46	.001,754,734	.102,960,7	.192,169,3	.807,830,7
26 15	.458,148,9	.442,288,7	.896,872,7	.103,127,3	.015,860,24	.001,822,962	.104,950,2	.195,619,3	.804,380,7
26 30	.462,512,3	.446,197,8	.894,934,4	.105,065,6	.016,314,44	.001,893,153	.106,958,8	.199,092,5	.800,907,5
26 45	.466,875,6	.450,098,4	.892,978,9	.107,021,1	.016,777,13	.001,965,345	.108,986,4	.202,588,6	.797,411,4
27 00	.471,238,9	.453,990,5	.891,006,5	.108,993,5	.017,248,40	.002,039,574	.111,033,0	.206,107,4	.793,892,6
27 15	.475,602,2	.457,873,9	.889,017,1	.110,982,9	.017,728,31	.002,115,878	.113,098,7	.209,648,5	.790,351,5
27 30	.479,965,5	.461,748,6	.887,010,8	.112,989,2	.018,216,93	.002,194,295	.115,183,5	.213,211,8	.786,788,2
27 45	.484,328,9	.465,614,5	.884,987,6	.115,012,4	.018,714,35	.002,274,863	.117,287,2	.216,796,9	.783,203,1
28 00	.488,692,2	.469,471,6	.882,947,6	.117,052,4	.019,220,63	.002,357,621	.119,410,0	.220,403,5	.779,596,5
28 15	.493,055,5	.473,319,7	.880,890,7	.119,109,3	.019,735,85	.002,442,608	.121,551,9	.224,031,5	.775,968,5
28 30	.497,418,8	.477,158,8	.878,817,1	.121,182,9	.020,260,08	.002,529,862	.123,712,7	.227,680,5	.772,319,5
28 45	.501,782,2	.480,988,8	.876,726,8	.123,273,2	.020,793,39	.002,619,424	.125,892,7	.231,350,2	.768,649,8
29 00	.506,145,5	.484,809,6	.874,619,7	.125,380,3	.021,335,86	.002,711,332	.128,091,6	.235,040,4	.764,959,6
29 15	.510,508,8	.488,621,2	.872,496,0	.127,504,0	.021,887,56	.002,805,628	.130,309,6	.238,750,7	.761,249,3
29 30	.514,872,1	.492,423,6	.870,355,7	.129,644,3	.022,448,57	.002,902,351	.132,546,7	.242,481,0	.757,519,0
29 45	.519,235,5	.496,216,5	.868,198,8	.131,801,2	.023,018,95	.003,001,542	.134,802,7	.246,230,8	.753,769,2
30 00	.523,598,8	.500,000,0	.866,025,4	.133,974,6	.023,598,78	.003,103,243	.137,077,8	.250,000,0	.750,000,0
30 15	.527,962,1	.503,774,0	.863,835,5	.136,164,5	.024,188,12	.003,207,494	.139,372,0	.253,788,2	.746,211,8
30 30	.532,325,4	.507,538,4	.861,629,2	.138,370,8	.024,787,06	.003,314,338	.141,685,2	.257,595,2	.742,404,8
30 45	.536,688,7	.511,293,1	.859,406,4	.140,593,6	.025,395,66	.003,423,816	.144,017,4	.261,420,6	.738,579,4
31 00	.541,052,1	.515,038,1	.857,167,3	.142,832,7	.026,013,99	.003,535,971	.146,368,7	.265,264,2	.734,735,8
31 15	.545,415,4	.518,773,3	.854,911,9	.145,088,1	.026,642,13	.003,650,845	.148,739,0	.269,125,7	.730,874,3
31 30	.549,778,7	.522,498,6	.852,640,2	.147,359,8	.027,280,15	.003,768,482	.151,128,3	.273,004,8	.726,995,2
31 45	.554,142,0	.526,213,9	.850,352,2	.149,647,8	.027,928,11	.003,888,924	.153,536,7	.276,901,1	.723,098,9
32 00	.558,505,4	.529,919,3	.848,048,1	.151,951,9	.028,586,10	.004,012,215	.155,964,1	.280,814,4	.719,185,6
32 15	.562,868,7	.533,614,5	.845,727,8	.154,272,2	.029,254,17	.004,138,399	.158,410,6	.284,744,5	.715,255,5
32 30	.567,232,0	.537,299,6	.843,391,4	.156,608,6	.029,932,40	.004,267,521	.160,876,1	.288,690,9	.711,309,1
32 45	.571,595,3	.540,974,5	.841,039,0	.158,961,0	.030,620,86	.004,399,624	.163,360,6	.292,653,4	.707,346,6
33 00	.575,958,7	.544,639,0	.838,670,6	.161,329,4	.031,319,62	.004,534,753	.165,864,2	.296,631,7	.703,368,3
33 15	.580,322,0	.548,293,2	.836,286,2	.163,713,8	.032,028,75	.004,672,954	.168,386,8	.300,625,5	.699,374,5
33 30	.584,685,3	.551,937,0	.833,885,8	.166,114,2	.032,748,31	.004,814,272	.170,928,4	.304,634,4	.695,365,6
33 45	.589,048,6	.555,570,2	.831,469,6	.168,530,4	.033,478,39	.004,958,752	.173,489,1	.308,658,3	.691,341,7
34 00	.593,411,9	.559,192,9	.829,037,6	.170,962,4	.034,219,04	.005,106,441	.176,068,9	.312,696,7	.687,303,3
34 15	.597,775,3	.562,804,9	.826,589,7	.173,410,3	.034,970,34	.005,257,385	.178,667,6	.316,749,4	.683,250,6
34 30	.602,138,6	.566,406,2	.824,126,2	.175,873,8	.035,732,36	.005,411,631	.181,285,4	.320,816,0	.679,184,0
34 45	.606,501,9	.569,996,8	.821,646,9	.178,353,1	.036,505,15	.005,569,224	.183,922,3	.324,896,3	.675,103,7
35 00	.610,865,2	.573,576,4	.819,152,0	.180,848,0	.037,288,80	.005,730,214	.186,578,2	.328,989,9	.671,010,1
35 15	.615,228,6	.577,145,2	.816,641,6	.183,358,4	.038,083,37	.005,894,646	.189,253,1	.333,096,6	.666,903,4
35 30	.619,591,9	.580,703,0	.814,115,5	.185,884,5	.038,888,93	.006,062,570	.191,947,1	.337,215,9	.662,784,1
35 45	.623,955,2	.584,249,7	.811,574,0	.188,426,0	.039,705,54	.006,234,033	.194,660,1	.341,347,7	.658,652,3
36 00	.628,318,5	.587,785,3	.809,017,0	.190,983,0	.040,533,28	.006,409,082	.197,392,1	.345,491,5	.654,508,5
36 15	.632,681,9	.591,309,6	.806,444,6	.193,555,4	.041,372,21	.006,587,768	.200,143,2	.349,647,1	.650,352,9
36 30	.637,045,2	.594,822,8	.803,856,9	.196,143,1	.042,222,39	.006,770,139	.202,913,3	.353,814,1	.646,185,9
36 45	.641,408,5	.598,324,6	.801,253,8	.198,746,2	.043,083,90	.006,956,245	.205,702,4	.357,992,3	.642,007,7
37 00	.645,771,8	.601,815,0	.798,635,5	.201,364,5	.043,956,80	.007,146,134	.208,510,6	.362,181,3	.637,818,7
37 15	.650,135,1	.605,294,0	.796,002,0	.203,998,0	.044,841,16	.007,339,857	.211,337,9	.366,380,8	.633,619,2
37 30	.654,498,5	.608,761,4	.793,353,3	.206,646,7	.045,737,04	.007,537,464	.214,184,1	.370,590,5	.629,409,5
37 45	.658,861,8	.612,217,3	.790,689,6	.209,310,4	.046,644,51	.007,739,005	.217,049,4	.374,810,0	.625,190,0
38 00	.663,225,1	.615,661,5	.788,010,8	.211,989,2	.047,563,64	.007,944,531	.219,933,8	.379,039,1	.620,960,9
38 15	.667,588,4	.619,093,9	.785,316,9	.214,683,1	.048,494,49	.008,154,093	.222,837,2	.383,277,3	.616,722,7
38 30	.671,951,8	.622,514,8	.782,608,2	.217,391,8	.049,437,13	.008,367,742	.225,759,6	.387,524,5	.612,475,5
38 45	.676,315,1	.625,923,5	.779,884,5	.220,115,5	.050,391,61	.008,585,530	.228,701,0	.391,780,2	.608,219,8
39 00	.680,678,4	.629,320,4	.777,146,0	.222,854,0	.051,358,02	.008,807,509	.231,661,5	.396,044,2	.603,955,8
39 15	.685,041,7	.632,705,3	.774,392,6	.225,607,4	.052,336,40	.009,033,731	.234,641,1	.400,316,0	.599,684,0
39 30	.689,405,1	.636,078,2	.771,624,8	.228,375,4	.053,326,83	.009,264,248	.237,639,7	.404,595,5	.595,404,5
39 45	.693,768,4	.639,439,0	.768,841,8	.231,158,2	.054,329,38	.009,499,113	.240,657,3	.408,882,2	.591,117,8
40 00	.698,131,7	.642,787,6	.766,0444	.233,955,6	.055,344,09	.009,738,379	.243,693,9	.413,175,9	.586,824,1
40 15	.702,495,0	.646,124,0	.763,232,5	.236,767,5	.056,371,04	.009,982,099	.246,749,6	.417,476,2	.582,523,8
40 30	.706,858,3	.649,448,0	.760,406,0	.239,594,0	.057,410,30	.010,230,33	.249,824,4	.421,782,8	.578,217,2
40 45	.711,221,7	.652,759,8	.757,565,0	.242,435,0	.058,461,92	.010,483,12	.252,918,1	.426,095,3	.573,904,7
41 00	.715,585,0	.656,059,0	.754,709,6	.245,290,4	.059,525,96	.010,740,52	.256,030,9	.430,413,4	.569,586,6
41 15	.719,948,3	.659,345,8	.751,839,8	.248,160,2	.060,602,50	.011,002,60	.259,162,8	.434,736,9	.565,263,1
41 30	.724,311,6	.662,620,0	.748,955,7	.251,044,3	.061,691,59	.011,269,40	.262,313,7	.439,065,3	.560,934,7
41 45	.728,675,0	.665,881,7	.746,057,4	.253,942,6	.062,793,30	.011,540,98	.265,483,6	.443,398,4	.556,601,6
42 00	.733,038,3	.669,130,6	.743,144,8	.256,855,2	.063,907,68	.011,817,39	.268,672,6	.447,735,8	.552,264,2
42 15	.737,401,6	.672,366,8	.740,218,1	.259,781,9	.065,034,80	.012,098,69	.271,880,6	.452,077,1	.547,922,9
42 30	.741,764,9	.675,590,2	.737,277,3	.262,722,7	.066,174,72	.012,384,94	.275,107,6	.456,422,1	.543,577,9
42 45	.746,128,3	.678,800,7	.734,322,5	.265,677,5	.067,327,51	.012,676,20	.278,353,7	.460,770,5	.539,229,5
43 00	.750,491,6	.681,998,4	.731,353,7	.268,646,3	.068,493,22	.012,972,51	.281,618,8	.465,121,8	.534,878,2
43 15	.754,854,9	.685,183,0	.728,371,0	.271,629,0	.069,671,91	.013,273,93	.284,903,0	.469,475,7	.530,524,3
43 30	.759,218,2	.688,354,6	.725,374,4	.274,625,6	.070,863,65	.013,580,53	.288,206,2	.473,832,0	.526,168,0
43 45	.763,581,5	.691,513,1	.722,364,0	.277,636,0	.072,068,49	.013,892,35	.291,528,4	.478,190,3	.521,809,7
44 00	.767,944,9	.694,658,4	.719,339,8	.280,660,2	.073,286,50	.014,209,46	.294,869,7	.482,550,3	.517,449,7
44 15	.772,308,2	.697,790,5	.716,301,9	.283,698,1	.074,517,73	.014,531,92	.298,230,0	.486,911,5	.513,088,5
44 30	.776,671,5	.700,909,3	.713,250,4	.286,749,6	.075,762,25	.014,859,77	.301,609,3	.491,273,8	.508,726,2
44 45	.781,034,8	.704,014,7	.710,185,4	.289,814,6	.077,020,12	.015,193,09	.305,007,7	.495,636,7	.504,363,3
45 00	.785,398,2	.707,106,8	.707,106,8	.292,893,2	.078,291,38	.015,531,92	.308,425,1	.500,000,0	.500,000,0

TABLE 22—Continued

ϕ Degrees	ϕ Radians	SIN ϕ	COS ϕ	VERS ϕ	ϕ-SIN ϕ	$\frac{\phi^2}{2}$-VERS ϕ	$\frac{\phi^2}{2}$	SIN^2 ϕ	COS^2 ϕ
45 15	.789,761,5	.710,185,4	.704,014,7	.295,985,3	.079,576,11	.015,876,33	.311,861,6	.504,363,3	.495,636,7
45 30	.794,124,8	.713,250,4	.700,909,3	.299,090,7	.080,874,36	.016,226,37	.315,317,1	.508,726,2	.491,273,8
45 45	.798,488,1	.716,301,9	.697,790,5	.302,209,5	.082,186,19	.016,582,11	.318,791,6	.513,088,5	.486,911,5
46 00	.802,851,5	.719,339,8	.694,658,4	.305,341,6	.083,511,66	.016,943,60	.322,285,2	.517,449,7	.482,550,3
46 15	.807,214,8	.722,364,0	.691,513,1	.308,486,9	.084,850,82	.017,310,91	.325,797,9	.521,809,7	.478,190,3
46 30	.811,578,1	.725,374,4	.688,354,6	.311,645,4	.086,203,73	.017,684,08	.329,329,5	.526,168,0	.473,832,0
46 45	.815,941,4	.728,371,0	.685,183,0	.314,817,0	.087,570,46	.018,063,20	.332,880,2	.530,524,3	.469,475,7
47 00	.820,304,7	.731,353,7	.681,998,4	.318,001,6	.088,951,05	.018,448,30	.336,449,9	.534,878,2	.465,121,8
47 15	.824,668,1	.734,322,5	.678,800,7	.321,199,3	.090,345,56	.018,839,46	.340,038,7	.539,229,5	.460,770,5
47 30	.829,031,4	.737,277,3	.675,590,2	.324,409,8	.091,754,06	.019,236,73	.343,646,5	.543,577,9	.456,422,1
47 45	.833,394,7	.740,218,1	.672,366,8	.327,633,2	.093,176,59	.019,640,19	.347,273,4	.547,922,9	.452,077,1
48 00	.837,758,0	.743,144,8	.669,130,6	.330,869,4	.094,613,22	.020,049,87	.350,919,3	.552,264,2	.447,735,8
48 15	.842,121,4	.746,057,4	.665,881,7	.334,118,3	.096,063,99	.020,465,86	.354,584,2	.556,601,6	.443,398,4
48 30	.846,484,7	.748,955,7	.662,620,0	.337,380,0	.097,528,97	.020,888,22	.358,268,2	.560,934,7	.439,065,3
48 45	.850,848,0	.751,839,8	.659,345,8	.340,654,2	.099,008,20	.021,316,98	.361,971,2	.565,263,1	.434,736,9
49 00	.855,211,3	.754,709,6	.656,059,0	.343,941,0	.100,501,8	.021,752,24	.365,693,2	.569,586,6	.430,413,4
49 15	.859,574,7	.757,565,0	.652,759,8	.347,240,2	.102,009,7	.022,194,05	.369,434,3	.573,904,7	.426,095,3
49 30	.863,938,0	.760,406,0	.649,448,0	.350,552,0	.103,532,0	.022,642,46	.373,194,4	.578,217,2	.421,782,8
49 45	.868,301,3	.763,232,5	.646,124,0	.353,876,0	.105,068,8	.023,097,56	.376,973,6	.582,523,8	.417,476,2
50 00	.872,664,6	.766,044,4	.642,787,6	.357,212,4	.106,620,2	.023,559,38	.380,771,8	.586,824,1	.413,175,9
50 15	.877,027,9	.768,841,8	.639,439,0	.360,561,0	.108,186,1	.024,028,01	.384,589,0	.591,117,8	.408,882,2
50 30	.881,391,3	.771,624,6	.636,078,2	.363,921,8	.109,766,7	.024,503,51	.388,425,3	.595,404,5	.404,595,5
50 45	.885,754,6	.774,392,6	.632,705,3	.367,294,7	.111,362,0	.024,985,93	.392,280,6	.599,684,0	.400,316,0
51 00	.890,117,9	.777,146,0	.629,320,4	.370,679,6	.112,972,0	.025,475,35	.396,155,0	.603,955,8	.396,044,2
51 15	.894,481,2	.779,884,5	.625,923,5	.374,076,5	.114,596,8	.025,971,82	.400,048,3	.608,219,8	.391,780,2
51 30	.898,844,6	.782,608,2	.622,514,6	.377,485,4	.116,236,4	.026,475,41	.403,960,8	.612,475,5	.387,524,5
51 45	.903,207,9	.785,316,9	.619,093,9	.380,906,1	.117,891,0	.026,986,19	.407,892,2	.616,722,7	.383,277,3
52 00	.907,571,2	.788,010,8	.615,661,5	.384,338,5	.119,560,5	.027,504,23	.411,842,8	.620,960,9	.379,039,1
52 15	.911,934,5	.790,689,6	.612,217,3	.387,782,7	.121,245,0	.028,029,58	.415,812,3	.625,190,0	.374,810,0
52 30	.916,297,9	.793,353,3	.608,761,4	.391,238,6	.122,944,5	.028,562,31	.419,800,9	.629,409,5	.370,590,5
52 45	.920,661,2	.796,002,0	.605,294,0	.394,706,0	.124,659,2	.029,102,49	.423,808,5	.633,619,2	.366,380,8
53 00	.925,024,5	.798,635,5	.601,815,0	.398,185,0	.126,389,0	.029,650,19	.427,835,2	.637,818,7	.362,181,3
53 15	.929,387,8	.801,253,8	.598,324,6	.401,675,4	.128,134,0	.030,205,47	.431,880,9	.642,007,7	.357,992,3
53 30	.933,751,1	.803,856,9	.594,822,8	.405,177,2	.129,894,3	.030,768,39	.435,945,6	.646,185,9	.353,814,1
53 45	.938,114,5	.806,444,6	.591,309,6	.408,690,4	.131,669,9	.031,339,03	.440,029,4	.650,352,9	.349,647,1
54 00	.942,477,8	.809,017,0	.587,785,3	.412,214,7	.133,460,8	.031,917,45	.444,132,2	.654,508,5	.345,491,5
54 15	.946,841,1	.811,574,0	.584,249,7	.415,750,3	.135,267,1	.032,503,72	.448,254,1	.658,652,3	.341,347,7
54 30	.951,204,4	.814,115,5	.580,703,0	.419,297,0	.137,088,9	.033,097,90	.452,394,9	.662,784,1	.337,215,9
54 45	.955,567,8	.816,641,6	.577,145,2	.422,854,8	.138,926,2	.033,700,07	.456,554,9	.666,903,4	.333,096,6
55 00	.959,931,1	.819,152,0	.573,576,4	.426,423,6	.140,779,0	.034,310,28	.460,733,8	.671,010,1	.328,989,9
55 15	.964,294,4	.821,646,9	.569,996,8	.430,003,2	.142,647,5	.034,928,62	.464,931,9	.675,103,7	.324,896,3
55 30	.968,657,7	.824,126,2	.566,406,2	.433,593,8	.144,531,5	.035,555,14	.469,148,9	.679,184,0	.320,816,0
55 45	.973,021,1	.826,589,7	.562,804,9	.437,195,1	.146,431,3	.036,189,92	.473,385,0	.683,250,6	.316,749,4
56 00	.977,384,4	.829,037,6	.559,192,9	.440,807,1	.148,346,8	.036,833,02	.477,640,1	.687,303,3	.312,696,7
56 15	.981,747,7	.831,469,6	.555,570,2	.444,429,8	.150,278,1	.037,484,51	.481,914,3	.691,341,7	.308,658,3
56 30	.986,111,0	.833,885,8	.551,937,0	.448,063,0	.152,225,2	.038,144,46	.486,207,5	.695,365,6	.304,634,4
56 45	.990,474,4	.836,286,2	.548,293,2	.451,706,8	.154,188,2	.038,812,95	.490,519,7	.699,374,5	.300,625,5
57 00	.994,837,7	.838,670,6	.544,639,0	.455,361,0	.156,167,1	.039,490,03	.494,851,0	.703,368,3	.296,631,7
57 15	.999,201,0	.841,039,0	.540,974,5	.459,025,5	.158,162,0	.040,175,79	.499,201,3	.707,346,6	.292,653,4
57 30	1.003,564,3	.843,391,4	.537,299,6	.462,700,4	.160,172,9	.040,870,28	.503,570,7	.711,309,1	.288,690,9
57 45	1.007,927,6	.845,727,8	.533,614,5	.466,385,5	.162,199,8	.041,573,58	.507,959,1	.715,255,5	.284,744,5
58 00	1.012,291,0	.848,048,1	.529,919,3	.470,080,7	.164,242,9	.042,285,76	.512,366,5	.719,185,6	.280,814,4
58 15	1.016,654,3	.850,352,2	.526,213,9	.473,786,1	.166,302,1	.043,006,90	.516,793,0	.723,098,9	.276,901,1
58 30	1.021,017,6	.852,640,2	.522,498,6	.477,501,4	.168,377,4	.043,737,05	.521,238,5	.726,995,2	.273,004,8
58 45	1.025,380,9	.854,911,9	.518,773,3	.481,226,7	.170,469,1	.044,476,29	.525,703,0	.730,874,3	.269,125,7
59 00	1.029,744,3	.857,167,3	.515,038,1	.484,961,9	.172,577,0	.045,224,69	.530,186,6	.734,735,8	.265,264,2
59 15	1.034,107,6	.859,406,4	.511,293,1	.488,706,9	.174,701,2	.045,982,33	.534,689,2	.738,579,4	.261,420,6
59 30	1.038,470,9	.861,629,2	.507,538,4	.492,461,6	.176,841,7	.046,749,27	.539,210,9	.742,404,8	.257,595,2
59 45	1.042,834,2	.863,835,5	.503,774,0	.496,226,0	.178,998,7	.047,525,59	.543,751,6	.746,211,8	.253,788,2
60 00	1.047,197,6	.866,025,4	.500,000,0	.500,000,0	.181,172,1	.048,311,36	.548,311,4	.750,000,0	.250,000,0
60 15	1.051,560,9	.868,198,8	.496,216,5	.503,783,5	.183,362,1	.049,106,64	.552,890,1	.753,769,2	.246,230,8
60 30	1.055,924,2	.870,355,7	.492,423,6	.507,576,4	.185,568,5	.049,911,52	.557,488,0	.757,519,0	.242,481,0
60 45	1.060,287,5	.872,496,0	.488,621,2	.511,378,8	.187,791,5	.050,726,05	.562,104,8	.761,249,3	.238,750,7
61 00	1.064,650,8	.874,619,7	.484,809,6	.515,190,4	.190,031,1	.051,550,33	.566,740,7	.764,959,6	.235,040,4
61 15	1.069,014,2	.876,726,8	.480,988,8	.519,011,2	.192,287,4	.052,384,41	.571,395,6	.768,649,8	.231,350,2
61 30	1.073,377,5	.878,817,1	.477,158,8	.522,841,2	.194,560,4	.053,228,38	.576,069,6	.772,319,5	.227,680,5
61 45	1.077,740,8	.880,890,7	.473,319,7	.526,680,3	.196,850,1	.054,082,30	.580,762,6	.775,968,5	.224,031,5
62 00	1.082,104,1	.882,947,6	.469,471,6	.530,528,4	.199,156,5	.054,946,24	.585,474,7	.779,596,5	.220,403,5
62 15	1.086,467,5	.884,987,6	.465,614,5	.534,385,5	.201,479,8	.055,820,29	.590,205,8	.783,203,1	.216,796,9
62 30	1.090,830,8	.887,010,8	.461,748,6	.538,251,4	.203,819,9	.056,704,51	.594,955,9	.786,788,2	.213,211,8
62 45	1.095,194,1	.889,017,1	.457,873,9	.542,126,1	.206,177,0	.057,598,98	.599,725,1	.790,351,5	.209,648,5
63 00	1.099,557,4	.891,006,5	.453,990,5	.546,009,5	.208,550,9	.058,503,77	.604,513,3	.793,892,6	.206,107,4
63 15	1.103,920,8	.892,978,9	.450,098,4	.549,901,6	.210,941,8	.059,418,95	.609,320,5	.797,411,4	.202,588,6
63 30	1.108,284,1	.894,934,4	.446,197,8	.553,802,2	.213,349,7	.060,344,61	.614,146,8	.800,907,5	.199,092,5
63 45	1.112,647,4	.896,872,7	.442,288,7	.557,711,3	.215,774,7	.061,280,81	.618,992,1	.804,380,7	.195,619,3
64 00	1.117,010,7	.898,794,0	.438,371,1	.561,628,9	.218,216,7	.062,227,62	.623,856,5	.807,830,7	.192,169,3
64 15	1.121,374,0	.900,698,2	.434,445,3	.565,554,7	.220,675,8	.063,185,13	.628,739,9	.811,257,3	.188,742,7
64 30	1.125,737,4	.902,585,3	.430,511,1	.569,488,9	.223,152,1	.064,153,41	.633,642,3	.814,660,2	.185,339,8
64 45	1.130,100,7	.904,455,1	.426,568,7	.573,431,3	.225,645,5	.065,132,53	.638,563,8	.818,039,1	.181,960,9
65 00	1.134,464,0	.906,307,8	.422,618,3	.577,381,7	.228,156,2	.066,122,56	.643,504,3	.821,393,8	.178,606,2
65 15	1.138,827,3	.908,143,2	.418,659,7	.581,340,3	.230,684,2	.067,123,59	.648,463,9	.824,724,0	.175,276,0
65 30	1.143,190,7	.909,961,3	.414,693,2	.585,306,8	.233,229,4	.068,135,69	.653,442,4	.828,029,5	.171,970,5
65 45	1.147,554,0	.911,762,0	.410,718,9	.589,281,1	.235,791,9	.069,158,92	.658,440,1	.831,310,0	.168,690,0
66 00	1.151,917,3	.913,545,5	.406,736,6	.593,263,4	.238,371,8	.070,193,38	.663,456,7	.834,565,3	.165,434,7
66 15	1.156,280,6	.915,311,5	.402,746,7	.597,253,3	.240,969,2	.071,239,14	.668,492,4	.837,795,1	.162,204,9
66 30	1.160,644,0	.917,060,1	.398,749,1	.601,250,9	.243,583,9	.072,296,26	.673,547,2	.840,999,2	.159,000,8
66 45	1.165,007,3	.918,791,2	.394,743,9	.605,256,1	.246,216,1	.073,364,83	.678,621,0	.844,177,3	.155,822,7
67 00	1.169,370,6	.920,504,9	.390,731,1	.609,268,9	.248,865,7	.074,444,93	.683,713,8	.847,329,2	.152,670,8
67 15	1.173,733,9	.922,201,0	.386,711,0	.613,289,0	.251,533,0	.075,536,62	.688,825,7	.850,454,6	.149,545,4
67 30	1.178,097,2	.923,879,5	.382,683,4	.617,316,6	.254,217,7	.076,639,99	.693,956,6	.853,553,4	.146,446,6

TABLE 22—Continued

ϕ Degrees	ϕ Radians	SIN ϕ	COS ϕ	VERS ϕ	ϕ-SIN ϕ	$\frac{\phi^2}{2}$-VERS ϕ	$\frac{\phi^2}{2}$	SIN^2 ϕ	COS^2 ϕ
67 45	1.182,460,6	.925,540,5	.378,648,6	.621,351,4	.256,920,1	.077,755,12	.699,106,5	.856,625,2	.143,374,8
68 00	1.186,823,9	.927,183,9	.374,606,6	.625,393,4	.259,640,0	.078,692,07	.704,275,5	.859,669,9	.140,330,1
68 15	1.191,187,2	.928,809,6	.370,557,4	.629,442,6	.262,377,7	.080,020,93	.709,463,5	.862,687,2	.137,312,8
68 30	1.195,550,5	.930,417,6	.366,501,2	.633,498,8	.265,133,0	.081,171,77	.714,670,5	.865,676,9	.134,323,1
68 45	1.199,913,9	.932,007,9	.362,438,0	.637,562,0	.267,906,0	.082,334,67	.719,896,8	.868,638,7	.131,361,3
69 00	1.204,277,2	.933,580,4	.358,367,9	.641,632,1	.270,696,8	.083,509,72	.725,141,8	.871,572,4	.128,427,6
69 15	1.208,640,5	.935,135,2	.354,291,0	.645,709,0	.273,505,3	.084,696,98	.730,405,9	.874,477,9	.125,522,1
69 30	1.213,003,8	.936,672,2	.350,207,4	.649,792,6	.276,331,6	.085,896,53	.735,689,1	.877,354,8	.122,645,2
69 45	1.217,367,2	.938,191,3	.346,117,1	.653,882,9	.279,175,8	.087,108,45	.740,991,4	.880,203,0	.119,797,0
70 00	1.221,730,5	.939,692,6	.342,020,1	.657,979,9	.282,037,9	.088,332,82	.746,312,7	.883,022,2	.116,977,8
70 15	1.226,093,8	.941,176,0	.337,916,7	.662,083,3	.284,917,8	.089,569,72	.751,653,0	.885,812,3	.114,187,7
70 30	1.230,457,1	.942,641,5	.333,806,9	.666,193,1	.287,815,6	.090,819,22	.757,012,4	.888,573,0	.111,427,0
70 45	1.234,820,4	.944,089,0	.329,690,6	.670,309,4	.290,731,4	.092,081,41	.762,390,8	.891,304,1	.108,695,9
71 00	1.239,183,8	.945,518,6	.325,568,2	.674,431,8	.293,665,2	.093,356,36	.767,788,2	.894,005,4	.105,994,6
71 15	1.243,547,1	.946,930,1	.321,439,5	.678,560,5	.296,817,0	.094,644,15	.773,204,7	.896,676,7	.103,323,3
71 30	1.247,910,4	.948,323,7	.317,304,7	.682,695,3	.299,586,8	.095,944,86	.778,640,2	.899,317,8	.100,682,2
71 45	1.252,273,7	.949,699,1	.313,163,8	.686,836,2	.302,574,6	.097,258,56	.784,094,8	.901,928,4	.098,071,57
72 00	1.256,637,1	.951,056,5	.309,017,0	.690,983,0	.305,580,5	.098,585,35	.789,568,4	.904,508,5	.095,491,50
72 15	1.261,000,4	.952,395,8	.304,864,3	.695,135,7	.308,604,6	.099,925,28	.795,061,0	.907,057,8	.092,942,24
72 30	1.265,363,7	.953,717,0	.300,705,8	.699,294,2	.311,646,8	.101,278,5	.800,572,7	.909,576,0	.090,423,98
72 45	1.269,727,0	.955,019,9	.296,541,6	.703,458,4	.314,707,1	.102,644,9	.806,103,4	.912,063,1	.087,936,91
73 00	1.274,090,4	.956,304,8	.292,371,7	.707,628,3	.317,785,6	.104,024,8	.811,653,1	.914,518,8	.085,481,21
73 15	1.278,453,7	.957,571,4	.288,196,3	.711,803,7	.320,882,3	.105,418,2	.817,221,9	.916,942,9	.083,057,09
73 30	1.282,817,0	.958,819,7	.284,015,3	.715,984,7	.323,997,3	.106,825,1	.822,809,7	.919,335,3	.080,664,72
73 45	1.287,180,3	.960,049,9	.279,829,0	.720,171,0	.327,130,5	.108,245,6	.828,416,6	.921,695,7	.078,304,28
74 00	1.291,543,6	.961,261,6	.275,637,4	.724,362,6	.330,282,0	.109,679,9	.834,042,5	.924,024,1	.075,975,95
74 15	1.295,907,0	.962,455,2	.271,440,4	.728,559,6	.333,451,7	.111,127,9	.839,687,4	.926,320,1	.073,679,92
74 30	1.300,270,3	.963,630,5	.267,238,4	.732,761,6	.336,639,8	.112,589,8	.845,351,4	.928,583,7	.071,416,35
74 45	1.304,633,6	.964,787,3	.263,031,2	.736,968,8	.339,846,3	.114,065,7	.851,034,4	.930,814,6	.069,185,42
75 00	1.308,996,9	.965,925,8	.258,819,0	.741,181,0	.343,071,1	.115,555,5	.856,736,5	.933,012,7	.066,987,30
75 15	1.313,360,3	.967,045,9	.254,601,9	.745,398,1	.346,314,3	.117,059,5	.862,457,6	.935,177,8	.064,822,15
75 30	1.317,723,6	.968,147,6	.250,380,0	.749,620,0	.349,575,9	.118,577,7	.868,197,7	.937,309,9	.062,690,15
75 45	1.322,086,9	.969,230,9	.246,153,3	.753,846,7	.352,856,0	.120,110,2	.873,956,9	.939,408,6	.060,591,44
76 00	1.326,450,2	.970,295,7	.241,921,9	.758,078,1	.356,154,5	.121,657,0	.879,735,1	.941,473,8	.058,526,20
76 15	1.330,813,6	.971,342,1	.237,685,9	.762,314,1	.359,471,5	.123,218,3	.885,532,4	.943,505,4	.056,494,58
76 30	1.335,176,9	.972,369,9	.233,445,4	.766,554,6	.362,807,0	.124,794,0	.891,348,6	.945,503,3	.054,496,74
76 45	1.339,540,2	.973,379,3	.229,200,4	.770,799,6	.366,160,9	.126,384,4	.897,184,0	.947,467,2	.052,532,82
77 00	1.343,903,5	.974,370,1	.224,951,1	.775,048,9	.369,533,5	.127,989,4	.903,038,3	.949,397,0	.050,602,98
77 15	1.348,266,8	.975,342,3	.220,697,4	.779,302,6	.372,924,5	.129,609,2	.908,911,7	.951,292,6	.048,707,36
77 30	1.352,630,2	.976,296,0	.216,439,6	.783,560,4	.376,334,2	.131,243,8	.914,804,2	.953,153,9	.046,846,11
77 45	1.356,993,5	.977,231,1	.212,177,7	.787,822,3	.379,762,4	.132,893,3	.920,715,7	.954,980,6	.045,019,36
78 00	1.361,356,8	.978,147,6	.207,911,7	.792,088,3	.383,209,2	.134,557,9	.926,646,2	.956,772,7	.043,227,27
78 15	1.365,720,1	.979,045,5	.203,641,8	.796,358,2	.386,674,7	.136,237,5	.932,595,7	.958,530,0	.041,469,96
78 30	1.370,083,5	.979,924,7	.199,367,9	.800,632,1	.390,158,8	.137,932,3	.938,564,3	.960,252,4	.039,747,57
78 45	1.374,446,8	.980,785,3	.195,090,3	.804,909,7	.393,661,5	.139,642,3	.944,552,0	.961,939,8	.038,060,23
79 00	1.378,810,1	.981,627,2	.190,809,0	.809,191,0	.397,182,9	.141,367,7	.950,558,7	.963,591,9	.036,408,07
79 15	1.383,173,4	.982,450,4	.186,524,0	.813,476,0	.400,723,0	.143,108,4	.956,584,4	.965,208,8	.034,791,22
79 30	1.387,536,8	.983,254,9	.182,235,5	.817,764,5	.404,281,8	.144,864,6	.962,629,1	.966,790,2	.033,209,79
79 45	1.391,900,1	.984,040,7	.177,943,5	.822,056,5	.407,859,4	.146,636,5	.968,692,9	.968,336,1	.031,663,91
80 00	1.396,263,4	.984,807,8	.173,648,2	.826,351,8	.411,455,6	.148,423,9	.974,775,7	.969,846,3	.030,153,69
80 15	1.400,626,7	.985,556,1	.169,349,5	.830,650,5	.415,070,7	.150,227,1	.980,877,6	.971,320,7	.028,679,25
80 30	1.404,990,0	.986,285,6	.165,047,6	.834,952,4	.418,704,4	.152,046,1	.986,998,5	.972,759,3	.027,240,71
80 45	1.409,353,4	.986,996,4	.160,742,6	.839,257,4	.422,357,0	.153,881,0	.993,138,5	.974,161,8	.025,838,17
81 00	1.413,716,7	.987,688,3	.156,434,5	.843,565,5	.426,028,4	.155,731,9	.999,297,4	.975,528,3	.024,471,74
81 15	1.418,080,0	.988,361,5	.152,123,4	.847,876,6	.429,718,5	.157,598,9	1.005,475,5	.976,858,5	.023,141,52
81 30	1.422,443,3	.989,015,9	.147,809,4	.852,190,6	.433,427,5	.159,481,9	1.011,672,5	.978,152,4	.021,847,62
81 45	1.426,806,7	.989,651,4	.143,492,6	.856,507,4	.437,155,3	.161,381,2	1.017,888,6	.979,409,9	.020,590,13
82 00	1.431,170,0	.990,268,1	.139,173,1	.860,826,9	.440,901,9	.163,296,9	1.024,123,8	.980,630,9	.019,369,15
82 15	1.435,533,3	.990,865,9	.134,850,9	.865,149,1	.444,667,4	.165,228,9	1.030,377,9	.981,815,2	.018,184,77
82 30	1.439,896,6	.991,444,9	.130,526,2	.869,473,8	.448,451,8	.167,177,3	1.036,651,2	.982,962,9	.017,037,09
82 45	1.444,260,0	.992,004,9	.126,199,0	.873,801,0	.452,255,0	.169,142,4	1.042,943,4	.984,073,8	.015,926,18
83 00	1.448,623,3	.992,546,2	.121,869,3	.878,130,7	.456,077,1	.171,124,0	1.049,254,7	.985,147,9	.014,852,14
83 15	1.452,986,6	.993,068,5	.117,537,4	.882,462,6	.459,918,1	.173,122,4	1.055,585,0	.986,185,0	.013,815,04
83 30	1.457,349,9	.993,571,9	.113,203,2	.886,796,8	.463,778,1	.175,137,6	1.061,934,4	.987,185,0	.012,814,97
83 45	1.461,713,2	.994,056,3	.108,866,9	.891,133,1	.467,656,9	.177,169,7	1.068,302,8	.988,148,0	.011,852,00
84 00	1.466,076,6	.994,521,9	.104,528,5	.895,471,5	.471,554,7	.179,218,7	1.074,690,3	.989,073,8	.010,926,20
84 15	1.470,439,9	.994,968,5	.100,188,1	.899,811,9	.475,471,4	.181,284,8	1.081,096,7	.989,962,4	.010,037,65
84 30	1.474,803,2	.995,396,2	.095,845,75	.904,154,2	.479,407,0	.183,368,0	1.087,522,3	.990,813,6	.009,186,408
84 45	1.479,166,5	.995,804,9	.091,501,62	.908,498,4	.483,361,6	.185,468,4	1.093,966,8	.991,627,5	.008,372,546
85 00	1.483,529,9	.996,194,7	.087,155,74	.912,844,3	.487,335,2	.187,586,2	1.100,430,4	.992,403,9	.007,596,124
85 15	1.487,893,2	.996,565,5	.082,808,21	.917,191,8	.491,327,7	.189,721,3	1.106,913,1	.993,142,8	.006,857,199
85 30	1.492,256,5	.996,917,3	.078,459,10	.921,540,9	.495,339,2	.191,873,8	1.113,414,7	.993,844,2	.006,155,830
85 45	1.496,619,8	.997,250,2	.074,108,49	.925,891,5	.499,369,6	.194,044,0	1.119,935,5	.994,507,9	.005,492,068
86 00	1.500,983,2	.997,564,1	.069,756,47	.930,243,5	.503,419,1	.196,231,7	1.126,475,2	.995,134,0	.004,865,966
86 15	1.505,346,5	.997,858,9	.065,403,13	.934,596,9	.507,487,6	.198,437,1	1.133,034,0	.995,722,4	.004,277,569
86 30	1.509,709,8	.998,134,8	.061,048,54	.938,951,5	.511,575,0	.200,660,4	1.139,611,8	.996,273,1	.003,726,924
86 45	1.514,073,1	.998,391,7	.056,692,79	.943,307,2	.515,681,5	.202,901,5	1.146,208,7	.996,785,9	.003,214,072
87 00	1.518,436,4	.998,629,4	.052,335,96	.947,664,0	.519,806,9	.205,160,6	1.152,824,6	.997,260,9	.002,739,052
87 15	1.522,799,8	.998,848,4	.047,978,13	.952,021,9	.523,951,4	.207,437,7	1.159,459,6	.997,698,1	.002,301,901
87 30	1.527,163,1	.999,048,2	.043,619,39	.956,380,6	.528,114,9	.209,733,0	1.166,113,6	.998,097,3	.001,902,651
87 45	1.531,526,4	.999,229,0	.039,259,82	.960,740,2	.532,297,4	.212,046,4	1.172,786,6	.998,458,7	.001,541,333
88 00	1.535,889,7	.999,390,8	.034,899,50	.965,100,5	.536,498,9	.214,378,1	1.179,478,8	.998,782,0	.001,217,975
88 15	1.540,253,1	.999,533,6	.030,538,51	.969,461,5	.540,719,5	.216,728,3	1.186,189,8	.999,067,4	.000,932,600,8
88 30	1.544,616,4	.999,657,3	.026,176,95	.973,823,1	.544,959,1	.219,096,8	1.192,919,9	.999,314,8	.000,685,232,6
88 45	1.548,979,7	.999,762,0	.021,814,89	.978,185,1	.549,217,7	.221,484,0	1.199,669,1	.999,524,1	.000,475,889,2
89 00	1.553,343,0	.999,847,7	.017,452,41	.982,547,6	.553,495,3	.223,889,7	1.206,437,3	.999,695,4	.000,304,586,5
89 15	1.557,706,4	.999,914,3	.013,089,60	.986,910,4	.557,792,0	.226,314,1	1.213,224,5	.999,828,7	.000,171,337,5
89 30	1.562,069,7	.999,961,9	.008,726,535	.991,273,5	.562,107,8	.228,757,4	1.220,030,8	.999,923,8	.0476,152,42
89 45	1.566,433,0	.999,990,5	.004,363,309	.995,636,7	.566,442,5	.231,219,5	1.226,856,2	.999,981,0	.0419,038,47
90 00	1.570,796,3	1.000,000,0	.000,000,000	1.000,000,0	.570,796,3	.233,700,6	1.233,700,6	1.000,000,0	0

LIST OF BULLETINS

The following list shows tentative titles of final reports on the Boulder Canyon Project now being prepared for publication. Titles and prices of printed reports now ready for distribution are also included. Appropriate announcements will be made in engineering periodicals as additional bulletins become available.

PART I—INTRODUCTORY

General Description of Project
History of Project
Legal and Financial Problems

PART II—HYDROLOGY

Stream Flow and Reservoir Operation
Lower Basin Utilization
Upper Basin Utilization

PART III—PREPARATORY EXAMINATIONS AND CONSTRUCTION

Geologic Investigations
Surveys and Preliminary Construction
(Topographic surveys, highways, railways, power lines, substations, and Boulder City).

PART IV—DESIGN AND CONSTRUCTION

General Features
Concrete Manufacture, Handling and Control
Boulder Dam
Diversion Structures and Spillways
Intake Towers and Outlet Works
Penstocks and Outlet Pipes
Hydraulic Valves and Gates
Handling Facilities
(Cableway, cranes, and other permanent handling facilities).
Power Plant Building
Generating, Transforming, and Switching Equipment
Turbines, Governors, and Mechanical Auxiliaries
Control, Communication, and Electrical Auxiliaries
Imperial Dam and Desilting Works
All-American Canal and Canal Structures

PART V—TECHNICAL INVESTIGATIONS

Trial Load Method of Analyzing Arch Dams*	(paper cover, $1.50) (cloth cover, $2.00)
Slab Analogy Experiments*	(paper cover, $1.00) (cloth cover, $1.50)

Model Tests of Boulder Dam
Stress Studies for Boulder Dam
Penstock Analysis and Stiffener Design
Model Tests of Arch and Cantilever Elements
Research Measurements at Dam

LIST OF BULLETINS—(Continued)

PART VI—HYDRAULIC INVESTIGATIONS

Model Studies of Spillways* (paper cover, $1.00) (cloth cover, $1.50)

Model Studies of Penstocks and Outlet Works* (paper cover, $1.00) (cloth cover, $1.50)

Studies of Crests for Overfall Dams
Model Studies of Imperial Dam and Desilting Works
Model Studies of All-American Canal Structures
Silt Movement in Colorado River

PART VII—CEMENT AND CONCRETE INVESTIGATIONS

Thermal Properties of Concrete
Investigation of Portland Cements
Cooling of Concrete Dams
Mass Concrete Investigations
Contraction Joint Grouting
Volume Changes in Mass Concrete
(Largely Investigations at Owyhee Dam, Owyhee Project, Oregon).

*For Sale at Offices of the U. S. Bureau of Reclamation in Denver, Colo., and Washington, D. C.

The W. H. Kistler Stationery Company, Denver, Colo. 7-21-38 3000